



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

October/November 2010

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

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Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

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The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



1 Find $\int \left(x + \frac{1}{x}\right)^2 dx$. [3]

2 In the expansion of $(1 + ax)^6$, where a is a constant, the coefficient of x is -30 . Find the coefficient of x^3 . [4]

3 Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 3,$$

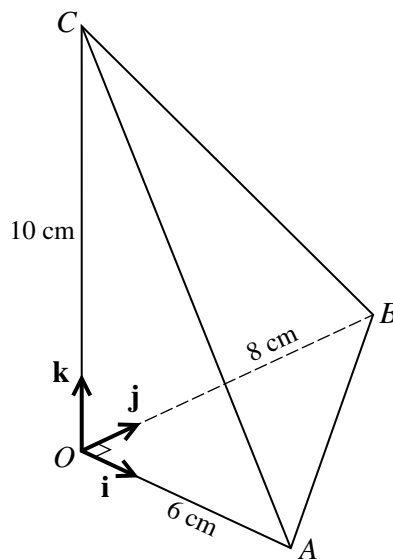
$$g : x \mapsto x^2 - 2x.$$

Express $gf(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [5]

4 (i) Prove the identity $\frac{\sin x \tan x}{1 - \cos x} \equiv 1 + \frac{1}{\cos x}$. [3]

(ii) Hence solve the equation $\frac{\sin x \tan x}{1 - \cos x} + 2 = 0$, for $0^\circ \leq x \leq 360^\circ$. [3]

5



The diagram shows a pyramid $OABC$ with a horizontal base OAB where $OA = 6$ cm, $OB = 8$ cm and angle $AOB = 90^\circ$. The point C is vertically above O and $OC = 10$ cm. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA , OB and OC as shown.

Use a scalar product to find angle ACB . [6]

6 (a) The fifth term of an arithmetic progression is 18 and the sum of the first 5 terms is 75. Find the first term and the common difference. [4]

(b) The first term of a geometric progression is 16 and the fourth term is $\frac{27}{4}$. Find the sum to infinity of the progression. [3]

7 A function f is defined by $f : x \mapsto 3 - 2 \tan\left(\frac{1}{2}x\right)$ for $0 \leq x < \pi$.

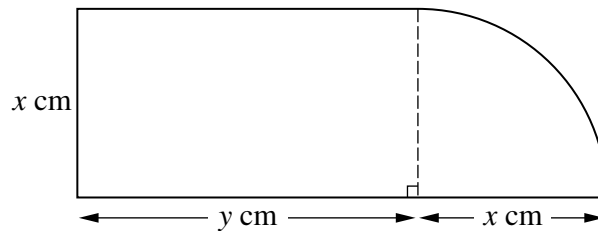
(i) State the range of f . [1]

(ii) State the exact value of $f\left(\frac{2}{3}\pi\right)$. [1]

(iii) Sketch the graph of $y = f(x)$. [2]

(iv) Obtain an expression, in terms of x , for $f^{-1}(x)$. [3]

8



The diagram shows a metal plate consisting of a rectangle with sides x cm and y cm and a quarter-circle of radius x cm. The perimeter of the plate is 60 cm.

(i) Express y in terms of x . [2]

