



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

October/November 2011

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

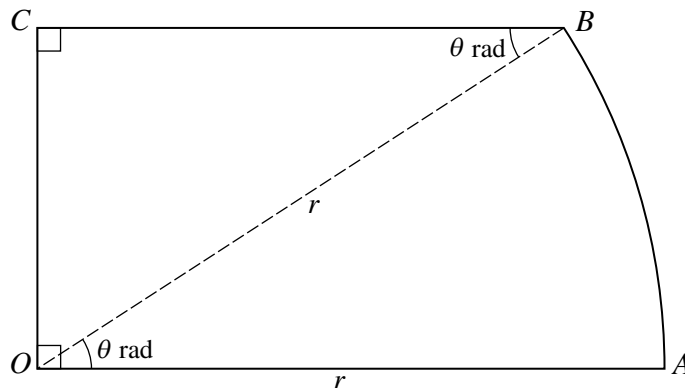
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

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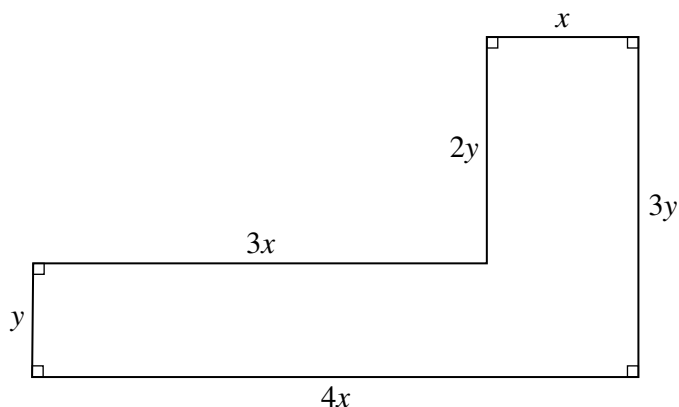
- 1 Find the term independent of x in the expansion of $\left(2x + \frac{1}{x^2}\right)^6$. [3]
- 2 A curve has equation $y = 3x^3 - 6x^2 + 4x + 2$. Show that the gradient of the curve is never negative. [3]
- 3 (i) Sketch, on a single diagram, the graphs of $y = \cos 2\theta$ and $y = \frac{1}{2}$ for $0 \leq \theta \leq 2\pi$. [3]
 (ii) Write down the number of roots of the equation $2 \cos 2\theta - 1 = 0$ in the interval $0 \leq \theta \leq 2\pi$. [1]
 (iii) Deduce the number of roots of the equation $2 \cos 2\theta - 1 = 0$ in the interval $10\pi \leq \theta \leq 20\pi$. [1]
- 4 A function f is defined for $x \in \mathbb{R}$ and is such that $f'(x) = 2x - 6$. The range of the function is given by $f(x) \geq -4$.
 (i) State the value of x for which $f(x)$ has a stationary value. [1]
 (ii) Find an expression for $f(x)$ in terms of x . [4]

5



The diagram represents a metal plate $OABC$, consisting of a sector OAB of a circle with centre O and radius r , together with a triangle OCB which is right-angled at C . Angle $AOB = \theta$ radians and OC is perpendicular to OA .

- (i) Find an expression in terms of r and θ for the perimeter of the plate. [3]
 (ii) For the case where $r = 10$ and $\theta = \frac{1}{5}\pi$, find the area of the plate. [3]
- 6 (a) The sixth term of an arithmetic progression is 23 and the sum of the first ten terms is 200. Find the seventh term. [4]
 (b) A geometric progression has first term 1 and common ratio r . A second geometric progression has first term 4 and common ratio $\frac{1}{4}r$. The two progressions have the same sum to infinity, S . Find the values of r and S . [3]



The diagram shows the dimensions in metres of an L-shaped garden. The perimeter of the garden is 48 m.

(i) Find an expression for y in terms of x . [1]

(ii) Given that the area of the garden is $A \text{ m}^2$, show that $A = 48x - 8x^2$. [2]

(iii) Given that x can vary, find the maximum area of the garden, showing that this is a maximum value rather than a minimum value. [4]

8 Relative to an origin O , the point A has position vector $4\mathbf{i} + 7\mathbf{j} - p\mathbf{k}$ and the point B has position vector $8\mathbf{i} - \mathbf{j} - p\mathbf{k}$, where p is a constant.

