

**MARK SCHEME for the October/November 2011 question paper
for the guidance of teachers**

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Mark Scheme Notes

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
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- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
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1	$6C4 \times [2(x)]^4 \times \left[\frac{1}{(x^2)} \right]^2$ 240	B2 B1	 [3]	B1 for 2/3 terms correct Identified as answer. Allow $240x^0$
2	$\frac{\partial y}{\partial x} = 9x^2 - 12x + 4$ $(3x - 2)^2 \geq 0$	M1A1 A1	 [3]	
3	(ii) Correct cosine curve for at least 1 oscillation Exactly 2 complete oscillations in $[0, 2\pi]$ Line $y = \frac{1}{2}$ correct	B1 B1 B1	 [3]	Range $-1 \rightarrow 1$. Ignore labels on θ axis
	(ii) 4	B1✓	[1]	Ft <i>their</i> graph. Accept 30° , 150° , 210° , 330°
	(iii) 20	B1✓	[1]	Or $5 \times$ <i>their</i> part (ii)
4	(i) 3	B1	[1]	
	(ii) $f(x) = x^2 - 6x(+c)$ Subst $(3, -4)$ $c = 5 \rightarrow f(x) = x^2 - 6x + 5$	M1A1 M1 A1	 [4]	Dependent on c present cao
5	(i) Arc $AB = r\theta$ $OC = r \sin \theta$ or $BC = r \cos \theta$ $r(1 + \theta + \cos \theta + \sin \theta)$ correctly derived	M1 M1 A1	 [3]	oe eg $BC = r \sin \frac{\theta}{\tan \theta}$ etc OC & BC reversed loses M1A1
	(ii) Sector $OAB = \frac{1}{2} \times 10^2 \times \frac{\pi}{5}$ ($= 31.42$) $\Delta OCB = \frac{1}{2 \left(10 \cos \frac{\pi}{5} \right) \left(10 \sin \frac{\pi}{5} \right)}$ $(= 23.78)$ Total area = 55.2	M1 M1 A1	 [3]	oe Δ in terms of π and 10 Allow OC & BC reversed (ie max 4/6)
6	(a) $a + 5d = 23$ $5(2a + 9d) = 200$ Attempt solution, expect $d = 6$ $a = -7$ 29	B1 B1 M1 A1	 [4]	Solution of 2 linear equations

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	(b) $\frac{1}{1-r} (=) \frac{4}{1-\frac{1}{4}r}$ $r = \frac{4}{5}$ oe $S = 5$	M1 A1A1		[3]	Use of S_{∞} formula twice
7	(i) $y = \frac{1}{6(48-8x)}$ oe	B1		[1]	
	(ii) $A = 4xy + 2xy$ or $3xy + 3xy = 6xy$ $A = x(48-8x) = 48x - 8x^2$	M1 A1		[2]	AG
	(iii) $\frac{\partial A}{\partial x} = 48 - 16x$ $A = 72$ cao $\frac{\partial^2 A}{\partial x^2} = -16$ (< 0) $\Rightarrow$ Maximum	B1 M1A1 B1		[4]	Attempt to solve derivative = 0 Expect $x = 3$ www Accept other complete methods
8	(i) $(4i + 7j - pk) \cdot (8i - j - pk) = 25 + p^2$	M1A1		[2]	$x_1x_2 + y_1y_2 + z_1z_2$ (Not $25 + (-p)^2$)
	(ii) $25 + p^2 = 0 \Rightarrow$ no real solutions	B1✓		[1]	Ft provided equation has no real solutions
	(iii) $\cos 60 = \frac{OA \cdot OB}{ OA OB }$ used $ OA = \sqrt{65 + p^2}$ or $ OB = \sqrt{65 + p^2}$ $\frac{25 + p^2}{65 + p^2} = \frac{1}{2}$ or $\frac{\text{his scalar (i)}}{65 + p^2} = \frac{1}{2}$ $p = \pm 3.87$ or $\pm \sqrt{15}$	M1 M1 A1✓ A1		[4]	$OA \cdot OB$ must be scalar Not $\sqrt{65 - p^2}$ unless follows $\sqrt{65 + (-p)^2}$ Scalar product = $25 + p^2$ can score here if not scored in part (i)
9	(i) $x^2 + 3x + 4 = 2x + 6 \Rightarrow x^2 + x - 2 (= 0)$ $(x-1)(x+2) = 0 \rightarrow (1,8), (-2,2)$ $AB = \sqrt{3^2 + 6^2} = 6.71$ or $\sqrt{45}$ or $3\sqrt{5}$ $\left(-\frac{1}{2}, 5\right)$	M1 DM1A1 B1 B1✓		[5]	3-term simplification DM1 for attempted solution for x cao ($\sqrt{45}$ from wrong points scores B0) Ft <i>their</i> coordinates