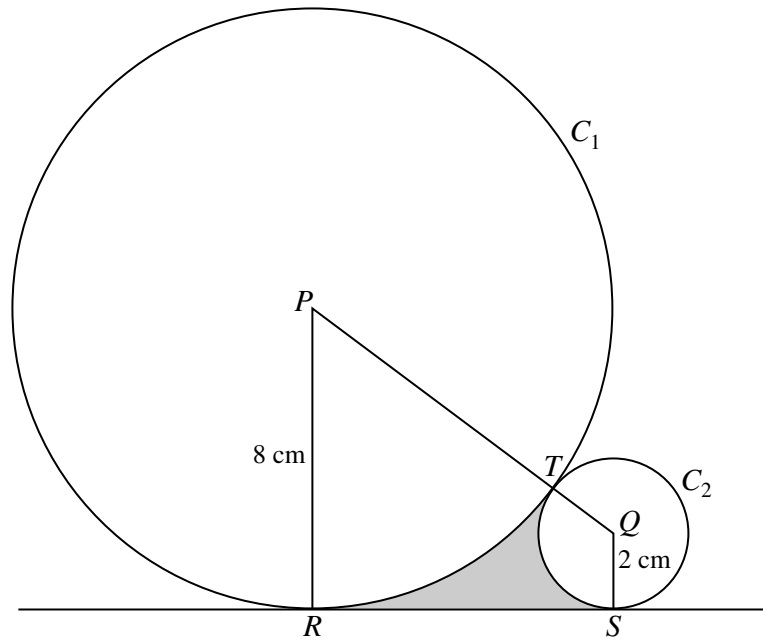
(ii) Show that the area of the plate, A cm², is given by $A = 30x - x^2$. [2]

Given that x can vary,

(iii) find the value of x at which A is stationary, [2]

(iv) find this stationary value of A , and determine whether it is a maximum or a minimum value. [2]

[Questions 9, 10 and 11 are printed on the next page.]



The diagram shows two circles, C_1 and C_2 , touching at the point T . Circle C_1 has centre P and radius 8 cm; circle C_2 has centre Q and radius 2 cm. Points R and S lie on C_1 and C_2 respectively, and RS is a tangent to both circles.

(i) Show that $RS = 8$ cm. [2]

(ii) Find angle RPQ in radians correct to 4 significant figures. [2]

(iii) Find the area of the shaded region. [4]

10 The equation of a curve is $y = 3 + 4x - x^2$.

(i) Show that the equation of the normal to the curve at the point $(3, 6)$ is $2y = x + 9$. [4]

(ii) Given that the normal meets the coordinate axes at points A and B , find the coordinates of the mid-point of AB . [2]

(iii) Find the coordinates of the point at which the normal meets the curve again. [4]

11 The equation of a curve is $y = \frac{9}{2-x}$.

(i) Find an expression for $\frac{dy}{dx}$ and determine, with a reason, whether the curve has any stationary points. [3]

(ii) Find the volume obtained when the region bounded by the curve, the coordinate axes and the line $x = 1$ is rotated through 360° about the x -axis. [4]

(iii) Find the set of values of k for which the line $y = x + k$ intersects the curve at two distinct points. [4]

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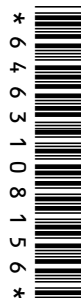
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- 1 (i) Find the first 3 terms in the expansion, in ascending powers of x , of $(1 - 2x^2)^8$. [2]
- (ii) Find the coefficient of x^4 in the expansion of $(2 - x^2)(1 - 2x^2)^8$. [2]

- 2 Prove the identity

$$\tan^2 x - \sin^2 x \equiv \tan^2 x \sin^2 x. \quad [4]$$

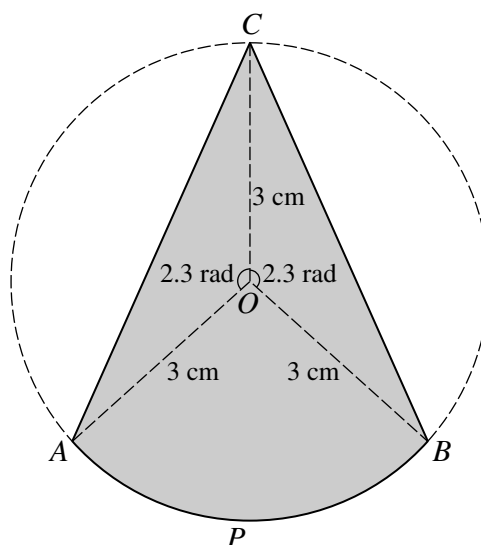
- 3 The length, x metres, of a Green Anaconda snake which is t years old is given approximately by the formula

$$x = 0.7 \sqrt{(2t - 1)},$$

where $1 \leq t \leq 10$. Using this formula, find

- (i) $\frac{dx}{dt}$, [2]
- (ii) the rate of growth of a Green Anaconda snake which is 5 years old. [2]

4



The diagram shows points A, C, B, P on the circumference of a circle with centre O and radius 3 cm. Angle $AOC = \text{angle } BOC = 2.3$ radians.