(i) Find $\overrightarrow{OA} \cdot \overrightarrow{OB}$. [2]

(ii) Hence show that there are no real values of p for which OA and OB are perpendicular to each other. [1]

(iii) Find the values of p for which angle $AOB = 60^\circ$. [4]

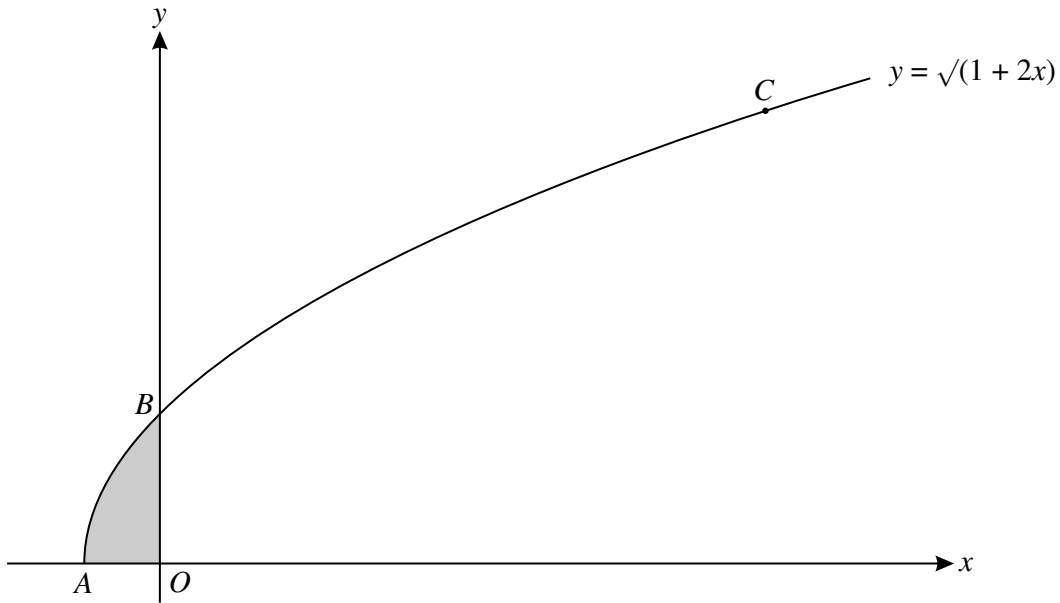
9 A line has equation $y = kx + 6$ and a curve has equation $y = x^2 + 3x + 2k$, where k is a constant.

(i) For the case where $k = 2$, the line and the curve intersect at points A and B . Find the distance AB and the coordinates of the mid-point of AB . [5]

(ii) Find the two values of k for which the line is a tangent to the curve. [4]

[Questions 10 and 11 are printed on the next page.]

10



The diagram shows the curve $y = \sqrt{1 + 2x}$ meeting the x -axis at A and the y -axis at B . The y -coordinate of the point C on the curve is 3.

(i) Find the coordinates of B and C . [2]

(ii) Find the equation of the normal to the curve at C . [4]

(iii) Find the volume obtained when the shaded region is rotated through 360° about the y -axis. [5]

11 Functions f and g are defined by

$$\begin{aligned} f : x &\mapsto 2x^2 - 8x + 10 && \text{for } 0 \leq x \leq 2, \\ g : x &\mapsto x && \text{for } 0 \leq x \leq 10. \end{aligned}$$

(i) Express $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

(ii) State the range of f . [1]

(iii) State the domain of f^{-1} . [1]

(iv) Sketch on the same diagram the graphs of $y = f(x)$, $y = g(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [4]

(v) Find an expression for $f^{-1}(x)$. [3]

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UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
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MATHEMATICS

9709/12

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- 1 (i) Find the first 3 terms in the expansion of $(2 - y)^5$ in ascending powers of y . [2]
- (ii) Use the result in part (i) to find the coefficient of x^2 in the expansion of $(2 - (2x - x^2))^5$. [3]

- 2 The functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 3x + a,$$

$$g : x \mapsto b - 2x,$$

where a and b are constants. Given that $ff(2) = 10$ and $g^{-1}(2) = 3$, find

- (i) the values of a and b , [4]
- (ii) an expression for $fg(x)$. [2]

- 3 Relative to an origin O , the position vectors of points A and B are given by

$$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k},$$

where p is a constant.

- (i) Find the value of p for which angle AOB is 90° . [3]
- (ii) In the case where $p = 4$, find the vector which has magnitude 28 and is in the same direction as $\overrightarrow{AB}$. [4]

- 4 The equation of a curve is $y^2 + 2x = 13$ and the equation of a line is $2y + x = k$, where k is a constant.

