



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

October/November 2012

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

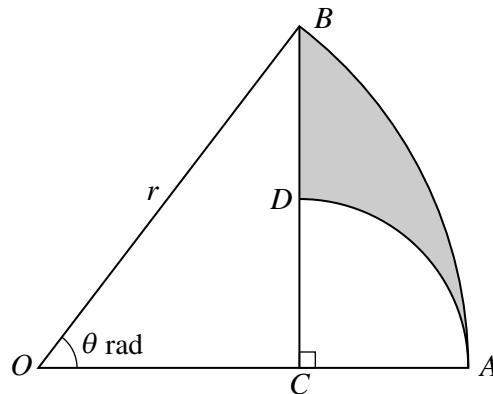
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



- 1 The first term of an arithmetic progression is 61 and the second term is 57. The sum of the first n terms is n . Find the value of the positive integer n . [4]
- 2 A curve is such that $\frac{dy}{dx} = -\frac{8}{x^3} - 1$ and the point $(2, 4)$ lies on the curve. Find the equation of the curve. [4]
- 3 An oil pipeline under the sea is leaking oil and a circular patch of oil has formed on the surface of the sea. At midday the radius of the patch of oil is 50 m and is increasing at a rate of 3 metres per hour. Find the rate at which the area of the oil is increasing at midday. [4]
- 4 (i) Find the first 3 terms in the expansion of $(2x - x^2)^6$ in ascending powers of x . [3]
- (ii) Hence find the coefficient of x^8 in the expansion of $(2 + x)(2x - x^2)^6$. [2]
- 5 A curve has equation $y = 2x + \frac{1}{(x-1)^2}$. Verify that the curve has a stationary point at $x = 2$ and determine its nature. [5]

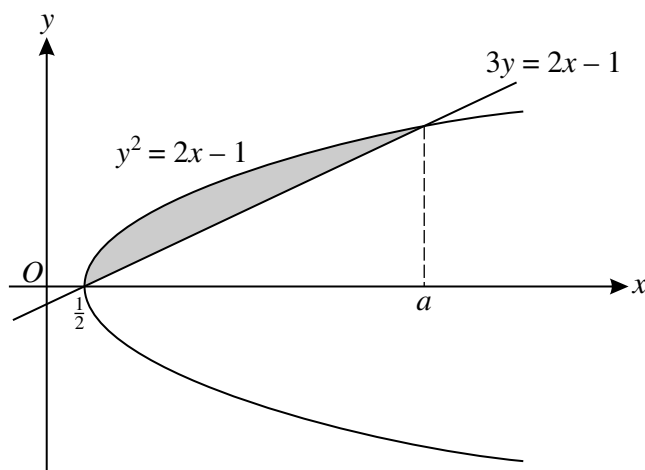
6



The diagram shows a sector OAB of a circle with centre O and radius r . Angle AOB is θ radians. The point C on OA is such that BC is perpendicular to OA . The point D is on BC and the circular arc AD has centre C .

- (i) Find AC in terms of r and θ . [1]
- (ii) Find the perimeter of the shaded region ABD when $\theta = \frac{1}{3}\pi$ and $r = 4$, giving your answer as an exact value. [6]
- 7 (i) Solve the equation $2\cos^2\theta = 3\sin\theta$, for $0^\circ \leq \theta \leq 360^\circ$. [4]
- (ii) The smallest positive solution of the equation $2\cos^2(n\theta) = 3\sin(n\theta)$, where n is a positive integer, is 10° . State the value of n and hence find the largest solution of this equation in the interval $0^\circ \leq \theta \leq 360^\circ$. [3]

8



The diagram shows the curve $y^2 = 2x - 1$ and the straight line $3y = 2x - 1$. The curve and straight line intersect at $x = \frac{1}{2}$ and $x = a$, where a is a constant.