- (i) Find angle AOB in radians, correct to 4 significant figures. [1]
- (ii) Find the area of the shaded region $ACBP$, correct to 3 significant figures. [4]
- 5 (a) The first and second terms of an arithmetic progression are 161 and 154 respectively. The sum of the first m terms is zero. Find the value of m . [3]
- (b) A geometric progression, in which all the terms are positive, has common ratio r . The sum of the first n terms is less than 90% of the sum to infinity. Show that $r^n > 0.1$. [3]

6 A curve has equation $y = kx^2 + 1$ and a line has equation $y = kx$, where k is a non-zero constant.

(i) Find the set of values of k for which the curve and the line have no common points. [3]

(ii) State the value of k for which the line is a tangent to the curve and, for this case, find the coordinates of the point where the line touches the curve. [4]

7 The function f is defined by

$$f(x) = x^2 - 4x + 7 \text{ for } x > 2.$$

(i) Express $f(x)$ in the form $(x - a)^2 + b$ and hence state the range of f . [3]

(ii) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [3]

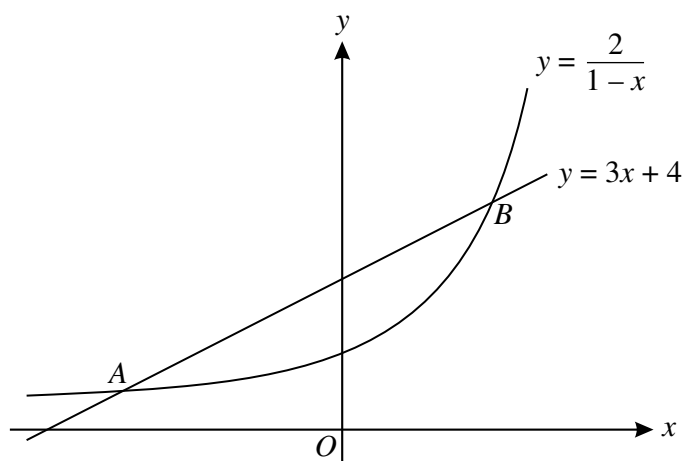
The function g is defined by

$$g(x) = x - 2 \text{ for } x > 2.$$

The function h is such that $f = hg$ and the domain of h is $x > 0$.

(iii) Obtain an expression for $h(x)$. [1]

8

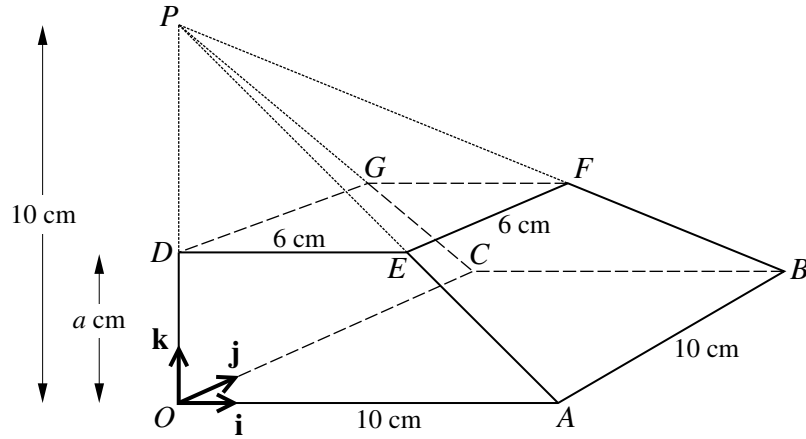


The diagram shows part of the curve $y = \frac{2}{1-x}$ and the line $y = 3x + 4$. The curve and the line meet at points A and B .

