

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the October/November 2014 series

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

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	Cambridge International AS/A Level – October/November 2014	9709	11

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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SOS	See Other Solution (the candidate makes a better attempt at the same question)
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1	${}^7C_1 \times 2^6 \times a (=) {}^7C_2 \times 2^5 \times a^2$ soi $a = \left(\frac{7 \times 2^6}{21 \times 2^5} \right) = \frac{2}{3}$ oe	B2, 1, 0 B1 [3]	Treat the same error in each expression as a single error
2	$\tan^{-1}(3) = 1.249$ or 71.565° $\sin 1.25$ or $\sin 71.6$ or 0.949 soi $(x =) 1.95$ cao, accept $1 + \frac{3}{\sqrt{10}}$ oe	M1 M1 A1 [3]	Attempt at $\tan^{-1}3$ or right angle triangle with attempt at hypotenuse = $\sqrt{10}$ Attempt at $\sin \tan^{-1}3$ Answer only B3
3	$13\sin^2 \theta + 2\cos \theta + \cos^2 \theta = 4 + 2\cos \theta$ $13\sin^2 \theta + 1 - \sin^2 \theta = 4 \rightarrow \sin^2 \theta = \frac{1}{4}$ or $13 - 13\cos^2 \theta + \cos^2 \theta = 4 \rightarrow \cos^2 \theta = \frac{3}{4}$ $30^\circ, 150^\circ$	M1 M1 A1A1 [4]	Attempt to multiply by $2 + \cos \theta$ Use of $s^2 + c^2$ appropriately SC both answers correct in radians, A1 only Ft on 180 – their first value of θ
4	(i) $32 - 4k = 20 \Rightarrow k = 3$ $4b + 3 \times 2b = 20$ $b = 2$ (ii) Mid-point = $(5, 0)$	M1A1 M1 A1 [4] B1 [1]	Sub $(8, -4)$ [alt: $(2b + 4)/(b - 8) = -4/k$ Sub $(b, 2b)$, $4b + 2bk = 20$ M1 both M1 solving A1 , A1] Ft on <i>their b</i>
5	$x^2 + x(k - 2) + (k - 2)(= 0)$ $(k - 2)^2 - 4(k - 2)(> 0)$ soi $(k - 2)(k - 6)(> 0)$ $k < 2$ or $k > 6$ (condone $\leq, \geq$) Allow $\{-\infty, 2\} \cup \{6, \infty\}$ etc.	M1 M1 DM1 A2 [5]	Equate and move terms to one side of equ. Apply $b^2 - 4ac (> 0)$. Allow $\geq$ at this stage. Attempt to factorise or solve or find 2 solns. SCA1 for 2, 6 seen with wrong inequalities
6	(i) $\overrightarrow{AB}$ or $\overrightarrow{BA} = \pm[(7\mathbf{i} - 3\mathbf{j} + \mathbf{k}) - (3\mathbf{i} + 2\mathbf{j} - \mathbf{k})] = \pm(4\mathbf{i} - 5\mathbf{j} + 2\mathbf{k})$ $(\overrightarrow{AO} \cdot \overrightarrow{AB}) = \pm(12 - 10 - 2)$ [allow as column if total given] $= 0$ hence $OAB = 90^\circ$ (ii) $ \overrightarrow{OA} = \sqrt{9 + 4 + 1} = \sqrt{14}$, $ \overrightarrow{AB} = \sqrt{16 + 25 + 4} = \sqrt{45}$ Area $\Delta = \frac{1}{2} \sqrt{14}(\sqrt{45}) = 12.5$	M1A1 DM1 A1 [4] B1 M1A1 [3]	May be seen in part (ii) OR $AB^2 = 45, AO^2 = 14, OB^2 = 59$ Hence $AB^2 + AO^2 = OB^2$ Hence $OAB = 90^\circ$ At least one magnitude correct in (i) or (ii) Accept $12.6, \frac{3\sqrt{70}}{2}$ oe