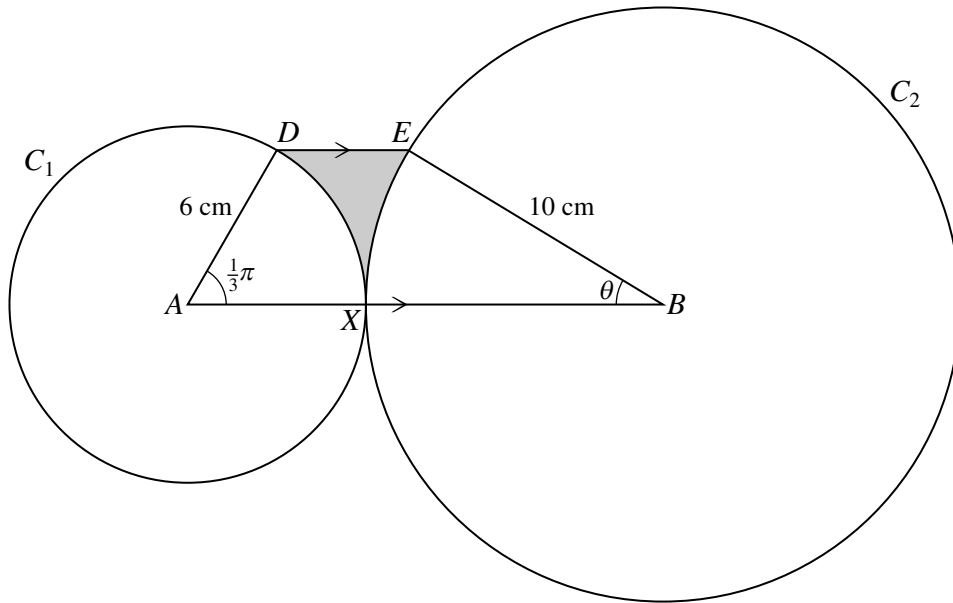
- (i) In the case where $k = 8$, find the coordinates of the points of intersection of the line and the curve. [4]
- (ii) Find the value of k for which the line is a tangent to the curve. [3]

- 5 (i) Sketch, on the same diagram, the graphs of $y = \sin x$ and $y = \cos 2x$ for $0^\circ \leq x \leq 180^\circ$. [3]

- (ii) Verify that $x = 30^\circ$ is a root of the equation $\sin x = \cos 2x$, and state the other root of this equation for which $0^\circ \leq x \leq 180^\circ$. [2]

- (iii) Hence state the set of values of x , for $0^\circ \leq x \leq 180^\circ$, for which $\sin x < \cos 2x$. [2]

6



The diagram shows a circle C_1 touching a circle C_2 at a point X . Circle C_1 has centre A and radius 6 cm, and circle C_2 has centre B and radius 10 cm. Points D and E lie on C_1 and C_2 respectively and DE is parallel to AB . Angle $DAX = \frac{1}{3}\pi$ radians and angle $EBX = \theta$ radians.

- (i) By considering the perpendicular distances of D and E from AB , show that the exact value of θ is $\sin^{-1}\left(\frac{3\sqrt{3}}{10}\right)$. [3]
- (ii) Find the perimeter of the shaded region, correct to 4 significant figures. [5]

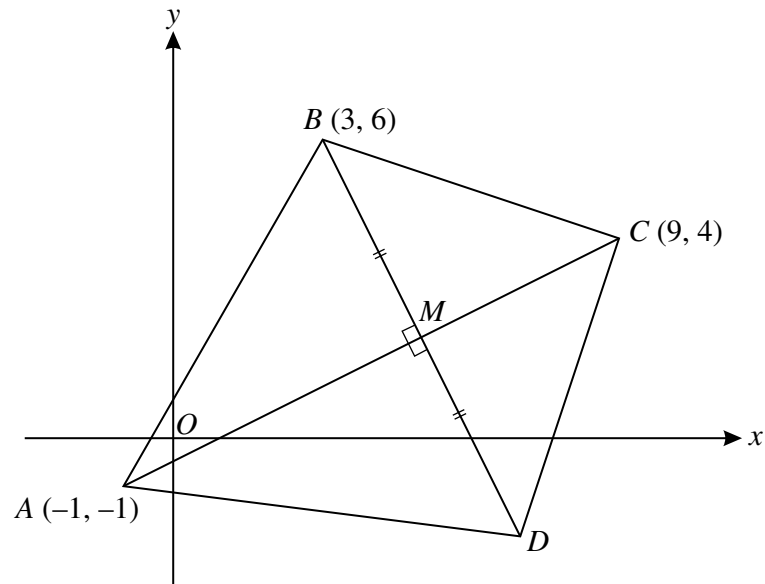
- 7 A curve is such that $\frac{dy}{dx} = 5 - \frac{8}{x^2}$. The line $3y + x = 17$ is the normal to the curve at the point P on the curve. Given that the x -coordinate of P is positive, find