(i) Find the coordinates of A and B . [4]

(ii) Find the length of the line AB and the coordinates of the mid-point of AB . [3]

9



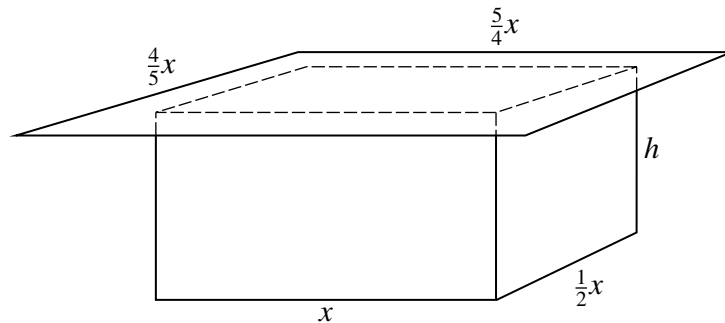
The diagram shows a pyramid $OABCP$ in which the horizontal base $OABC$ is a square of side 10 cm and the vertex P is 10 cm vertically above O . The points D, E, F, G lie on OP, AP, BP, CP respectively and $DEFG$ is a horizontal square of side 6 cm. The height of $DEFG$ above the base is a cm. Unit vectors $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$ are parallel to OA, OC and OD respectively.

(i) Show that $a = 4$. [2]

(ii) Express the vector $\overrightarrow{BG}$ in terms of $\mathbf{i}, \mathbf{j}$ and $\mathbf{k}$. [2]

(iii) Use a scalar product to find angle GBA . [4]

10

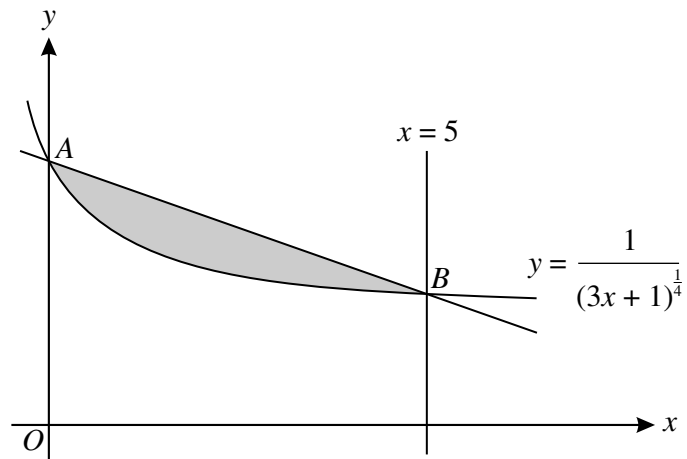


The diagram shows an open rectangular tank of height h metres covered with a lid. The base of the tank has sides of length x metres and $\frac{1}{2}x$ metres and the lid is a rectangle with sides of length $\frac{5}{4}x$ metres and $\frac{4}{5}x$ metres. When full the tank holds 4 m^3 of water. The material from which the tank is made is of negligible thickness. The external surface area of the tank together with the area of the top of the lid is $A \text{ m}^2$.

(i) Express h in terms of x and hence show that $A = \frac{3}{2}x^2 + \frac{24}{x}$. [5]

(ii) Given that x can vary, find the value of x for which A is a minimum, showing clearly that A is a minimum and not a maximum. [5]

11



The diagram shows part of the curve $y = \frac{1}{(3x+1)^{\frac{1}{4}}}$. The curve cuts the y -axis at A and the line $x = 5$ at B .