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7	(i)	$S = \frac{a}{1-r}, \quad 3S = \frac{a}{1-2r}$ $1-r = 3-6r$ $r = \frac{2}{5}$	B1 M1 A1 [3]	At least $3S = \frac{a}{1-2r}$ Eliminate S
	(ii)	$7 + (n-1)d = 84$ and/or $7 + (3n-1)d = 245$ $[(n-1)d = 77, (3n-1)d = 238, 2nd = 161]$ $\frac{n-1}{3n-1} = \frac{77}{238}$ (must be from the correct u_n formula) $n = 23$ ($d = \frac{77}{22} = 3.5$)	B1 B1 M1 A1 [4]	At least one of these equations seen Two different seen – unsimplified ok Or other attempt to elim d . E.g. sub $d = \frac{161}{2n}$ (if n is eliminated d must be found)
8	(i)	Arc $AB = 4\alpha$ Arc $DC = (4 \cos \alpha)\alpha$ AC (or DB) $= 4 - 4 \cos \alpha$ Perimeter $= 4\alpha \cos \alpha + 4\alpha + 8 - 8 \cos \alpha$	B1 B1 B1 B1 [4]	
	(ii)	$OD = 4 \cos \frac{\pi}{6} (= 2\sqrt{3})$ Shaded area $= \left[\frac{1}{2} \times 4^2 \times \frac{\pi}{6} \right] - \left[\frac{1}{2} (2\sqrt{3})^2 \times \frac{\pi}{6} \right]$ $\frac{\pi}{3}$	B1 B1B1 B1 [4]	Or $k = \frac{1}{3}$
9	(i)	$f'(2) = 4 - \frac{1}{2} = \frac{7}{2} \rightarrow$ gradient of normal $= -\frac{2}{7}$ $y - 6 = -\frac{2}{7}(x - 2)$ AEF	B1M1 A1 [3]	Ft from their $f'(2)$
	(ii)	$f(x) = x^2 + \frac{2}{x} (+c)$ $6 = 4 + 1 + c \Rightarrow c = 1$	B1B1 M1A1 [4]	Sub (2, 6) – dependent on c being present
	(iii)	$2x - \frac{2}{x^2} = 0 \Rightarrow 2x^3 - 2 = 0$ $x = 1$ $f''(x) = 2 + \frac{4}{x^3}$ or any valid method $f''(1) = 6$ OR > 0 hence minimum	M1 A1 M1 A1 [4]	Put $f'(x) = 0$ and attempt to solve Not necessary for last A mark as $x > 0$ given Dependent on everything correct

