

**MARK SCHEME for the October/November 2010 question paper
for the guidance of teachers**

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through √” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
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1	$\int \left(x + \frac{1}{x}\right)^2 dx$ $= \frac{x^3}{3} - \frac{1}{x} + 2x + (c)$	B1 $\times$ 3	[3]	co. Omission of middle term of expansion can still get 2/3.
2	$(1 + ax)^6$ Term in $x = 6ax$ Equate with $-30 \rightarrow a = -5$ Term in $x^3 = \frac{6.5.4}{3!} a^3$ $\rightarrow$ coefficient of -2500	B1 B1 $\checkmark$ B1 B1 $\checkmark$	[4]	co $\checkmark$ from his answer for $6ax$ co For $20 \times a^3$
3	$f: x \mapsto 2x + 3,$ $g: x \mapsto x^2 - 2x,$ $gf(x) = (2x + 3)^2 - 2(2x + 3)$ $= 4x^2 + 8x + 3$ $= 4(x + 1)^2 - 1$	M1 A1 $3 \times$ B1 $\checkmark$	[5]	Must be f into g, not g into f. co Allow all these as $\checkmark$ for either fg or gf.
4	(i) $\frac{\sin x \tan x}{1 - \cos x} = \frac{\sin^2 x}{\cos x(1 - \cos x)}$ $= \frac{1 - \cos^2 x}{\cos x(1 - \cos x)}$ $= \frac{(1 - \cos x)(1 + \cos x)}{\cos x(1 - \cos x)} = \frac{1}{\cos x} + 1$ (ii) $\frac{1}{\cos x} + 1 + 2 = 0$ $\rightarrow \cos x = -\frac{1}{3}$ $\rightarrow x = 109.5^\circ$ or 250.5°	M1 M1 M1 M1 A1 A1 $\checkmark$	[3] [3]	Use of $\tan x = \sin x \div \cos x$ Use of $\sin^2 x = 1 - \cos^2 x$ Realising the need to use difference of 2 squares. Answer given. Uses part (i) with $\cos x$ as subject. co. $\checkmark$ for $360^\circ - 1^{\text{st}}$ answer.
5	$\overrightarrow{AC} = -6\mathbf{i} + 10\mathbf{k}$ $\overrightarrow{BC} = -8\mathbf{j} + 10\mathbf{k}$ $\overrightarrow{AC} \cdot \overrightarrow{BC} = 100$ $\overrightarrow{AC} \cdot \overrightarrow{BC} = \sqrt{136}\sqrt{164} \cos ACB$ Angle $ACB = 48.0^\circ$	B1 B1 M1 M1 M1 A1	[6]	co (or $\overrightarrow{CA}$) co (or $\overrightarrow{CB}$) Must be scalar – available for any pair For modulus – available for any vector All linked correctly – for ACB only co

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6	(a) $a + 4d = 18$ $\frac{5}{2}(2a + 4d) = 75$ Solution $\rightarrow a = 12, d = 1\frac{1}{2}$	B1 B1 M1 A1	[4]	co or $75 = 5/2(a + 18) \rightarrow a = 12$ etc co Solution of sim equations co for both
	(b) $a = 16$ and $ar^3 = \frac{27}{4}$ $r = \frac{3}{4}$ Sum to infinity = 64	B1 M1 A1		Needs both of these Correct formula and $ r < 1$
7	$x \mapsto 3 - 2 \tan(\frac{1}{2}x)$ (i) Range of $f \leq 3$	B1	[1]	co. Allow $<$
	(ii) $f(\frac{2}{3}\pi) = 3 - 2\sqrt{3}$ (iii)	B1 B2, 1, 0 Indep.		co Starting at $y = 3$ Shape correct – no turning points. Tending tangentially towards $x = \pi$
	(iv) $y = 3 - 2 \tan\left(\frac{x}{2}\right)$ $\rightarrow f^{-1}(x) = 2 \tan^{-1}\left(\frac{3-x}{2}\right)$	M1 M1 A1	[2] [3]	Attempt at making x the subject. Order of operations all ok. co – but with x , not y .
8	(i) $2x + 2y + \frac{\pi x}{2} = 60$ $\rightarrow y = 30 - x - \frac{\pi x}{4}$	M1 A1	[2]	Linking 60 with sum of at least 4 sides and use of radians co
	(ii) $A = xy + \frac{\pi x^2}{4}$ $= x(30 - x - \frac{\pi x}{4}) + \frac{\pi x^2}{4}$ $= 30x - x^2$	M1 A1		Subs “y” into area eqn and use $\frac{1}{2}r^2\theta$ co.
	(iii) $\frac{dA}{dx} = 30 - 2x$ $= 0$ when $x = 15$ cm	M1 A1	[2]	Knowing to differentiate Sets differential to 0 + solution. co.
	(iv) Max.	M1 A1	[2]	Any valid method. co.