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	(ii) $x^2 + (3-k)x + 2k - 6 (= 0)$ $(3-k)^2 - 4(2k-6) = 0$ $(3-k)(11-k) = 0$ $k = 3$ or 11	M1 DM1 DM1 A1	[4]	Simplified to 3-term quadratic Apply $b^2 - 4ac = 0$ as function of k only Attempt factorisation or use formula Both correct NB Alternative methods for (ii) possible
10	(i) $B = (0,1)$ $C = (4,3)$	B1, B1	[2]	If B0B0 then SCB1 for both $y = 1$ & $x = 4$
	(ii) $\frac{\delta y}{\delta x} = \frac{1}{2} \times 2(1+2x)^{-\frac{1}{2}}$ Grad. of normal $= -3$ $y - 3 = -3(x - 4)$ or $y = -3x + 15$ oe	M1A1 B1 B1✓	[4]	$-\frac{1}{2}$ required & at least one of $\frac{1}{2} \times 2$ for M1 Ft only from <i>their</i> C
	(iii) $y^2 = 1 + 2x \Rightarrow x = \frac{1}{2(y^2 - 1)}$ SOI $(\pi) \times \frac{1}{4} \times \int (y^4 - 2y^2 + 1) \delta y$ $(\pi) \times \frac{1}{4} \left[\frac{y^5}{5} - \frac{2y^3}{3} + y \right]$ $(\pi) \times \frac{1}{4} \left[\frac{1}{5} - \frac{2}{3} + 1 \right]$ $\frac{2}{15} \pi$	B1 M1 A1 DM1 A1	[5]	$\int x^2 \delta y$, square $\frac{1}{2}(y^2 - 1)$ & attempt int^n Apply limits $0 \rightarrow \text{their } 1$ (from <i>their</i> B) cao SCB1 for $\int y^2 \delta x \rightarrow \frac{\pi}{4}$ (scores 1/5)
11	(i) $2(x-2)^2 + 2$	B 1, B1, B1	[3]	For 2, -2, 2
	(ii) $2 \leq f(x) \leq 10$ oe	B1	[1]	Allow < etc. Ignore notation
	(iii) $2 \leq x \leq 10$	B1✓	[1]	Ft from part (ii). Ignore notation
	(iv) $f(x)$: $\approx$ half parabola from (0,10) to (2,2) $g(x)$: line through 0 at $\approx 45^\circ$ $f^{-1}(x)$: reflection of <i>their</i> $f(x)$ in $g(x)$ Everything totally correct	B1 B1 B1✓ B1	[4]	Or from int with y axis to int with <i>their</i> $y = x$