- (i) Show that the equation of the line AB is $y = -\frac{1}{10}x + 1$. [4]
- (ii) Find the volume obtained when the shaded region is rotated through 360° about the x -axis. [9]

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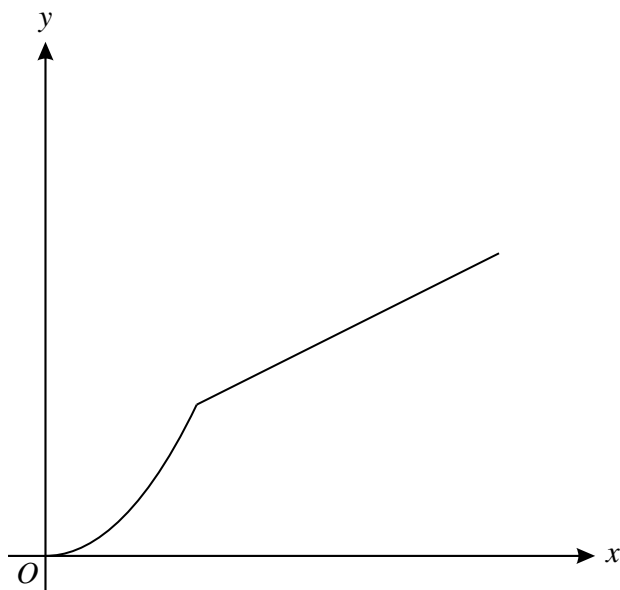
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- 1** Find the term independent of x in the expansion of $\left(x - \frac{1}{x^2}\right)^9$. [3]
- 2** Points A , B and C have coordinates $(2, 5)$, $(5, -1)$ and $(8, 6)$ respectively.
- (i) Find the coordinates of the mid-point of AB . [1]
- (ii) Find the equation of the line through C perpendicular to AB . Give your answer in the form $ax + by + c = 0$. [3]
- 3** Solve the equation $15 \sin^2 x = 13 + \cos x$ for $0^\circ \leq x \leq 180^\circ$. [4]
- 4** (i) Sketch the curve $y = 2 \sin x$ for $0 \leq x \leq 2\pi$. [1]
- (ii) By adding a suitable straight line to your sketch, determine the number of real roots of the equation
- $$2\pi \sin x = \pi - x.$$
- State the equation of the straight line. [3]
- 5** A curve has equation $y = \frac{1}{x-3} + x$.
- (i) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [2]
- (ii) Find the coordinates of the maximum point A and the minimum point B on the curve. [5]
- 6** A curve has equation $y = f(x)$. It is given that $f'(x) = 3x^2 + 2x - 5$.
- (i) Find the set of values of x for which f is an increasing function. [3]
- (ii) Given that the curve passes through $(1, 3)$, find $f(x)$. [4]

7

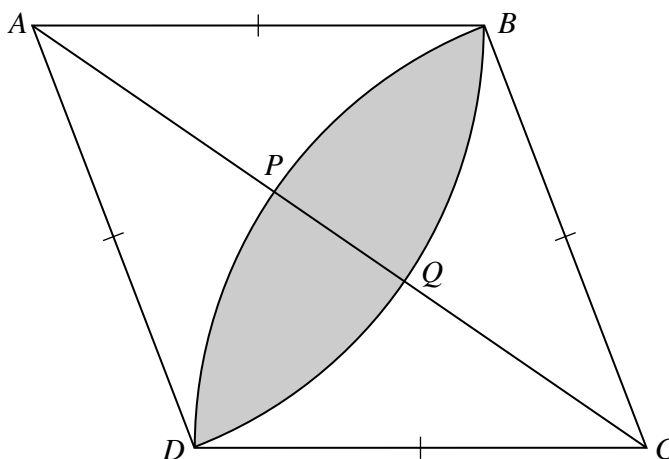


The diagram shows the function f defined for $0 \leq x \leq 6$ by

$$\begin{aligned} x &\mapsto \frac{1}{2}x^2 && \text{for } 0 \leq x \leq 2, \\ x &\mapsto \frac{1}{2}x + 1 && \text{for } 2 < x \leq 6. \end{aligned}$$