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10	(i)	$(x-1)^2 - 16$	B1B1 [2]	
	(ii)	-16	B1 ^{ft} [1]	Ft from (i)
	(iii)	$9 \leq (x-1)^2 - 16 \leq 65$ OR $x^2 - 2x - 15 = 9 \rightarrow 6, -4$ $25 \leq (x-1)^2 \leq 81$ $x^2 - 2x - 15 = 65 \rightarrow 10, -8$ $5 \leq x-1 \leq 9$ $p = 6$ $6 \leq x \leq 10$ $q = 10$	M1 M1 A1 A1 [4]	OR $x^2 - 2x - 24 \geq 0$, $x^2 - 2x - 80 \leq 0$, $(x-6)(x+4) \geq 0$ $(x-10)(x+8) \leq 0$ $x \geq 6$ $x \leq 10$ SC B2, B2 for trial/improvement
	(iv)	$x = (y-1)^2 - 16$ [interchange x/y] $y-1 = (\pm)\sqrt{x+16}$ $f^{-1}(x) = 1 + \sqrt{x+16}$	M1 M1 A1 [3]	OR $(x-1)^2 = y+16$ $x = 1 + (\pm)\sqrt{y+16}$ $f^{-1}(x) = 1 + \sqrt{x+16}$
11	(i)	For $y = (4x+1)^{\frac{1}{2}}$, $\frac{dy}{dx} = \left[\frac{1}{2}(4x+1)^{-\frac{1}{2}} \right] \times [4]$ When $x = 2$, gradient $m_1 = \frac{2}{3}$ For $y = \frac{1}{2}x^2 + 1$, $\frac{dy}{dx} = x \rightarrow$ gradient $m_2 = 2$ $\alpha = \tan^{-1} m_2 - \tan^{-1} m_1$ $\alpha = 63.43 - 33.69 = 29.7$ cao	B1B1 B1 ^{ft} B1 M1 A1 [6]	Ft from <i>their</i> derivative above
	(ii)	$\int (4x+1)^{\frac{1}{2}} dx = \left[\frac{(4x+1)^{\frac{3}{2}}}{2/3} \right] \div [4]$ $\int \left(\frac{1}{2}x^2 + 1 \right) dx = \frac{1}{6}x^3 + x$ $\int_0^2 (4x+1)^{\frac{1}{2}} dx = \frac{1}{6}[27-1]$, $\int_0^2 \left(\frac{1}{2}x^2 + 1 \right) dx = \left[\frac{8}{6} + 2 \right]$ $\frac{13}{3} - \frac{10}{3}$ 1	B1B1 B1 M1 M1 A1 [6]	Apply limits $0 \rightarrow 2$ to at least the 1 st integral Subtract the integrals (at some stage)

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	GCE AS/A LEVEL – May/June 2012	9709	12

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1 $\text{Vol} = (\pi) \int x^2 dy = (\pi) \int (y-1) dy$ Integral is $\frac{1}{2}y^2 - y$ or $\frac{(y-1)^2}{2}$ Limits for y are 1 to 5 $\rightarrow 8\pi$ or 25.1(AWRT)	M1 A1 B1 A1 [4]	Use of $\int x^2$ – not $\int y^2$ – ignore π co Sight of an integral sign with 1 and 5 co (no π max 3/4)
2 (i) $\tan \theta = \frac{5}{12}$ $\rightarrow (\theta = 0.3948)$ (ii) Other angle in triangle $= \frac{1}{2}\pi - 0.3948$ Area of triangle $AOB = \frac{1}{2} \times 12 \times 5 (= 30)$ Use of $\frac{1}{2}r^2\theta$ once Shaded area = sector + sector – triangle $= \frac{1}{2} \times 12^2 \times 0.3948 + \frac{1}{2}5^2\theta - 30$ $= 28.43 + 14.70 - 30 = 13.1$	M1 [1] B1 B1 M1 DM1 A1 [5]	Any valid trig method ag Unsimplified OK co With θ in radians and $r = 5$ or 12 Sum of 2 sectors – triangle or any other valid method using the given angle and a different one. co
3 (i) $(1+x)^5 = 1 + 5x + 10x^2$ (ii) $(1+px+x^2)^5$ $(1+) 5(px+x^2) + 10(px+x^2)^2$ Coeff of $x^2 = 5 + 10p^2$ $= 95 \rightarrow p = 3$	B2,1 [2] M1 DM1 A1 [3]	Loses 1 for each error Replace x by $(px+x^2)$ in their expansion Considers 2 terms co – no penalty for ± 3
4 $y = \frac{12}{3-2x}$ (i) Differential $= -12(3-2x)^{-2} \times -2$ (ii) $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt} = 0.4 \div 0.15$ $\rightarrow \frac{24}{(3-2x)^2} = \frac{8}{3}$ $\rightarrow x = 0$ or 3	B1 B1 [2] M1 M1 A1 A1 [4]	co co (even if 1st B mark lost) Chain rule used correctly (AEF) Equates their $\frac{dy}{dx}$ with their $\frac{8}{3}$ or $\frac{3}{8}$ co co