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	(v) $(x-2)^2 = \frac{1}{2}(y-2)$ $x = 2 \pm \sqrt{\frac{1}{2}(y-2)}$ $f^{-1}(x) = 2 - \sqrt{\frac{1}{2}(x-2)}$	M1 M1 A1	[3]	Allow $+\sqrt{\quad}$ or $-\sqrt{\quad}$. Dep on final ans as f^n of x cao
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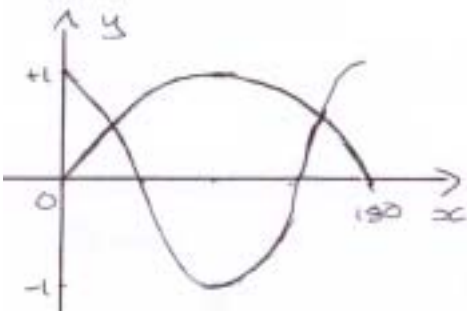
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1	(i) $(2 - y)^5 = 32 - 80y + 80y^2$ (ii) $(2 - (2x - x^2))^5$ “y” = “$2x - x^2$” $\rightarrow 80 + 320 = 400$	B2,1 [2] M1 M1 A1 [3]	–1 for each error. Accept 2^5. Allow for $y = 2x + x^2$ Needs to consider exactly 2 terms. CO – accept $400x^2$, accept full expansion if $400x^2$ is part of it.
2	$f: x \mapsto 3x + a, g: x \mapsto b - 2x$ (i) $f^2(x) = 3(3x + a) + a$ $f^2(2) = 18 + 4a = 10 \rightarrow a = -2$ $g^{-1}(x) = \frac{b-x}{2} \rightarrow \frac{b-2}{2} = 3 \quad b = 8$ or $g(3) = 2 \rightarrow b - 6 = 2 \quad b = 8$ (ii) $fg(x) = 3(b - 2x) + a$ $= 22 - 6x$	B1 B1 M1 A1 [4] M1 A1✓ [2]	Must be correct – unsimplified ok co Correct method leading to a value for b co Must be fg not gf. ✓ on a and b ($3b + a - 6x$) must be two term answer.
3	$\overrightarrow{OA} = 5\mathbf{i} + \mathbf{j} + 2\mathbf{k}, \overrightarrow{OB} = 2\mathbf{i} + 7\mathbf{j} + p\mathbf{k}$ (i) $\overrightarrow{OA} \cdot \overrightarrow{OB} = 10 + 7 + 2p$ $= 0 \rightarrow p = -8\frac{1}{2}$ (ii) $\mathbf{AB} = -3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}$ Modulus = $\sqrt{9+36+4}$ Magnitude 28 $\rightarrow 28 \times$unit vector $\rightarrow -12\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}$.	M1 DM1 A1 [3] B1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ =0 co co (accept negative) For modulus Scales by $\times 28 \div$ modulus. Co – could leave as “$4 \times \dots$”.
4	(i) $y^2 + 2x = 13, 2y + x = 8$ $\rightarrow y^2 - 4y + 3 = 0, x^2 - 8x + 12 = 0$ $\rightarrow (2, 3)$ and $(6, 1)$ (ii) Removes $x \rightarrow y^2 + 2(k - 2y) = 13$ Uses $b^2 - 4ac$ on “quadratic = 0” $\rightarrow k = 8\frac{1}{2}$ or $\frac{dy}{dx} = -\frac{1}{2} = \frac{-1}{y} \rightarrow y=2, x=4\frac{1}{2}, k=8\frac{1}{2}$	M1 A1 DM1 A1 [4] M1 DM1 A1 [3]	Complete elimination of x or y co (allow multiples) – needs 3 terms Solution of quadratic = 0 Needs all 4 coordinates. Complete elimination of x or y. Use of discriminant $=0, <0$ or >0 Co (M1 equating m of line and curve M1 x to y A1 for k)

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5 (i)  (ii) Evidence of $\sin 30 = \cos 60 = 0.5$ Other root is 150° (iii) $0 \leq x < 30$ and $150 < x \leq 180$ ($x < 30$ or $x > 150$ ok)	B1 B1 B1 [3] B1 B1 [2] B1 B1✓ [2]	$y = \sin x$ (0,0). (π,0) + curve $y = \cos 2x$ One full cycle. $y = \cos 2x$ starts and finishes at (0, 1) and oscillates between -1 and +1. Do not penalise graphs from 0 to 360. co co Condone $<$ or $\leq$ throughout
6 (i) D to AX = $6 \sin \frac{\pi}{3} = 6\sqrt{3} \div 2$ E to AX = $10 \sin \theta$ Equate these $\rightarrow \theta = \sin^{-1} \frac{3\sqrt{3}}{10}$. (ii) Arc DX = $6 \cdot \frac{1}{3}\pi = 2\pi$ Arc EX = $10 \times 0.5464 = 5.464$ Horizontal steps = $6 \cos \frac{1}{3}\pi$ and $10 \cos \theta$ $DE = 10 + 6 - 6 \cos \frac{1}{3}\pi - 10 \cos \theta$ Perimeter = arc DX + arc BX + DE $\rightarrow 16.20$	B1 B1 B1 [3] B1 M1 M1 M1 A1 [5]	co Needs $-\sqrt{3} \div 2$ not just $3\sqrt{3}$. co Correct method. ag. Use of decimals loses this B mark. co Use of $s=r\theta$ radians. Attempt at both steps needed Full method for DE. Co – must be exactly 16.20, not more or less places.
7 $\frac{dy}{dx} = 5 - \frac{8}{x^2}$, Normal $3y + x = 17$ (i) Gradient of line = $-\frac{1}{3}$ $\frac{dy}{dx} = 3 \rightarrow x = 2, y = 5$ (ii) $y = 5x + 8x^{-1} (+c)$ Uses (2, 5) $\rightarrow c = -9$	B1 M1 DM1 A1 [4] B1 B1 M1 A1 [4]	co Use of $m_1 m_2 = -1$ DM1 solution. A1 co. co.co. doesn't need +c. Use of +c following integration. co.