- (i) the coordinates of P , [4]
- (ii) the equation of the curve. [4]

- 8 The equation of a curve is $y = \sqrt{(8x - x^2)}$. Find

- (i) an expression for $\frac{dy}{dx}$, and the coordinates of the stationary point on the curve, [4]
- (ii) the volume obtained when the region bounded by the curve and the x -axis is rotated through 360° about the x -axis. [4]

[Questions 9 and 10 are printed on the next page.]



The diagram shows a quadrilateral $ABCD$ in which the point A is $(-1, -1)$, the point B is $(3, 6)$ and the point C is $(9, 4)$. The diagonals AC and BD intersect at M . Angle $BMA = 90^\circ$ and $BM = MD$. Calculate

- (i) the coordinates of M and D , [7]
- (ii) the ratio $AM : MC$. [2]

10 (a) An arithmetic progression contains 25 terms and the first term is -15 . The sum of all the terms in the progression is 525. Calculate

- (i) the common difference of the progression, [2]
- (ii) the last term in the progression, [2]
- (iii) the sum of all the positive terms in the progression. [2]

(b) A college agrees a sponsorship deal in which grants will be received each year for sports equipment. This grant will be \$4000 in 2012 and will increase by 5% each year. Calculate

- (i) the value of the grant in 2022, [2]
- (ii) the total amount the college will receive in the years 2012 to 2022 inclusive. [2]



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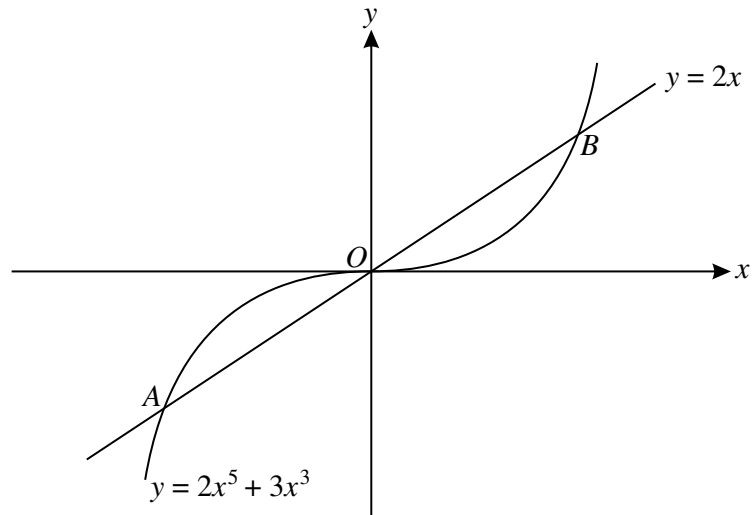
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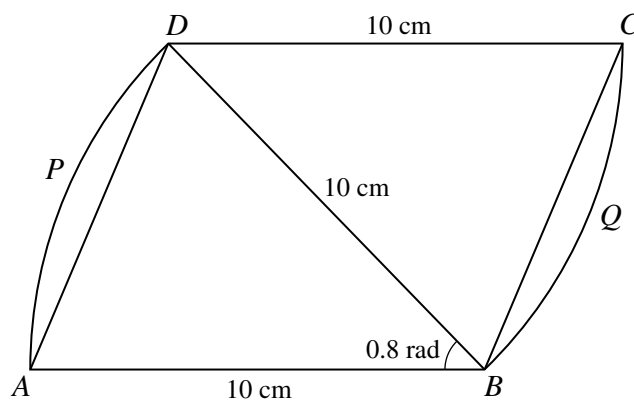
- 1 The coefficient of x^2 in the expansion of $(k + \frac{1}{3}x)^5$ is 30. Find the value of the constant k . [3]
- 2 The first and second terms of a progression are 4 and 8 respectively. Find the sum of the first 10 terms given that the progression is
- (i) an arithmetic progression, [2]
- (ii) a geometric progression. [2]
- 3



The diagram shows the curve $y = 2x^5 + 3x^3$ and the line $y = 2x$ intersecting at points A , O and B .

