UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS

GCE Advanced Subsidiary Level and GCE Advanced Level

MARK SCHEME for the October/November 2007 question paper

9709 MATHEMATICS

9709/01 Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began.

All Examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes must be read in conjunction with the question papers and the report on the examination.

- CIE will not enter into discussions or correspondence in connection with these mark schemes.

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UNIVERSITY of CAMBRIDGE
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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.

- The symbol √ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.

- Note: B2 or A2 means that the candidate can earn 2 or 0.
  B2/1/0 means that the candidate can earn anything from 0 to 2.

  The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking *g* equal to 9.8 or 9.81 instead of 10.

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through √" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

1. $y = 4x + k$ and $y = x^2$ $\rightarrow x^2 - 4x - k = 0$ $b^2 - 4ac < 0 \rightarrow$ 16+4*k* < 0 $\rightarrow$ *k* < –4 "2*x*=4⇒*x* =2" M1 "*y*=4 ⇒ *k*=–4" M1⇒ A1	M1 M1 A1 [3]	Complete elimination of *x* or *y* Any use of $b^2 - 4ac$ (=0, >0 etc) Co - condone ≤.
2. Area = integral of $2\sqrt{x}$ attempted. $\rightarrow \dfrac{2x^{1.5}}{1.5}$ Uses limits 1 to 4 correctly $\rightarrow \frac{32}{3} - \frac{4}{3}$ = 9⅓ or 9.33 or $\frac{28}{3}$	B1 B1 M1 A1 [4]	Correct power of *x* Coefficient correct unsimplified (Value at *x* = 4) – (value at *x* = 1) co
3. (i) $(2+u)^5$ = 32 + 80*u* + 80*u*² (ii) … $80(x+x^2)+80(x+x^2)^2$ $\rightarrow$ coeff of *x*² of 80 + 80 = 160	B1 × 3 [3] M1 A1√ [2]	Co. Allow 2^5 for B1. Knows what to do – looks at more than 1 term. √ "coeff of *x* + coeff of *x*²".
4. (i) $a+4d$ and $a+14d$ (ii) *a*+4*d* = ar, *a*+14*d*=*ar*² or $\dfrac{a}{a+4d} = \dfrac{a+4d}{a+14d}$ or "*ac*=*b*²" $\rightarrow$ 3*a* = 8*d* (iii) r = $\dfrac{a+4d}{a}$ or $\dfrac{a+14d}{a+4d}$ = 2.5	B1 [1] M1 M1 A1 [3] M1 A1 [2]	Both correct. Correct first step – award the mark for both of these starts. Correct elimination of *r*. co. nb answer was given. Statement + some substitution. co.
5. (i) $3\sin x \tan x = 8$ Uses tan = sine ÷ cosine Uses sin² = 1 – cos ² $\rightarrow 3\cos^2 x + 8\cos - 3 = 0$ (ii) (3c – 1)(*c* + 3) =0 or formula $\rightarrow$ cos*x* = ⅓ as only solution. *x* = 70.5° or 289.5° only.	M1 M1 A1 [3] M1 A1 A1√ [3]	Replaces t by s/c. Uses sin² = 1–cos² for eqn in cosine. Answer given. Correct means of solution of quad. co. For 360° – 1st ans + no others in range.

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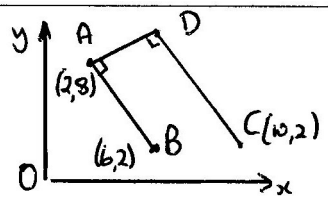
6. 		
Gradient of $AB = -2$	B1	co
Eqn of CD $\quad y-2=-2(x-10)$ $(y+2x=22)$	M1 A1√	correct form of eqn (inc $y=mx+c$) – awarded for either CD or AD. accept any form for A mark.
Uses $m_1m_2=-1$	M1	Use of $m_1m_2=-1$
Eqn of DA $\quad y-8=\frac{1}{2}(x-3)$ $(2y=x+13)$	A1√	Any correct form.
Sim eqns $\rightarrow$ (6.2, 9.6)	M1A1 [7]	Reasonable algebra. co.
7. (i) area of sector = $\frac{1}{2}r^2\theta$ used.	M1	Used correctly for the sector.
$AX = r\sin\theta \quad OX = r\cos\theta$ Area of $\Delta\frac{1}{2}bh$ used	M1	Realises the need to use trig in ΔOAX. (beware AX, OX wrong way round)
$\rightarrow \quad A=\frac{r^2}{2}(\theta-\sin\theta\cos\theta)$	A1 [3]	ag- be careful of above error that scores 2/3
(ii) $AX = 12\sin\frac{1}{6}\pi = 6$	B1	co – anywhere even if in an area.
$OX = 12\cos\frac{1}{6}\pi = 6\sqrt{3}$	B1	co – anywhere (for $6\sqrt{3}$ or $\frac{1}{2}\times12\sqrt{3}$)
$BX = 12 - 6\sqrt{3}$		
Arc $AB = 12\times\frac{1}{6}\pi = 2\pi$	M1	Use of $s=r\theta$
$\rightarrow$ P = $18 - 6\sqrt{3} + 2\pi$ (17.0)	A1 [4]	co. allow 6 + 12 instead of 18.
8. $y=(2x-3)^3-6x$		
(i) $\frac{dy}{dx} = 3\times(2x-3)^2\times2 \quad -6$	B1 B1	For $3\times(2x-3)^2-6$ For $\times2$ Nb $24x^2-72x+48$ B2,1
$\frac{d^2y}{dx^2} = 12\times(2x-3)\times2$	B1√ [3]	√ from his dy/dx.
(ii) s.p $\rightarrow \frac{dy}{dx}=0 \rightarrow (2x-3)^2=1$	M1 DM1	Sets dy/dx to 0 Solution to give **two** values of x.
$\rightarrow x=2$ or $x=1$	A1	For both values correct.
If $x=2$, 2nd diff = +ve $\rightarrow$ MIN	M1	Looks at sign of 2nd differential or other method with one x value.
If $x=1$, 2nd diff = –ve $\rightarrow$ MAX	A1 [5]	A1 for correct conclusions