(i) Show that $a = 5$. [2]

(ii) Find, showing all necessary working, the area of the shaded region. [6]

- 9 The position vectors of points A and B relative to an origin O are $\mathbf{a}$ and $\mathbf{b}$ respectively. The position vectors of points C and D relative to O are $3\mathbf{a}$ and $2\mathbf{b}$ respectively. It is given that

$$\mathbf{a} = \begin{pmatrix} 2 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 4 \\ 0 \\ 6 \end{pmatrix}.$$

(i) Find the unit vector in the direction of $\overrightarrow{CD}$. [3]

(ii) The point E is the mid-point of CD . Find angle EOD . [6]

- 10 The function f is defined by $f(x) = 4x^2 - 24x + 11$, for $x \in \mathbb{R}$.

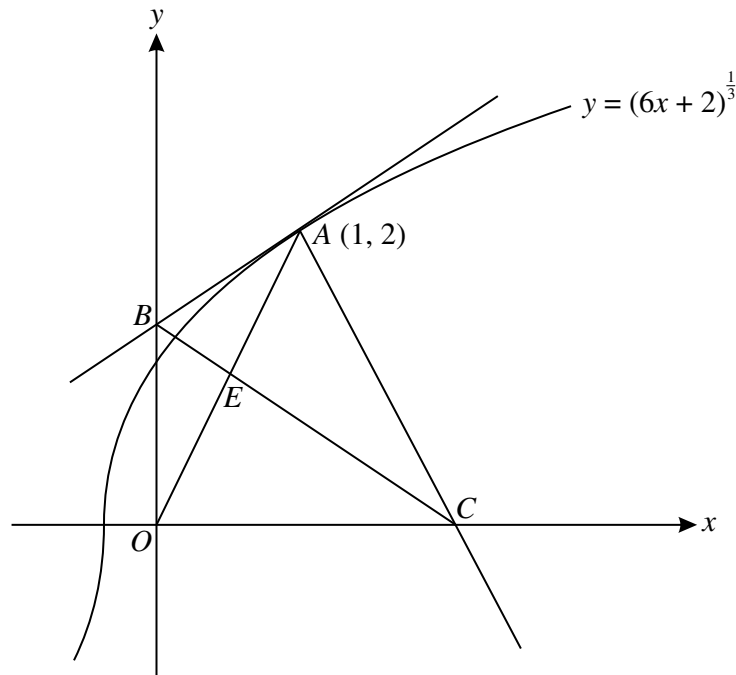
(i) Express $f(x)$ in the form $a(x - b)^2 + c$ and hence state the coordinates of the vertex of the graph of $y = f(x)$. [4]

The function g is defined by $g(x) = 4x^2 - 24x + 11$, for $x \leq 1$.

(ii) State the range of g . [2]

(iii) Find an expression for $g^{-1}(x)$ and state the domain of g^{-1} . [4]

[Question 11 is printed on the next page.]



The diagram shows the curve $y = (6x + 2)^{\frac{1}{3}}$ and the point $A(1, 2)$ which lies on the curve. The tangent to the curve at A cuts the y -axis at B and the normal to the curve at A cuts the x -axis at C .

- (i) Find the equation of the tangent AB and the equation of the normal AC . [5]
- (ii) Find the distance BC . [3]
- (iii) Find the coordinates of the point of intersection, E , of OA and BC , and determine whether E is the mid-point of OA . [4]



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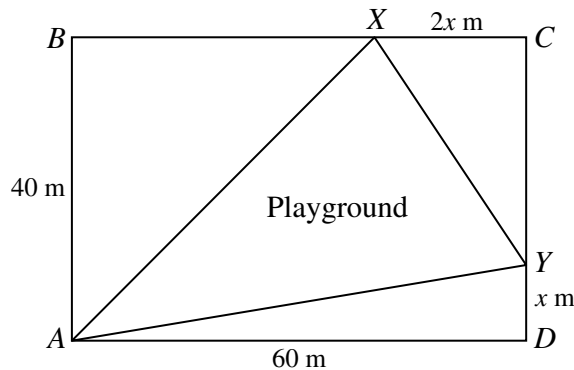
1 In the expansion of $\left(x^2 - \frac{a}{x}\right)^7$, the coefficient of x^5 is -280 . Find the value of the constant a . [3]

2 A function f is such that $f(x) = \sqrt{\left(\frac{x+3}{2}\right)} + 1$, for $x \geq -3$. Find

(i) $f^{-1}(x)$ in the form $ax^2 + bx + c$, where a , b and c are constants, [3]

(ii) the domain of f^{-1} . [1]

3



The diagram shows a plan for a rectangular park $ABCD$, in which $AB = 40$ m and $AD = 60$ m. Points X and Y lie on BC and CD respectively and AX , XY and YA are paths that surround a triangular playground. The length of DY is x m and the length of XC is $2x$ m.