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5 $1 + \sin x \tan x = 5 \cos x$ (i) Replaces t by s/c $1 + \frac{s^2}{c} = 5c$ Replace s^2 by $1 - c^2$ $\rightarrow 6c^2 - c - 1 (= 0)$ (ii) Soln of quadratic $\rightarrow (c = -\frac{1}{3} \text{ or } \frac{1}{2})$ $\rightarrow x = 60^\circ \text{ or } 109.5^\circ$	M1 M1 A1 [3] M1 A1 A1 [3]	Correct formula Correct formula used in appropriate place AG Correct method co co
6 $y = x^3 + ax^2 + bx$ (i) $\frac{dy}{dx} = 3x^2 + 2ax + b$ (ii) $b^2 - 4ac = 4a^2 - 12b (< 0)$ $\rightarrow a^2 < 3b$ (iii) $y = x^3 - 6x^2 + 9x$ $\frac{dy}{dx} = 3x^2 - 12x + 9 < 0$ $= 0$ when $x = 1$ and 3 $\rightarrow 1 < x < 3$	B1 M1 A1 [3] M1 A1 A1 [3]	co Use of discriminant on their quadratic $\frac{dy}{dx}$ or other valid method co – answer given Attempt at differentiation co condone $\leq$
7 (i) $\mathbf{AM} = -6\mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$ $\mathbf{AC} = -8\mathbf{i} + 8\mathbf{j}$ (ii) $\mathbf{AM} \cdot \mathbf{AC} = 48 + 16 = 64$ $64 = \sqrt{128}\sqrt{65}\cos\theta$ $\rightarrow \theta = 45.4^\circ$	B2,1 B1 [3] M1 M1 M1 A1 [4]	co -1 each error co Use of $x_1y_1 + \text{etc.}$ with suitable vectors Product of moduli. Correct link. co

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8 (a) $S_n = 32n - n^2$. Set n to 1, a or $S_1 = 31$ Set n to 2 or other value $S_2 = 60$ $\rightarrow$ 2nd term = 29 $\rightarrow d = -2$ (or equates formulae – compares coeffs n^2, n) [M1 comparing, A1 d A1 a] (b) $\frac{a}{1-r} = 20$, $\frac{a(1-r)^2}{1-r}$, or $a + ar = 12.8$ Elimination of $\frac{a}{1-r}$ or a or r $\rightarrow (r = 0.6) \rightarrow a = 8$	B1 M1 A1 [3] B1 B1 M1 DM1 A1 [5]	co Correct method. co [M1 only when coeffs compared] co co ‘Correct’ elimination to form equation in a or r Complete method leading to $a =$ Condone $a = 8$ and 32
9 (i) $m_{AB} = -3$ or $\frac{-9}{3}$ $m_{AD} = \frac{1}{3}$ Eqn AD $y - 6 = \frac{1}{3}(x - 2)$ or $3y = x + 16$ (ii) Eqn CD $y - 3 = -3(x - 8)$ or $y = -3x + 27$ Sim Eqns $\rightarrow D(6\frac{1}{2}, 7\frac{1}{2})$ (iii) Use of vectors or mid-point $\rightarrow E(5, 12)$ or mid-point $(5, 4.5)$ Length of $BE = 15$	B1 M1 A1 [3] B1 $\frac{1}{3}$ M1 A1 [3] B1 B1 [2]	oe use of $m_1 m_2 = -1$ with grad AB co – OK unsimplified OK unsimplified. $\frac{1}{3}$ on m of AB. Reasonable algebra leading to $x =$ or $y =$ with AD and CD May be implied co
10 $\frac{d^2 y}{dx^2} = \frac{24}{x^3} - 4$ (i) (If $x = 2$) it's negative $\rightarrow$ Max (ii) $\left(\frac{dy}{dx}\right) = -12x^{-2} - 4x + (A)$ $= 0$ when $x = 2$ $\rightarrow A = 11$ (iii) $(y =) 12x^{-1} - 2x^2 + Ax + (c)$ $y = 13$ when $x = 1 \rightarrow c = -8$ (If $x = 2$) $y = 12$	B1 [1] B2,1,0 M1 A1 [4] B2,1,0 $\frac{1}{3}$ M1 A1 [4]	www oe one per term Attempt at the constant A after $\int n$ co oe Doesn't need $+c$, but does need a term A to give “Ax”. Attempt at c after $\int n$ co