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8 $y = \sqrt{8x - x^2}$ (i) $\frac{dy}{dx} = \frac{1}{2}(8x - x^2)^{-\frac{1}{2}} \times (8 - 2x)$ $= 0$ when $x = 4$. $\rightarrow (4, 4)$ (ii) $y = 0$ when $x = 0$ or 8 $\text{Vol} = \pi \int (8x - x^2) dx$ $= \pi \left[4x^2 - \frac{x^3}{3} \right]$ $\rightarrow \frac{256\pi}{3}$	B1 B1 M1 A1 [4] B1 B2,1 B1 [4]	B1 for everything but $\times(8-2x)$ B1 for $\times(8-2x)$, even if B0 Sets to 0 + attempt at solution. Co – A0 if fortuitous because of B0 earlier. Anywhere –1 for each error (not including π) co
9 (i) Gradient of $AC = \frac{1}{2}$ Gradient of $BD = -2$ Eqn of BD is $y - 6 = -2(x - 3)$ Eqn of AC is $y + 1 = \frac{1}{2}(x + 1)$ Sim eqns $\rightarrow M(5, 2)$ Vector move – or midpoint back $\rightarrow D(7, -2)$ (ii) Ratio of $AM : MC = \sqrt{45} : \sqrt{20}$ or Vector step $\rightarrow 3 : 2$	B1 M1 M1 M1 A1 M1 A1√ [7] M1 A1 [2]	co Use of $m_1 m_2 = -1$ with AC Correct formula for straight line Solution. co Correct method. $\sqrt{}$ on M. Correct distance formula. Looks at the two x or y steps. Must be numerical, 1.5 ok, not as roots
10 (a) $a = -15, n = 25$ (i) Use of $S_n \rightarrow d = 3$. (ii) Last term $= a + 24d$ $\rightarrow 57$ (or $525 = \frac{1}{2} \times 25 \times (-15 + l) \rightarrow l = 57$) (iii) Positive terms are 3, 6, ..., 57 Either $a = 0$ or 3, $n = 19$ or 20 Use of S_{19} or S_{20} $\rightarrow 570$ (b) $r = 1.05$ (i) 11th term $= ar^{10} = \\$6516$ or $\\$6520$ (ii) $S_{11} = \frac{4000 \times (1.05^{11} - 1)}{.05}$ $= \\$56800$ or (56827)	M1 A1 [2] M1 A1√ [2] M1 A1 [2] B1 B1 [2] M1 A1 [2]	Must be correct formula. co Must be $a + 24d$ $\sqrt{}$ for his d. Correct use of formula for S_n. co In either part (i) or (ii). co Correct sum formula with their r. co