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9	(i) $RS^2 = 10^2 - 6^2$ $\rightarrow RS = 8 \text{ cm.}$	M1 A1	[2]	Use of Pythagoras (or other) Answer given.
	(ii) $\sin \theta = 8/10$ oe $\rightarrow \text{angle } RPQ = 0.9273 \text{ radians}$	M1 A1	[2]	Use of trig – even if with degrees. co in radians. (Accept 0.927)
	(iii) Region = trapezium – 2 sectors Area of trapezium = 40 cm^2 Large sector = $\frac{1}{2} \times 8^2 \times 0.9273$ Small sector angle = $(\pi - 0.9273)$ Small sector = $\frac{1}{2} \times 2^2 \times 2.214$ $\rightarrow 5.90 \text{ cm}^2$	B1 M1 M1 A1	 [4]	co Use of $\frac{1}{2}r^2\theta$. Use of $\frac{1}{2}r^2\theta$ with angle = $\pi - \text{(ii)}$ co
10	$y = 4x - x^2 + 3$			
	(i) $\frac{dy}{dx} = 4 - 2x$ At $x = 3$, $m = -2$ Gradient of normal = $\frac{1}{2}$ Eqn of normal $y - 6 = \frac{1}{2}(x - 3)$ $\rightarrow 2y = x + 9$	B1 M1 M1 A1	 [4]	co Use of $m_1m_2 = -1$ Use of $y - k = m(x - h)$ or $y = mx + c$ (where m is gradient of normal)
	(ii) Meets axes at $(0, \frac{9}{2})$ and $(-9, 0)$ Mid-point is $(\frac{-9}{2}, \frac{9}{4})$	M1 A1	 [2]	Sets x and y to 0 + midpoint formula. co.
	(iii) $2y = x + 9$, $y = 4x - x^2 + 3$ $\rightarrow 2x^2 - 7x + 3 = 0$ oe $\rightarrow (\frac{1}{2}, \frac{3}{4})$	M1 A1 M1 A1	 [4]	Eliminates x completely. Correct eqn. Solution of quadratic. co

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11	$y = \frac{9}{2-x}$ (i) $\frac{dy}{dx} = -9(2-x)^{-2} \times -1$ $\frac{9}{(2-x)^2} \neq 0$. No turning points.	B1 B1 B1√	[3]	Without the “ $\times -1$ ” Indep. With the “ $\times -1$ ”. Indep. √ provided of form $k \div (2-x)^2$.
	(ii) $V = \pi \int \frac{81}{(2-x)^2} dx$ $\int y^2 dx = -81(2-x)^{-1} \div (-1)$ Use of limits 0 to 1 $\rightarrow \frac{81\pi}{2}$ (or 127)	B1 B1 M1 A1		Answer without the “ $\div -1$ including π For “ $\div -1$ ”. Uses both limits in an integral of y^2 – if “0” ignored, M0. co (If π omitted – max 3/4)
	(iii) $\frac{9}{2-x} = x + k$ $\rightarrow x^2 - 2x + kx - 2k + 9 = 0$ Uses $b^2 - 4ac$ $\rightarrow k^2 + 4k - 32$ $\rightarrow$ end-points of 4 and -8 Range for 2 points of intersection $\rightarrow k < -8, k > 4$.	M1 M1 A1 A1	[4]	Elimination of y Uses discriminant End-values correct. Accept $\leq, \geq$.