(i) Show that the area, A m², of the playground is given by

$$A = x^2 - 30x + 1200. \quad [2]$$

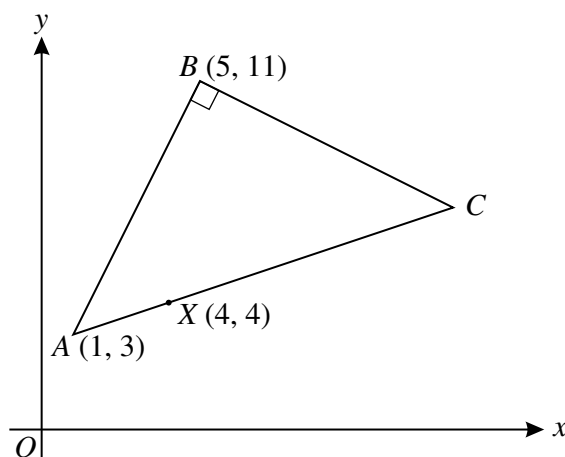
(ii) Given that x can vary, find the minimum area of the playground. [3]

4 The line $y = \frac{x}{k} + k$, where k is a constant, is a tangent to the curve $4y = x^2$ at the point P . Find

(i) the value of k , [3]

(ii) the coordinates of P . [3]

5



The diagram shows a triangle ABC in which A has coordinates $(1, 3)$, B has coordinates $(5, 11)$ and angle ABC is 90° . The point $X(4, 4)$ lies on AC . Find

(i) the equation of BC , [3]

(ii) the coordinates of C . [3]

6 (i) Show that the equation $2 \cos x = 3 \tan x$ can be written as a quadratic equation in $\sin x$. [3]

(ii) Solve the equation $2 \cos 2y = 3 \tan 2y$, for $0^\circ \leq y \leq 180^\circ$. [4]

7 The position vectors of the points A and B , relative to an origin O , are given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} k \\ -k \\ 2k \end{pmatrix},$$

where k is a constant.

(i) In the case where $k = 2$, calculate angle AOB . [4]

(ii) Find the values of k for which $\overrightarrow{AB}$ is a unit vector. [4]

8 (a) In a geometric progression, all the terms are positive, the second term is 24 and the fourth term is $13\frac{1}{2}$. Find

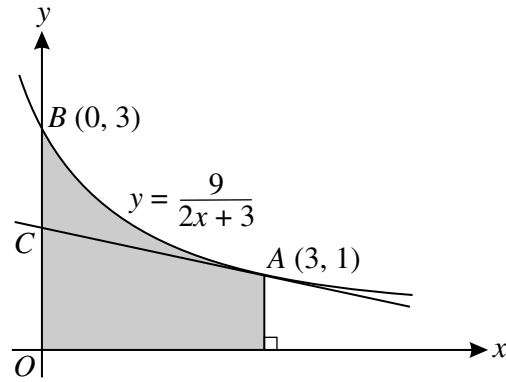
(i) the first term, [3]

(ii) the sum to infinity of the progression. [2]

(b) A circle is divided into n sectors in such a way that the angles of the sectors are in arithmetic progression. The smallest two angles are 3° and 5° . Find the value of n . [4]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows part of the curve $y = \frac{9}{2x+3}$, crossing the y -axis at the point $B(0, 3)$. The point A on the curve has coordinates $(3, 1)$ and the tangent to the curve at A crosses the y -axis at C .