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11 $f: x \mapsto 6 - 4\cos\left(\frac{1}{2}x\right)$ (i) $6 - 4\cos\left(\frac{1}{2}x\right) = 4 \rightarrow 4\cos\left(\frac{1}{2}x\right) = 2$ $\frac{1}{2}x = \frac{1}{3}\pi \quad x = \frac{2}{3}\pi$ (ii) Range is $2 \leq f(x) \leq 10$ (iii) (iv) $\cos\left(\frac{1}{2}x\right) = \frac{1}{4}(6 - y)$ $\frac{1}{2}x = \cos^{-1}\left(\frac{1}{4}(6 - y)\right)$ $f^{-1}(x) = 2\cos^{-1}\left(\frac{6 - x}{4}\right)$	M1 M1 A1 [3] B1 B1 [2] B1 B1 [2] M1 M1 A1 [3]	Makes $\cos\left(\frac{1}{2}x\right)$ the subject. Looks up "$\frac{1}{2}x$" before $\times 2$ co (120° gets A0 – decimals A0) condone < Point of inflexion at π Fully correct Makes $\cos\left(\frac{1}{2}x\right)$ the subject Order of operations correct (M marks allowed if + for -) oe – needs to be a function of x not y
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The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only – often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 $(15 \text{ or } {}^{16}C_2) \times 2^4 \times (ax)^2, (20 \text{ or } {}^6C_3) \times 2^3 \times (ax)^3$ $a = \frac{15 \times 2^4}{20 \times 2^3} = \frac{3}{2}$	B1B1 M1A1 [4]	$240a = 160a$ is M0
2 (i) $CB \text{ or } AB = \frac{3}{\tan \frac{\pi}{6}} \text{ or } 3 \tan \frac{\pi}{3}$ Arc or $AC = 3 \times \left[\frac{2\pi}{3} \text{ or } \frac{\pi}{3} \right] \quad (= 2\pi \text{ or } \pi)$ Perimeter $= 6\sqrt{3} + 2\pi$ oe (ii) Area $OABC \quad (2) \times \frac{1}{2} \times 3 \times \text{their } AB$ $ (= 9\sqrt{3} \text{ or } \frac{9\sqrt{3}}{2})$ Area $OADC \quad \frac{1}{2} \times 3^2 \times \left(\frac{2\pi}{2} \text{ or } \frac{\pi}{3} \right) \quad \left(= 3\pi \text{ or } \frac{3\pi}{2} \right)$ Shaded area $9\sqrt{3} - 3\pi$ oe	B1 B1 B1 [3] B1^h B1 B1 [3]	Allow throughout for e.g. $3\sqrt{3}, \sqrt{27}, \sqrt{3^3}, (\sqrt{3})^3, \frac{9}{\sqrt{3}}$ After B0B0 SCB1 for 16.7 Their AB in form $k\sqrt{3}$ After B0B0 SCB1 for 6.16 or 6.17. Allow $(\sqrt{3})^5 - 3\pi$
3 (i) $(3x - 2)^2 + 1$ (ii) $f'(x) = 9x^2 - 12x + 5$ $= \text{their } (3x - 2)^2 + 1$ $> 0 \text{ (or } \geq 1) \text{ hence an increasing function}$	B1B1B1 [3] B1 M1 A1 [3]	For either of 1st 2 marks bracket must be in the form $(ax + b)^2$ except for SCB2 for $9\left(x - \frac{2}{3}\right)^2 + 1$ Ft from (i). Some reference/recognition Allow > 1. Allow <i>their</i> 1 provided positive. Allow a complete alt method (2/2 or 0/2)
4 (i) $S_P = \frac{2}{1 - \frac{1}{2}}, S_P = \frac{3}{1 - \frac{1}{3}}$ $S_P = 4, S_Q = \frac{9}{2}$ $S_R = 5$ cao (ii) $\frac{4}{1 - r} = \text{their } S_R$ $r = \frac{1}{5}$	M1 A1 A1 [3] M1 A1	At least one correct At least one correct