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1	(i) $1 + 8(-2x^2) + {}^8C_2(-2x^2)^2$ $1 - 16x^2 + 112x^4$	B2, 1	[2]	Loses 1 for each error
	(ii) $(2 - x^2) \times \text{their } (1 - 16x^2 + 112x^4)$ $(2 \times \text{their } 112) - \text{their } (-16)$ 240	M1 A1✓		Must consider exactly 2 terms
2	LHS = $\sin^2 x / \cos^2 x - \sin^2 x$ $\sin^2 x (1 - \cos^2 x) / \cos^2 x$ $\frac{\sin^2 x \sin^2 x}{\cos^2 x}$ oe $\tan^2 x \sin^2 x$ OR RHS = $\frac{\sin^2 x}{\cos^2 x} \cdot \sin^2 x$ $\sin^2 x (1 - \cos^2 x) / \cos^2 x$ $(\sin^2 x / \cos^2 x) - \sin^2 x$ $\tan^2 x - \sin^2 x$	M1 M1	[4]	Replace t^2 by s^2/c^2 or $\sec^2 - 1$ Use of $1 - \cos^2 x = \sin^2 x$
		M1 A1		Valid overall method AG
3	(i) $(k(2t - 1))^{-1/2}$ $0.7(2t - 1)^{-1/2}$	M1 A1	[2]	$k \neq 1$ oe
	(ii) Sub $t = 5$ into <i>their</i> deriv 0.23(3)	M1 A1		Ignore units
4	(i) 1.683(18...)	B1	[1]	Condone omission of factor 2 NB M0 if using angle of 2.3 Two correct triangles + sector co
	(ii) $(2) \times \frac{1}{2} \times 3^2 \sin 2.3$ $\frac{1}{2} \times 3^2 \times \text{their } 1.683$ Triangle $AOC + COB + \text{sector}$ 14.3	M1 M1 M1 A1		
5	(a) $d = -7$ used $(m/2)[322 + (m - 1)(-7)] = 0$ 47	B1 M1 A1	[3]	co Condone omission of $(m/2)$. Statement co (condone $m = 0$)
	(b) $\frac{a(1 - r^n)}{1 - r} < \frac{0.9a}{1 - r}$ $1 - r^n < 0.9$ $r^n > 0.1$	M1 M1 A1		Allow for $=, <, >, \leq, \geq$ Needs inequality sign correct co

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6	(i) $kx^2 - kx + 1 = 0$ $k^2 - 4k < 0$ $0 < k < 4$	M1 M1 A1	[3]	y eliminated Applying $b^2 - 4ac < 0$ or = or $\leq$ or $\geq$ co
	(ii) $k = 4$ only $(2x - 1)^2 = 0$ $x = \frac{1}{2}, y = 2$ or $(\frac{1}{2}, 2)$	B1✓ M1 A1, A1	[4]	ft from <i>their</i> $k^2 - 4k = 0$. (Not $k = 0$) ft from <i>their</i> k
7	(i) $(x - 2)^2$ $(x - 2)^2 + 3$ $f(x) > 3$	M1 A1 B1✓	[3]	Must be “-2” $\pm k$ co ft on <i>their</i> ‘3’
	(ii) $x - 2 = (\pm)\sqrt{y - 3}$ $f^{-1}(x) = 2 + \sqrt{x - 3}$ domain is $x > 3$	M1 A1 B1✓	[3]	$\pm$ not required for M mark f(x) + removal of minus sign needed ft domain of f^{-1} = range of f or for f^{-1}
	(iii) $h(x) = x^2 + 3$	B1	[1]	co
8	(i) $3x^2 + x - 2 = 0$ $(x + 1)(3x - 2) \rightarrow x = -1$ or $\frac{2}{3}$ $(-1, 1), (\frac{2}{3}, 6)$	M1A1 M1 A1	[4]	Eliminates x or y . Sets quadratic to 0. Attempt to solve <i>their</i> equation co
	(ii) $AB^2 = (5/3)^2 + 5^2$ $AB = 5.27(0\dots)$ mid-point = $(-1/6, 7/2)$	M1 A1 B1✓	[3]	✓ <i>their</i> coordinates from (i) Or $(5\sqrt{10})/3$ oe ft from <i>their</i> (i)
9	(i) $\frac{10 - a}{10} = \frac{6}{10}$ oe $a = 4$	M1 A1	[2]	or PDE is isos hence $PD = 6$ (M1) AG
	(ii) $\overrightarrow{BG} = -10\mathbf{j} - 10\mathbf{i} + 4\mathbf{k} + 6\mathbf{j}$ $= -10\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$	B2,1	[2]	Any acceptable notation. Loses 1 for each error.
	(iii) $\overrightarrow{BG} \cdot \overrightarrow{BA} = 40$ $\cos GBA = \frac{40}{\sqrt{132}\sqrt{100}}$ $GBA = 69.6^\circ$	M1 M1 DM1 A1	[4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Modulus worked correctly for either All ok – must be using $\pm \overrightarrow{BG} \cdot \pm \overrightarrow{AB}$. Must be the acute angle