(i) Find the equation of the tangent to the curve at A . [4]

(ii) Determine, showing all necessary working, whether C is nearer to B or to O . [1]

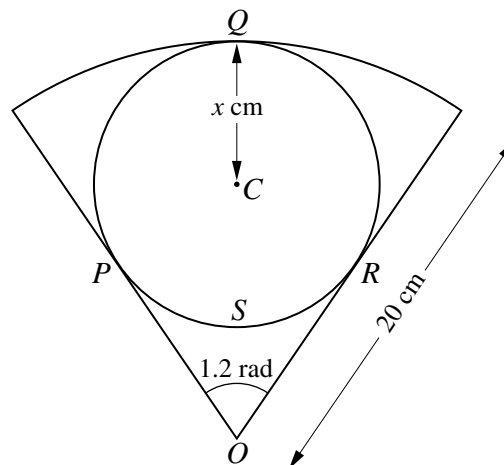
(iii) Find, showing all necessary working, the exact volume obtained when the shaded region is rotated through 360° about the x -axis. [4]

10 A curve is defined for $x > 0$ and is such that $\frac{dy}{dx} = x + \frac{4}{x^2}$. The point $P(4, 8)$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Show that the gradient of the curve has a minimum value when $x = 2$ and state this minimum value. [4]

11



The diagram shows a sector of a circle with centre O and radius 20 cm. A circle with centre C and radius x cm lies within the sector and touches it at P , Q and R . Angle $POR = 1.2$ radians.

(i) Show that $x = 7.218$, correct to 3 decimal places. [4]

(ii) Find the total area of the three parts of the sector lying outside the circle with centre C . [2]

(iii) Find the perimeter of the region $OPSR$ bounded by the arc PSR and the lines OP and OR . [4]



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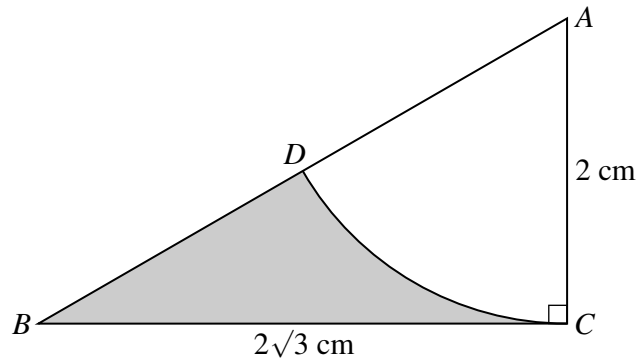
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- 1 Find the coefficient of x^3 in the expansion of $(2 - \frac{1}{2}x)^7$. [3]
- 2 It is given that $f(x) = \frac{1}{x^3} - x^3$, for $x > 0$. Show that f is a decreasing function. [3]
- 3 Solve the equation $7 \cos x + 5 = 2 \sin^2 x$, for $0^\circ \leq x \leq 360^\circ$. [4]

4



In the diagram, D lies on the side AB of triangle ABC and CD is an arc of a circle with centre A and radius 2 cm. The line BC is of length $2\sqrt{3}$ cm and is perpendicular to AC . Find the area of the shaded region BDC , giving your answer in terms of π and $\sqrt{3}$. [4]

- 5 The first term of a geometric progression is $5\frac{1}{3}$ and the fourth term is $2\frac{1}{4}$. Find
 (i) the common ratio, [3]
 (ii) the sum to infinity. [2]

- 6 The functions f and g are defined for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$ by

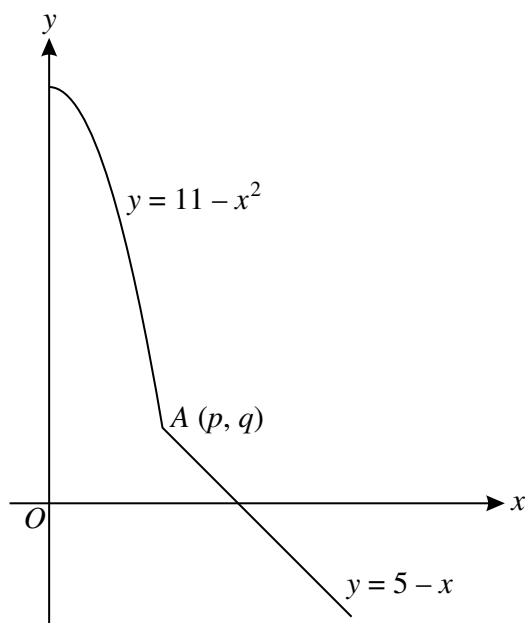
$$f(x) = \frac{1}{2}x + \frac{1}{6}\pi,$$

$$g(x) = \cos x.$$

Solve the following equations for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