- (i) State the range of f . [1]
- (ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]
- (iii) Obtain expressions to define $f^{-1}(x)$, giving the set of values of x for which each expression is valid. [4]

8

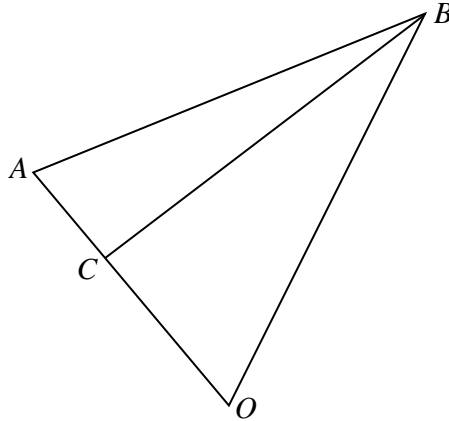


The diagram shows a rhombus $ABCD$. Points P and Q lie on the diagonal AC such that BPD is an arc of a circle with centre C and BQD is an arc of a circle with centre A . Each side of the rhombus has length 5 cm and angle $BAD = 1.2$ radians.

- (i) Find the area of the shaded region $BPDQ$. [4]
- (ii) Find the length of PQ . [4]

- 9 (a) A geometric progression has first term 100 and sum to infinity 2000. Find the second term. [3]
- (b) An arithmetic progression has third term 90 and fifth term 80.
- (i) Find the first term and the common difference. [2]
- (ii) Find the value of m given that the sum of the first m terms is equal to the sum of the first $(m + 1)$ terms. [2]
- (iii) Find the value of n given that the sum of the first n terms is zero. [2]

10



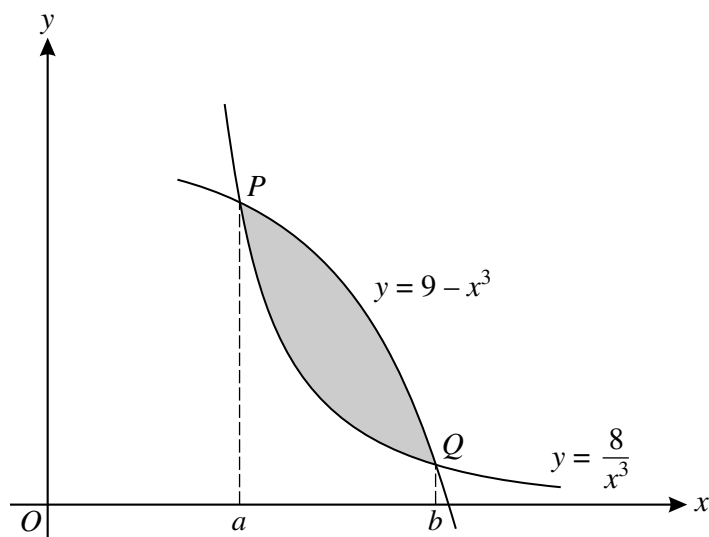
The diagram shows triangle OAB , in which the position vectors of A and B with respect to O are given by

$$\overrightarrow{OA} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = -3\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}.$$

C is a point on OA such that $\overrightarrow{OC} = p\overrightarrow{OA}$, where p is a constant.

- (i) Find angle AOB . [4]
- (ii) Find $\overrightarrow{BC}$ in terms of p and vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]
- (iii) Find the value of p given that BC is perpendicular to OA . [4]

11



The diagram shows parts of the curves $y = 9 - x^3$ and $y = \frac{8}{x^3}$ and their points of intersection P and Q . The x -coordinates of P and Q are a and b respectively.

- (i) Show that $x = a$ and $x = b$ are roots of the equation $x^6 - 9x^3 + 8 = 0$. Solve this equation and hence state the value of a and the value of b . [4]
- (ii) Find the area of the shaded region between the two curves. [5]
- (iii) The tangents to the two curves at $x = c$ (where $a < c < b$) are parallel to each other. Find the value of c . [4]

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