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10	(i) $h = \frac{8}{x^2}$ $A = \frac{1}{2}x^2 + 2 \times \frac{1}{2}xh + 2xh + \frac{5}{4}x \times \frac{4}{5}x$ $A = (3/2)x^2 + 3xh$ $A = \frac{3}{2}x^2 + 3x \times \frac{8}{x^2}$ $A = \frac{3}{2}x^2 + \frac{24}{x}$ (ii) $\frac{dA}{dx} = 3x - \frac{24}{x^2} = 0$ $x = 2$ $\frac{d^2A}{dx^2} = 3 + \frac{48}{x^3}$ > 0 when $x = 2$ hence minimum	M1 A1 M1 M1 A1 B1 M1 A1 M1 A1	[5] [5]	Uses $lbh = 4$ co Allow 1 error but needs the lid For substitution of h as $f(x)$ AG Correct derivative. Sets to 0 and attempts to solve. co Reasonable attempt – allow 1 error co AG (Result consistent with their f')
	(i) $A = (0, 1)$ $B = (5, \frac{1}{2})$ $y - 1 = -\frac{1}{10}(x - 0)$ $y = -\frac{1}{10}x + 1$ (ii) Curve: $(\pi) \int_0^5 (3x + 1)^{-1/2} dx$ $\frac{2\pi}{3}[(3x + 1)^{1/2}]_0^5$ $\frac{2\pi}{3}[4 - 1]$ [2π] Line: $(\pi) \int_0^5 (\frac{1}{100}x^2 - \frac{1}{5}x + 1) dx$ $(\pi)[\frac{1}{300}x^3 - \frac{1}{10}x^2 + x]_0^5$ $(\pi)[\frac{125}{300} - \frac{25}{10} + 5]$ [$\frac{35\pi}{12}$] Volume = $\frac{35\pi}{12} - 2\pi = \frac{11\pi}{12}$	B1 B1 M1 A1 M1 A1A1 DM1 M1 A2,1 DM1 A1	[4] [9]	fit <i>their</i> A, B AG Attempt $\int_0^5 y^2 dx$ (π not vital) (π not vital). 2nd A mark is for $\div 3$. Application of limits to <i>their</i> integral (in either integral). Limits 0 to 5 only. Attempt $\int_0^5 y^2 dx$ (π not vital) Also directly $-\frac{10}{3}(-\frac{1}{10}x + 1)^3$ or $-\frac{10}{3}\left[(-\frac{1}{2} + 1)^3 - 1^3\right]$ (π not vital) – applying limits to <i>their</i> integral Subtraction of <i>their</i> volumes co

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AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	${}^9C_6$ or ${}^9C_3$ used $\left(\frac{1}{x^2}\right)^3$ seen -84	M1 B1 A1	[3]	Correct answer only $\Rightarrow$ 3marks
2	(i) $(3\frac{1}{2}, 2)$ (ii) $m = \frac{-1-5}{5-2} = -2$ $y-6 = \frac{-1}{m}(x-8)$ $x-2y+4=0$	B1 B1 M1 A1	[1] [3]	Use of $m_1m_2 = -1$ and $y-k = m(x-h)$ Accept any form
3	$15\cos^2x + \cos x - 2 = 0$ $(5\cos x + 2)(3\cos x - 1) = 0$ 113(.6), 70.5	M1 M1 A1A1	[4]	$1 - \cos^2x = \sin^2x$ & attempt simplify Attempt to solve 3-term quadratic for $\cos x$ SC 1.98, 1.23 scores 1/2
4	(i) Correct sine curve (ii) Required line $y = 1 - \frac{x}{\pi}$ Line through $(0, 1), (\pi, 0)$ drawn 3 roots	B1 B1 B1 B1✓	[1] [3]	2 shown or implied SC B1 for correct graphs without 1 or 2 marked fit on trig curve and line
5	(i) $\frac{dy}{dx} = \frac{-1}{(x-3)^2} + 1$ $\frac{d^2y}{dx^2} = \frac{2}{(x-3)^3}$ (ii) $(x-3)^2 = 1 \Rightarrow x-3 = \pm 1$ $x = 4, 2$ $y = 5, 1$ When $x = 4$ $\frac{d^2y}{dx^2} > 0 (=2) \Rightarrow \text{min}$ When $x = 2$ $\frac{d^2y}{dx^2} < 0 (= -2) \Rightarrow \text{max}$	B1 B1 M1 A1 A1 M1 A1	[2] [5]	oe oe Set $\frac{dy}{dx} = 0$ & reasonable attempt to solve Investigate signs of f'' at a point or other method