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BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\sqrt{}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 $k^3 \times \left(\frac{1}{3}x\right)^2 \times 10$ (or correct factorials) $10 \times k^3 \times \frac{1}{9} = 30 \Rightarrow k = 3$	B2 B1 [3]	B1 for 2/3 terms correct cao
2 (i) $5[8 + 9 \times 4]$ 220 (ii) $\frac{4(2^{10} - 1)}{2 - 1}$ 4092	M1 A1 [2] M1 A1 [2]	Use correct formula with $a=4$, $d=4$ Use correct formula with $a=4$, $r=2$ or $\frac{1}{2}$ 4090 without 4092 A0
3 (i) $2x^5 + 3x^2 = 2x \Rightarrow 2x^5 + 3x^2 - 2x = 0$ $[x(2x)^4 + 3x^2 - 2] = 0$ $2x^4 + 3x^2 - 2 = 0$ (ii) $(x^2 + 2)(2x^2 - 1) = 0$ $x = \pm \frac{1}{\sqrt{2}}$ only $\left(\frac{1}{\sqrt{2}}, \frac{2}{\sqrt{2}}\right), \left(-\frac{1}{\sqrt{2}}, -\frac{2}{\sqrt{2}}\right)$	M1 A1 [2] M1 A1 A1 [3]	First line essential AG Factorising needed for A1 Reasonable attempt at solving a quadratic in x^2 For a correct pair of solutions, either 2 x's or 1 x and 1 y SC ($\pm 0.707, \pm 1.41$) AWRT B1
4 (i) $10^2 \sin 0.8 = 71.7$ (ii) sector(s) = $(2) \times \frac{1}{2} \times 10^2 \times 0.8 = (2) \times 40$ Total area = 80 (iii) arc(s) = $(2) \times 10 \times 0.8$ 16+20 = 36	M1A1 [2] M1 A1 [2] M1 A1 [2]	Completely correct method for a triangle Correct formula used for a sector Correct formula used for an arc
5 (i) $3\cos^2 x + 8\cos x + 4 = 0$ $(3\cos x + 2)(\cos x + 2) = 0$ $\cos x = -\frac{2}{3}$ (ii) $\cos(\theta + 70) = -\frac{2}{3}$, $\theta = 61.8$ $\theta + 70 = 131.8$ (or 228.2) $\theta = 158.2$	M1 M1 A1 [3] M1 A1 M1 A1 [4]	Use of $c^2 + s^2 = 1$ Factorising, formula or completing the square needed AG Ignore $\cos x = -2$ also offered SC B1 if $-2/3$ and -2 seen

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6 (i) Scalar product = $15 - 8 + 3$ $10 = \mathbf{OA} \mathbf{OB} \cos \theta$ $\mathbf{OA} = \sqrt{26}$, $\mathbf{OB} = \sqrt{38}$ Angle $BOA = 71.4$ or 71.5 or 1.25 radians (ii) $\mathbf{a} + \frac{1}{2}(\mathbf{b} - \mathbf{a})$ or $\mathbf{b} + \frac{1}{2}(\mathbf{a} - \mathbf{b})$ or $\frac{1}{2}(\mathbf{a} + \mathbf{b})$ $-2\mathbf{b} + \text{their } \mathbf{c}$ oe $-6\mathbf{i} + 5\mathbf{j} + 4\mathbf{k}$	M1 M1 M1 A1 [4] M1 M1 A2,1,0 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct magnitude for either Linking everything correctly cao
7 (i) $y = m(x - 2)$ oe (ii) $x^2 - 4x + 5 = mx - 2m \Rightarrow x^2 - x(4 + m) + 5 + 2m = 0$ $(4 + m)^2 - 4(5 + 2m) = 0 \Rightarrow m^2 - 4 = 0$ $m = \pm 2$ $m = 2 \Rightarrow x^2 - 6x + 9 = 0 \Rightarrow x = 3$ $m = -2 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow x = 1$ $(3, 2), (1, 2)$ OR $m = 2x - 4$ $y = mx - 2m, y = x^2 - 4x + 5$ (iii) $(x - 2)^2 + 1, (2, 1)$	B1 [1] M1 DM1 A1 DM1 A1 A1 [6] M1 M1 M1 A1 A1 A1 B1,B1 [2]	Accept $y = mx + c$, $c = -2m$ Apply $b^2 - 4ac$ Substitute their m and attempt to solve for x Allow for a pair of x values or 1 x and 1 y. Eliminating 2 variables from 3 equations. Obtaining a quadratic in x or y. Solving their quadratic correctly. A pair of x values or 1 x and 1 y.. $m = 2, -2$ also needed for final mark.

