

**MARK SCHEME for the October/November 2009 question paper
for the guidance of teachers**

9709 MATHEMATICS

9709/11

Paper 11, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 $3 \tan(2x + 15^\circ) = 4$ $\tan(2x + 15^\circ) = 1\frac{1}{3}$ Sets the bracket to $\tan^{-1}(1\frac{1}{3})$ $2x + 15 = 53.13^\circ$ or 233.13° $\rightarrow x = 19.1^\circ$ or 109.1°	M1 M1 A1 A1√ [4]	Removes the “3” first by division. Looks up $\tan^{-1}1\frac{1}{3}$, then uses bracket co. √ for $(90 + 1^{\text{st}} \text{ answer})$ and no other answers in the range.
2	B1 B1 B1 B1 [4]	1 complete oscillation $0 \rightarrow \pi$ Range from -3 to 3 All correct (V shape B0) Line correct.
3 (i) $(2 - x)^6$ $64 - 192x + 240x^2$ (ii) $(1 + 2x + ax^2)(2 - x)^6$ Coeff of $x^2 = 240 - 384 + 64a$ Equates to 48 $\rightarrow a = 3$	$3 \times B1$ [3] M1 M1 A1 [3]	co Allow 2^6 . Considers at least 2 terms in x^2 . Considers exactly 3 terms + solution co
4 $y = x^4 + 4x + 9$ (i) Differential $= 4x^3 + 4$ Sets to 0 + solution $\rightarrow (-1, 6)$ 2 nd differential $= 12x^2$ Positive, $\rightarrow$ Minimum (ii) $A = \left[\frac{x^5}{5} + 2x^2 + 9x \right]$ Limits from 0 to 1 $\rightarrow 11.2$	B1 M1 A1 B1 [4] B1 M1 A1 [3]	co Differentiates and sets to 0. co. Statement only. co. Value at “1” – value at “0” in integral of y .
5 $r = 6 \text{ cm}$ (i) $AB = \sqrt{6^2 + 6^2} = \sqrt{72}$ Angle $BAD = \frac{1}{4}\pi$ or 45° Arc length $= \sqrt{72} \times \frac{1}{4} \pi = 6.66(7)$ (ii) Sector area $= \frac{1}{2}r^2\theta = \frac{1}{2} \times 72 \times \frac{1}{4}\pi$ Area of triangle $= \frac{1}{2} \times 6 \times 6$ Shaded area $= 10.3$ or $9\pi - 18$.	B1 B1 M1 A1 [4] M1 B1 A1 [3]	Use of Pythagoras – or trig (8.5 ok) In degrees or radians Use of $s=r\theta$ with θ in rads only – or correct with degrees. Use of $r = 6$ M0. Use of $\frac{1}{2}r^2\theta$ with θ in rad, and $r \neq 6$. co co

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6 $\frac{dy}{dx} = k - 2x$ (i) At $x = 2$, $m = (k - 4)$ $x = 3$ $m = (k - 6)$ $(k - 4)(k - 6) = -1$ $\rightarrow k = 5$ (ii) $y = kx - x^2 (+ c)$ Substitutes (4, 9) $\rightarrow c = 5$	M1 M1 DM1A1 [4] B1√ M1 A1 [3]	Obtains either gradient as $f(k)$. Uses $m_1m_2 = -1$ with gradients $f(k)$ Soln of quadratic = 0. co (watch for fortuitous answers) For integration without c Realises need to substitute for x and y co (nb If $k = 5$ is fortuitous, loses last A1)
7 $y = \frac{12}{x^2 + 3}$ (i) $\frac{dy}{dx} = -12(x^2 + 3)^{-2} \times 2x$ (ii) At $x = 1$, $m = -\frac{3}{2}$ m of normal = $\frac{2}{3}$ Eqn of normal $y - 3 = \frac{2}{3}(x - 1)$ (iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} = -\frac{3}{2} \times 0.012$ $\rightarrow -0.018$	B1 B1 [2] M1 M1 A1 [3] M1 A1√ [2]	Without the “$\times 2x$”. For “$\times 2x$”. Accept unsimplified answer Uses $m_1m_2 = -1$ algebraic ok. Correct form of equation. co unsimplified Correct link between differentials co to his $\frac{dy}{dx}$. (Omission of x in part (i) causes fortuitous results in (ii) and (iii).)
8 (i) $8 + 4d = 8r$ $8 + 7d = 8r^2$ Eliminates one of the variables $\rightarrow 4r^2 - 7r + 3 = 0$ Solution $\rightarrow r = \frac{3}{4} \rightarrow d = -\frac{1}{2}$ (ii) $S_\infty = \frac{a}{1-r} \rightarrow 32$ (iii) $S_8 = 4(16 + 7d)$ $= 50$	B1 B1 M1 DM1 A1 A1 [6] M1 A1 [2] M1 A1 [2]	co – but allow if a in place of 8. co – but allow if a in place of 8. Complete elimination of either r or d. Correct method of solution. nb answer for r given. co (assumes $r = \frac{3}{4}$, give B1B1 for equations, B1 for d) Correct formula used. Correct formula used. $64 + 28d$ ok co