- (i) Show that the x -coordinates of A and B satisfy the equation $2x^4 + 3x^2 - 2 = 0$. [2]
- (ii) Solve the equation $2x^4 + 3x^2 - 2 = 0$ and hence find the coordinates of A and B , giving your answers in an exact form. [3]

4



In the diagram, $ABCD$ is a parallelogram with $AB = BD = DC = 10$ cm and angle $ABD = 0.8$ radians. APD and BQC are arcs of circles with centres B and D respectively.

- (i) Find the area of the parallelogram $ABCD$. [2]
- (ii) Find the area of the complete figure $ABQCDP$. [2]
- (iii) Find the perimeter of the complete figure $ABQCDP$. [2]

- 5 (i) Given that

$$3 \sin^2 x - 8 \cos x - 7 = 0,$$

show that, for real values of x ,

$$\cos x = -\frac{2}{3}. \quad [3]$$

- (ii) Hence solve the equation

$$3 \sin^2(\theta + 70^\circ) - 8 \cos(\theta + 70^\circ) - 7 = 0$$

$$\text{for } 0^\circ \leq \theta \leq 180^\circ. \quad [4]$$

- 6 Relative to an origin O , the position vectors of points A and B are $3\mathbf{i} + 4\mathbf{j} - \mathbf{k}$ and $5\mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$ respectively.

- (i) Use a scalar product to find angle BOA . [4]

The point C is the mid-point of AB . The point D is such that $\overrightarrow{OD} = 2\overrightarrow{OB}$.

- (ii) Find $\overrightarrow{DC}$. [4]

- 7 (i) A straight line passes through the point $(2, 0)$ and has gradient m . Write down the equation of the line. [1]

- (ii) Find the two values of m for which the line is a tangent to the curve $y = x^2 - 4x + 5$. For each value of m , find the coordinates of the point where the line touches the curve. [6]

- (iii) Express $x^2 - 4x + 5$ in the form $(x + a)^2 + b$ and hence, or otherwise, write down the coordinates of the minimum point on the curve. [2]

- 8 A curve $y = f(x)$ has a stationary point at $P(3, -10)$. It is given that $f'(x) = 2x^2 + kx - 12$, where k is a constant.

- (i) Show that $k = -2$ and hence find the x -coordinate of the other stationary point, Q . [4]

- (ii) Find $f''(x)$ and determine the nature of each of the stationary points P and Q . [2]

- (iii) Find $f(x)$. [4]

- 9 Functions f and g are defined by

$$f : x \mapsto 2x + 3 \quad \text{for } x \leq 0,$$

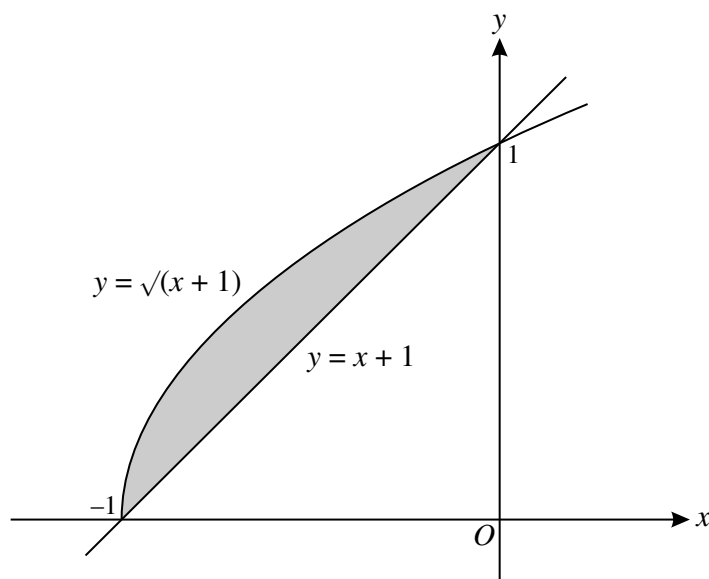
$$g : x \mapsto x^2 - 6x \quad \text{for } x \leq 3.$$

- (i) Express $f^{-1}(x)$ in terms of x and solve the equation $f(x) = f^{-1}(x)$. [3]

- (ii) On the same diagram sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$, showing the coordinates of their point of intersection and the relationship between the graphs. [3]

- (iii) Find the set of values of x which satisfy $gf(x) \leq 16$. [5]

10



The diagram shows the line $y = x + 1$ and the curve $y = \sqrt{x + 1}$, meeting at $(-1, 0)$ and $(0, 1)$.

- (i) Find the area of the shaded region. [5]
- (ii) Find the volume obtained when the shaded region is rotated through 360° about the **y-axis**. [7]

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