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$R = 4 + \frac{4}{5} + \frac{4}{25} = 4\frac{24}{25}$ or 4.96 cao	A1 [3]	
5 (i) $(s^2 - c^2)(s^2 + c^2)$ OR $s^2(1 - c^2) - c^2(1 - s^2)$ $\sin^2 \theta - \cos^2 \theta$ $2\sin^2 \theta - 1$ www AG	M1 A1 A1 [3]	OR $\sin^4 \theta - (1 - \sin^2 \theta)^2$ $\sin^4 \theta - (1 - 2\sin^2 \theta + \sin^4 \theta)$ $= 2\sin^2 \theta - 1$ AG
(ii) $2\sin^2 \theta - 1 = \frac{1}{2} \Rightarrow \sin \theta = (\pm)\frac{\sqrt{3}}{2}$ or $(\pm)0.866$ $\theta = 60^\circ$ $\theta = 120^\circ$ $\theta = 240^\circ, 300^\circ$	B1 B1 ^½ B1 ^½ [4]	OR $\cos 2\theta = -\frac{1}{2} \rightarrow 2\theta = 120, 240$ etc. Ft for 180 – <i>their</i> 60 Ft for 180 + <i>their</i> 60, 360 – <i>their</i> 60 Allow $\frac{\pi}{3}, \frac{2\pi}{3}$ etc. Extra sols in range –1
6 (i) $m = \frac{3a+9-(2a-1)}{2a+4-a} = \frac{a+10}{a+4}$ oe e.g. $\frac{-a-10}{-a-4}$ Gradient of perpendicular = $\frac{-(a+4)}{a+10}$ oe but not $\frac{-1}{\left(\frac{a+10}{a+4}\right)}$	M1A1 A1 ^½ [3]	cao Allow omission of brackets for M1 Do not ISW. Max penalty for erroneous cancellation 1 mark
(ii) $(\sqrt{})[(a+4)^2 + (a+10)^2] = (\sqrt{})260$ $(\sqrt{})[(a+4)^2 + (a+10)^2]$ cao $(2)(a^2 + 14a - 72) (= 0)$ $a = 4$ or -18 cao	M1 A1 A1 A1 [4]	Allow <i>their</i> $(a+4), (a+10)$ from (i). Allow $(-a-4)^2$ etc. Allow omission of brackets

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7 (i) $OA \cdot OB = -7 + 3 - 3p + p^2$ $(p+1)(p-4) = 0$ $p = -1$ or 4 (ii) $49 + (1 - p^2) + p^2 = 2(1 + 9 + p^2)$ $p = 15$ (iii) $AB = -8\mathbf{i} + 6\mathbf{j}$ Divide AB by $AB = \sqrt{(-8)^2 + 6^2} = 10$ soi Unit vector $= \frac{1}{10}(-8\mathbf{i} + 6\mathbf{j})$ oe cao	M1 DM1 A1 [3] M1 A1 [2] B1 M1 A1 [3]	Correct method for scalar product Equate to zero & attempt to factorise/solve ‘= 0’ implied by answers Scalar result required $p = 15$ used – treat as MR $\rightarrow \frac{1}{\sqrt{353}} \begin{pmatrix} -8 \\ -17 \\ 0 \end{pmatrix}$
8 (i) Minimum since $f''(3) (= 4/3) > 0$ www (ii) $f'(x) = -18x^{-2} (+c)$ $0 = -2 + c$ $c = 2 \rightarrow f'(x) = -18x^{-2} + 2$ $f(x) = 18x^{-1} + 2x (+k)$ $7 = 6 + 6 + k$ $k = -5 \rightarrow f(x) = 18x^{-1} + 2x - 5$ cao	B1 [1] B1 M1 A1 B1[✓] B1[✓] M1 A1 [7]	Sub $f'(3) = 0$. (dep c present) $c = 2$ sufficient at this stage Allow cx at this stage Sub $f(3) = 3$ (k present & numeric (or no) c)
9 (i) $x - 3\sqrt{x} + 2$ or $k^2 - 3k + 2$ or $(3\sqrt{x})^2 = (x+2)^2$ $\sqrt{x} = 1$ or 2 or $k = 1$ or 2 or $x^2 - 5x + 4 (= 0)$ $x = 1$ or 4 $y = 3$ or 6	M1 A1 A1 A1 [4]	OR attempt to eliminate x eg sub $x = \frac{y^2}{9}$ $y^2 - 9y + 18 = 0$ $y = 3$ or 6 $x = 1$ or 4