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6	(i) $(3x + 5)(x - 1)(> 0)$ $-5/3, 1$ $x < -5/3, x > 1$	M1 A1 A1	[3]	Attempt at factorisation Both required Ignore any words between answers Condone < >
	(ii) $f(x) = x^3 + x^2 - 5x (+ c)$ $3 = 1 + 1 - 5 + c$ $f(x) = x^3 + x^2 - 5x + 6$	M1 A1 M1 A1		Attempt at integration Any unsimplified expression ok Sub. (1, 3) Accept $c = 6$
7	(i) Range is $0 < f(x) < 4$, 0 to 4	B1	[1]	Accept in two parts. Condone <
	(ii) $y = x$ drawn or implied Correct sketch of f^{-1}	B1 B1	[2]	SC if f missing, (2, 2) (4, 6) must be shown
	(iii) $(x \mapsto) \sqrt{2x}$ for $0 < x < 2$ $(x \mapsto) 2x - 2$ for $2 < x < 4$	B1B1 B1B1	[4]	Condone < <
8	(i) $1/2 \times 5^2 \times 1.2$ $1/2 \times 5^2 \times \sin 1.2$ $2[1/2 \times 5^2 \times 1.2 - 1/2 \times 5^2 \times \sin 1.2]$ 6.70	B1 B1 M1 A1	[4]	Subtraction and multiplication by 2 Accept 6.7 or anything rounding to 6.70
	(ii) $5\cos 0.6$ $5 - "5\cos 0.6"$ $10(1 - \cos 0.6)$ 1.75	M1 M1 M1 A1		Subtraction from 5 Multiplication by 2
9	(a) $\frac{100}{1-r} = 2000$ $r = 19/20$ $ar = 95$	M1 A1 A1✓	[3]	Correct formula and attempt to solve For $100 \times r$
	(b) (i) $a + 2d = 90, a + 4d = 80$ $d = -5, a = 100$	B1B1	[2]	
	(ii) $a + md = 0$ $m = 20$	M1 A1	[2]	Or use correct sum formula $m = 20$ with no working scores 2
	(iii) $\frac{n}{2}[200 + (n-1)(-5)] = 0$ $n = 41$	M1 A1	[2]	$n = 41$ with no working scores 2 Do not penalise $n = 0$

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10	(i) $\overrightarrow{OA} \cdot \overrightarrow{OB} = -6 + 2 + 12 = 8$ $\cos AOB = \frac{8}{\sqrt{14}\sqrt{29}}$ $AOB = 66.6^\circ$	M1 M1 M1 A1	[4] [1]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Mod worked correctly for either one Division of “8” by product of mods
	(ii) $3\mathbf{i} - 2\mathbf{j} + 4\mathbf{k} + p(2\mathbf{i} + \mathbf{j} - 3\mathbf{k})$ (iii) $\overrightarrow{BC} = \mathbf{i}(3 + 2p) + \mathbf{j}(-2 + p) + \mathbf{k}(4 - 3p)$ Their $\overrightarrow{BC} \cdot [2\mathbf{i} + \mathbf{j} - 3\mathbf{k}] = 0$ $2(3 + 2p) + (p - 2) - 3(4 - 3p) = 0$ $p = 4/7 \quad 0.571$	B1 M1 M1 A1√ A1		In any unsimplified form Scalar product = 0 used ft from their BC cao
11	(i) $9 - x^3 = \frac{8}{x^3}$ $x^6 - 9x^3 + 8 = 0$ $(X - 1)(X - 8) = 0 \rightarrow X = 1 \text{ or } 8$ $a = 1, b = 2$	M1 A1 M1 A1	[4] [5]	Together with attempt to mult by x^3 AG completely correct working Attempt to solve quadratic in X or x^3
	(ii) $\int_1^2 \left[(9 - x^3) - \frac{8}{x^3} \right] dx$ $\left[9x - \frac{x^4}{4} \right] \cdot \left[\frac{-4}{x^2} \right]$ $18 - 4 + 1 - (9 - \frac{1}{4} + 4)$ $2\frac{1}{4}$ (iii) $\frac{dy}{dx} = \frac{-24}{x^4}, \frac{dy}{dx} = -3x^2$ $\frac{-24}{c^4} = -3c^2$ $c^6 = 8$ $c = \sqrt{2} \text{ or } 8^{1/6} \text{ or } 1.41(4\dots)$	M1 B1 B1 M1 A1 B1, B1 M1 A1		Intention to integrate the difference $y_1 - y_2$ not $\pi(y_1 - y_2)$ Correct use of their limits once cao Equating and solution Accept x or c