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9 $\vec{OA} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}, \vec{OB} = \begin{pmatrix} 0 \\ -6 \\ 8 \end{pmatrix}, \vec{OC} = \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix}.$ (i) Scalar product = $-18 - 48$ $-66 = \mathbf{a} \mathbf{b} \cos \theta$ $\mathbf{a} = 7$ and $\mathbf{b} = 10$ $\rightarrow$ Angle $AOB = 160.5^\circ$ (ii) $\vec{AC} = \mathbf{c} - \mathbf{a} = \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ Modulus = 6 Vector = $5 \times \begin{pmatrix} -4 \\ 2 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} -20 \\ 10 \\ 20 \end{pmatrix}$ (iii) $\begin{pmatrix} 2 \\ 3-6p \\ -6+8p \end{pmatrix} \cdot \begin{pmatrix} -2 \\ 5 \\ -2 \end{pmatrix} = 0$ $\rightarrow p = \frac{1}{2}$	M1 M1 M1 A1 [4] B1 M1 A1 [3] B1 M1 A1 [3]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Linking everything correctly Correct modulus of either a or b. co co. allow $\pm$. For modulus and multiplying by “5” co For $\vec{OA} + p\vec{OB}$ as single vector. Scalar product = 0. Co (beware fortuitous answers)
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10 $f: x \mapsto 2x + 1, x \in \mathbb{R}, x > 0$ $g: x \mapsto \frac{2x-1}{x+3}, x \in \mathbb{R}, x \neq -3.$ (i) $gf(x) = \frac{2(2x+1)-1}{2x+1+3} = \frac{4x+1}{2x+4}$ Equates to $x \rightarrow 2x^2 = 1$ $\rightarrow x = \frac{1}{2}\sqrt{2}$ (ii) $f^{-1}(x) = \frac{1}{2}(x-1)$ To find $g^{-1}(x)$, make x the subject Order must be correct $\rightarrow g^{-1}(x) = \frac{-1-3x}{x-2}$ or $\frac{1+3x}{2-x}$ (iii) $\frac{1+3x}{2-x} = x \rightarrow x^2 + x + 1 = 0$ Looks at $b^2 - 4ac$ $\rightarrow$ negative $\rightarrow$ no roots. (iv)	M1 M1 A1 [3] B1 M1 M1 A1 [4] M1 M1 A1 [3] B1 B1 B1 [3]	Must be gf, needs x replacing twice. Forms quadratic + solution Co. condone $\pm$. Co Attempt at x as the subject. Order correct. Allow for sign errors. Co – must be $f(x)$. Forms quadratic equation. Looks at discriminant or attempts to solve and finds $\sqrt{(\text{negative})}$. Co Correct $y = 2x + 1$ on graph from (0, 1) Correct $y = \frac{1}{2}(x - 1)$ on graph from (1, 0) (if –ve x plotted, B1 s.c. for both) Shows or states or implies that f, f^{-1} are reflections in $y = x$.
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5 (i) $(\sin x + \cos x)(1 - \sin x \cos x)$ $= \sin x + \cos x - \sin^2 x \cos x - \cos^2 x \sin x$ $\sin^2 x = 1 - \cos^2 x$ and $\cos^2 x = 1 - \sin^2 x$ $\rightarrow \sin^3 x + \cos^3 x$ (ii) $(\sin x + \cos x)(1 - \sin x \cos x) = 9 \sin^3 x$ Uses part (i) $\rightarrow 8 \sin^3 x = \cos^3 x$ $\rightarrow \tan^3 x = \frac{1}{8} \rightarrow \tan x = \frac{1}{2}$ $\rightarrow x = 26.6^\circ$ and 206.6°	M1 M1 A1 [3] M1 A1 B1√ [3]	Needs 4 terms from the product. Needs to be used once. All ok. Uses $\tan x = \sin x \div \cos x \rightarrow \tan^3 x = k$. Co. $\sqrt{\quad}$ for $180^\circ +$ first answer and providing there are no other answers in range.
6 (i) $\overrightarrow{OQ} = 3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ $\overrightarrow{PQ} = -3\mathbf{i} + \mathbf{j} + 6\mathbf{k}$ (ii) $(3\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) \cdot (-3\mathbf{i} + \mathbf{j} + 6\mathbf{k})$ $= -9 + 3 + 36 = 30$ $30 = \sqrt{54}\sqrt{46}\cos\theta$ $\theta = 53.0^\circ$ Cosine rule M1 modulus M1 attempt at 3 sides M1 A1 answer.	B1 B2, 1 [3] M1 M1 M1A1 [4]	co Loses one for each error. Use of $x_1x_2 + y_1y_2 + z_1z_2$ co. Correct method for modulus (once) and all correctly linked. co. nb $\overrightarrow{QO} \cdot \overrightarrow{QP}$ can gain 4/4. but $\overrightarrow{OQ} \cdot \overrightarrow{PO}$ can only gain 3/4. Use of other vectors (e.g. $\overrightarrow{OP} \cdot \overrightarrow{OQ}$) M3 ok.
7 (i) $2r + r\theta = 50$ $\theta = \frac{1}{r} (50 - 2r)$ $A = \frac{1}{2} r^2 \theta$ $\rightarrow A = 25r - r^2$ (ii) $\frac{dA}{dr} = 25 - 2r$ $= 0$ when $r = 12.5$ $A = 156\frac{1}{4}$ 2^{nd} differential negative $\rightarrow$ Maximum	M1 A1 M1 A1 [4] B1 M1 A1 B1 [4]	Must use $s = r\theta$ and link with perimeter co Used with θ as $f(r)$ co (answer given) co sets differential to 0 + solution co Could be quoted directly from quadratic.