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(ii) $\int 3x^{\frac{1}{2}} dx - \left[\int (x+2) dx \text{ or attempt at trapezium} \right]$ $2x^{\frac{3}{2}} - \left[\left(\frac{1}{2}x^2 + 2x \right) \text{ or } \frac{1}{2}(y_2 + y_1)(x_2 - x_1) \right]$ $(16 - 2) - \left[\left[(8 + 8) - \left(\frac{1}{2} + 2 \right) \right] \text{ or } \textit{their} \frac{1}{2} \times 9 \times 3 \right]$ $\frac{1}{2}$ OR $\left[\int (y-2) dy \text{ or attempt at trap} \right] - \int \frac{y^2}{9} dy$ $\left[\frac{1}{2}y^2 - 2y \text{ or } \frac{1}{2}(x_1 + x_2)(y_2 - y_1) \right] - \frac{y^3}{27}$ $\left[(18 - 12) - \left(4\frac{1}{2} - 6 \right) \text{ or } \frac{1}{2} \times 5 \times 3 \right] - [8 - 1]$ $\frac{1}{2}$	M1DM1 A1A1 DM1 A1 [6] M1DM1 A1A1 DM1 A1	Attempt to integrate. Subtract at some stage Where $(x_1, y_1), (x_2, y_2)$ is <i>their</i> (1, 3), (4, 6) Apply <i>their</i> 1→4 limits correctly to curve For A mark allow reverse subtn→ $-\frac{1}{2} \rightarrow \frac{1}{2}$ but not reversed limits Apply <i>their</i> 3→6 limits correctly to curve
10 (a) (i) $(a+b)^{\frac{1}{3}} = 2, (9a+b)^{\frac{2}{3}} = 16$ $a+b=8, 9a+b=64$ $a=7, b=1$ (ii) $x = (7y+1)^{\frac{1}{3}}$ (x/y interchange as first or last step) $x^3 = 7y+1$ or $y^3 = 7x+1$ $f^{-1}(x) = \frac{1}{7}(x^3 - 1)$ cao Domain of f^{-1} is $x \geq 1$ cao (b) $\frac{dy}{dx} = \left[\frac{1}{3}(7x^2 + 1)^{-\frac{2}{3}} \right] \times [14x]$ When $x=3, \frac{dy}{dx} = \frac{1}{3} \times (64)^{\frac{2}{3}} \times 42 \quad \left(= \frac{7}{8} \right)$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = \frac{7}{8} \times 8$ 7	B1B1 M1 A1 [4] B1 B1 B1 B1 [4] B1B1 M1 DM1 A1 [5]	Ignore 2nd soln (−9, 17) throughout Cube etc. & attempt to solve Correct answers without any working 0/4 ft on from <i>their</i> a, b or in terms of a, b ft on from <i>their</i> a, b or in terms of a, b A function of x required Accept $>$. Must be x Use chain rule