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8 (i) $f'(3) = 0 \Rightarrow 18 + 3k - 12 = 0$ $k = -2$ $(x - 3)(x + 2) = 0$ $x = -2$, (Allow also $= 3$) (ii) $f''(x) = 4x - 2$ $f''(3) > 0$ hence min at P $f''(-2) < 0$ hence max at Q (iii) $f(x) = \frac{2}{3}x^3 - x^2 - 12x (+ c)$ Sub $(3, -10) \rightarrow -10 = 18 - 9 - 36 + c$ $c = 17$	M1 A1 M1 A1 [4] B1 B1 [2] B2,1,0 M1 A1 [4]	AG 3 min, -2 max independent of $f''(x)$ Accept anywhere in question Dependent on c present Condone $y =$, or equation $=$
9 (i) $f^{-1}(x) = \frac{1}{2}x - \frac{3}{2}$ $2x + 3 = \frac{1}{2}x - \frac{3}{2} \Rightarrow x = -3$ (ii) 2 lines approximately correct, reflected in $y=x$ & meeting at $(-3, -3)$ (iii) $gf(x) = (2x + 3)^2 - 6(2x + 3)$ $4x^2 - 9$ $4x^2 - 9 \leq 16 \Rightarrow x^2 \leq \frac{25}{4}$ $-\frac{5}{2} \leq x \leq 0$	B1 M1A1 [3] B3,2,1,0 [3] M1 A1 M1 A1A1 [5]	Can be implied by graph or in writing. Ignore lines extended Solving any quadratic to do with f and g ≤ 16, to $x =$ Condone $<$ and $>$

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10 (i) $\int (x+1)^{\frac{1}{2}} - (x+1) \text{ or } \int (y^2 - 1) - (y - 1)$ $\frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{1}{2}x^2 - x \text{ or } \frac{1}{3}y^2 - \frac{1}{2}y^2$ $\frac{2}{3} - \left(0 - \frac{1}{2} + 1\right) \text{ or } \frac{1}{3} - \frac{1}{2}$ $\frac{1}{6}$ (ii) $V_1 = (\pi) \int (y^2 - 1)^2 = (\pi) \int y^4 - 2y^2 + 1$ $(\pi) \left[\frac{y^5}{5} - \frac{2y^2}{3} + y \right]$ $(\pi) \left[\frac{1}{5} - \frac{2}{3} + 1 \right]$ $V_1 = \frac{8}{15(\pi)} \text{ or } 0.533(\pi) \text{ (AWRT)}$ or $(\pi) \left[\frac{y^3}{3} - y^2 + y \right]$ $V_2 = \frac{1}{3}\pi$ Volume = $\frac{8}{15}\pi - \frac{1}{3}\pi = \frac{1}{5}\pi \text{ (or } 0.628)$ OR $(y^4 - 2y^2 + 1) - (y^2 - 2y + 1)$ $(\pi) \int y^4 - 3y^2 + 2y$ $(\pi) [y^{\uparrow 5} / 5 - y^{\uparrow 3} / 3 + y^{\uparrow 2} / 2]$ $(\pi) \left[\frac{1}{5} - 1 + 1 \right]$ $\frac{1}{5}\pi$	M1	Dealing with line as a triangle or integral with correct limits.
	M1A1	Attempt at integral of curve.
	DM1	Applying limits $-1 \rightarrow 0$ or $0 \rightarrow 1$ to curve
	A1	π included loses last mark.
	[5]	
	M1	Attempt at $\int x^2 \text{ dy}$ for curve
	A1	
	DM1	Apply limits $0 \rightarrow 1$
	A1	
	M1	Or $\frac{1}{3} \times \pi (\times 1^2 \times 1)$
	A1	Vol of cone or attempt to $\int x^2 \text{ dy}$ for line
	A1	
	[7]	
	M1	Attempt to $\int x^2 \text{ dy}$
	M1	Attempt to $\int (x_1^2 - x_2^2)$
	A1,A1,A1	
	DM1	Apply limits $0 \rightarrow 1$ dependent on first M1
	A1	