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8 $x \mapsto \frac{3}{2x+5}$ (i) $f'(x) = -3(2x+5)^{-2} \times 2$ $f'(x)$ is negative $\rightarrow$ decreasing (ii) $y = \frac{3}{2x+5} \rightarrow 2x+5 = \frac{3}{y}$ $\rightarrow f^{-1}(x) = \frac{1}{2}\left(\frac{3}{x} - 5\right)$ or $\frac{3-5x}{2x}$ (iii) $\int \pi \frac{9}{(2x+5)^2} dx$ $= (-9\pi(2x+5)^{-1} \div 2)$ Limits 0 to 2 $\rightarrow \pi(-\frac{1}{2} - -0.9)$ $\rightarrow = 0.4\pi$ (or 1.26)	B1 B1 B1√ [3] M1 A1 [2] B1 B1 M1 A1 [4]	B1 for $-3(2x+5)^{-2}$. B1 for $\times 2$ √ providing bracket is squared. (using value or values only B0) Attempt at making x the subject. co including $f(x)$ not $f(y)$ For $-9(2x+5)^{-1}$ For $\div 2$ in $\int$ of y^2 Use of correct limits with $\int$ of y^2. co
9 (i) y-coordinate same as the y-coordinate of the mid-point of AC. (ii) m of $AD = \frac{8}{h}$ or $\frac{h-12}{8}$ m of $CD = \frac{8}{12-h}$ or $\frac{-h}{8}$ nb $AC = 20$, $M(6, 6)$ $MD = 10 \rightarrow$ $D(16, 6)$ and $B(-4, 6)$ (iii) Product of gradients $= -1$ $\rightarrow h^2 - 12h - 64 = 0$ $\rightarrow h = 16$ or -4 so $x_D = 16$ and $x_B = -4$ or Pyth $h^2 + 8^2 + 8^2 + (12-h)^2 = 400$ (iv) Area $= \sqrt{320} \times \sqrt{80}$ $\rightarrow 160$ (or Area $= 2 \times$ area of a triangle with base $= BD$, $\rightarrow 2 \times \frac{1}{2} \times 20 \times 8$ $= 160$) (or matrix method)	B1 [1] M1 A1 A1 [3] M1 M1 DM1A1 [4] M1 M1 A1 [3]	co any use of y-step $\div x$-step for M mark co Used correctly with the two gradients Forming a quadratic equation Solution of equation. co M1 for method for one of the lengths M1 for base $\times$ height. co

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10	(i) (a) $2y = x + 5, y = x^2 - 4x + 7$ Sim equations $\rightarrow 2x^2 - 9x + 9 = 0$ $\rightarrow x = 3$ or $x = 1\frac{1}{2}$. (b) $\frac{dy}{dx} = 2x - 4$ $\rightarrow y - 4 = 2(x - 3)$ nb use of $y + 4$ or x, y interchanged M1 A0 (c) $m = 2 \rightarrow$ angle of 63.4° $m = \frac{1}{2} \rightarrow$ angle of 26.6° $\rightarrow$ angle between = 37° $(i+2j).(2i+j) \rightarrow 4=\sqrt{5}\sqrt{5}\cos\theta$ M1M1A1 or use of $\tan(A-B)$ M2A1 or Cosine rule with 3 sides found. (ii) $y = x^2 - 4x + 7$ $2y = x + k$ Sim eqns $\rightarrow 2x^2 - 9x + 14 - k = 0$ Uses $b^2 - 4ac$, $81 - 8(14 - k)$ Key value is $k = 3.875$ or $31/8$. $k < 3.875$	M1 DM1 A1 [3] B1 M1 A1 [3] M1 M1A1 [3] M1 A1 M1 A1 [4]	Complete elimination of x or y Correct method for quadratic. co. co Correct form of eqn with m numeric. co Finds angle with x-axis once. Subtracts two angles. co. Eliminates y or x completely. Co (= 0) Uses $b^2 - 4ac = 0$, or < 0 or > 0 Co condone $\leq$.
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