- (i) $gf(x) = 1$, giving your answer in terms of π . [2]
- (ii) $fg(x) = 1$, giving your answers correct to 2 decimal places. [4]

7



- (i) The diagram shows part of the curve $y = 11 - x^2$ and part of the straight line $y = 5 - x$ meeting at the point A(p, q), where p and q are positive constants. Find the values of p and q . [3]

- (ii) The function f is defined for the domain $x \geq 0$ by

$$f(x) = \begin{cases} 11 - x^2 & \text{for } 0 \leq x \leq p, \\ 5 - x & \text{for } x > p. \end{cases}$$

Express $f^{-1}(x)$ in a similar way. [5]

- 8 A curve is such that

$$\frac{dy}{dx} = 2(3x + 4)^{\frac{3}{2}} - 6x - 8.$$

- (i) Find $\frac{d^2y}{dx^2}$. [2]

- (ii) Verify that the curve has a stationary point when $x = -1$ and determine its nature. [2]

- (iii) It is now given that the stationary point on the curve has coordinates $(-1, 5)$. Find the equation of the curve. [5]

- 9 The position vectors of points A and B relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} p \\ 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ p \end{pmatrix},$$

where p is a constant.

- (i) In the case where OAB is a straight line, state the value of p and find the unit vector in the direction of $\overrightarrow{OA}$. [3]
- (ii) In the case where OA is perpendicular to AB, find the possible values of p . [5]
- (iii) In the case where $p = 3$, the point C is such that OABC is a parallelogram. Find the position vector of C. [2]

10 A straight line has equation $y = -2x + k$, where k is a constant, and a curve has equation $y = \frac{2}{x-3}$.

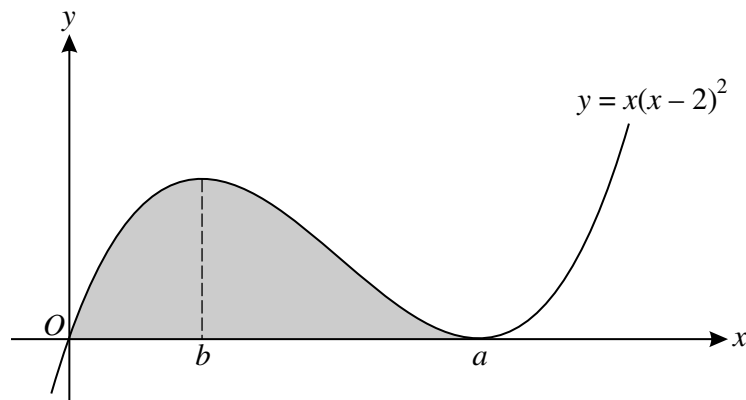
(i) Show that the x -coordinates of any points of intersection of the line and curve are given by the equation $2x^2 - (6+k)x + (2+3k) = 0$. [1]

(ii) Find the two values of k for which the line is a tangent to the curve. [3]

The two tangents, given by the values of k found in part (ii), touch the curve at points A and B .

(iii) Find the coordinates of A and B and the equation of the line AB . [6]

11



The diagram shows the curve with equation $y = x(x-2)^2$. The minimum point on the curve has coordinates $(a, 0)$ and the x -coordinate of the maximum point is b , where a and b are constants.

(i) State the value of a . [1]

(ii) Find the value of b . [4]

(iii) Find the area of the shaded region. [4]

(iv) The gradient, $\frac{dy}{dx}$, of the curve has a minimum value m . Find the value of m . [4]