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9. (i) $y = 4x - \frac{1}{2}x^2$ + c Uses (2,9) → c = 3	B1 M1 A1 [3]	For $y = 4x - \frac{1}{2}x^2$ Introduces +c and attempts to evaluate
(ii) grad of tan = 2, normal = −½ Eqn $y - 9 = -\frac{1}{2}(x - 2)$	M1 M1 A1 [3]	Uses $m_1m_2 = -1$, m_1 = dy/dx = number Any correct method – not for tangent.
(iii) $y = 4x - \frac{1}{2}x^2 + 3$, $2y + x = 20$ eliminates $y \rightarrow$ $x^2 - 9x + 14 = 0$ eliminates $x \rightarrow$ $2y^2 - 31y + 117 = 0$	M1	Eliminates one variable completely – needs a linear and quadratic eqn.
Soln of quadratic → $x = 7$, $y = 6.5$	DM1 A1 [3]	Correct method for quad. co.
10 (i) $\overrightarrow{PR} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}$ $\overrightarrow{PQ} = \begin{pmatrix} -2 \\ 2 \\ 4 \end{pmatrix}$	B1 B2,1 [3]	All elements of $\overrightarrow{PR}$ – any notation ok. Loses one mark for each error in $\overrightarrow{PQ}$
(ii) $\overrightarrow{PQ}.\overrightarrow{PR}$ = -4 + 4 + 8 = 8 $\lvert\overrightarrow{PQ}\rvert$ = √24 $\lvert\overrightarrow{PR}\rvert$ = √12 $\overrightarrow{PQ}.\overrightarrow{PR}$ = √12 √24 cos QPR Angle QPR = 61.9° or 1.08 rad	M1 M1 M1 A1 [4]	Must be scalar As long as this is used with dot product Everything linked ($\overrightarrow{QP}.\overrightarrow{PR}$ used – still gains all M marks) Co
(iii) $\overrightarrow{QR} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$ $\lvert\overrightarrow{QR}\rvert$ = √ 20	M1	For correct $\overrightarrow{QR}$ - cosine rule ok.
Perimeter = √12 + √24 + √20 = 12.8 cm	M1 A1 [3]	Adds three roots. co – beware fortuitous answers from incorrect sign in vectors.

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11. f(x) = $2x^2 - 8x + 11$		
(i) f(x) = $2(x-2)^2 + 3$	3 × B1 [3]	B1 for each of 2, −2 and 3. – no need to equate with *a*, *b* and *c*.
(ii) Range is ≥3	B1√ [1]	Condone >3. √ for ≥ *c*.
(iii) Not 1:1 (2 *x*-values for 1 *y*-value) or curve is quadratic – or has a minimum value	B1 [1]	
(iv) $A = 2$	B1√ [1]	co - √ for *x* value from (i)
(v) $y = 2(x-2)^2 + 3$ $\rightarrow \frac{y-3}{2} = (x-2)^2$	M1	attempt to make *x* the subject – or *y* if *x* and *y* interchanged at start
$\rightarrow x = 2 \pm \sqrt{\left(\frac{y-3}{2}\right)}$	M1	order correct ±3, ÷2, √, ±2
$\rightarrow g^{-1}(x) = 2 - \sqrt{\left(\frac{x-3}{2}\right)}$	A1	co – must be f(*x*)
Range of g^{-1} ≤2	B1√ [4]	condone < or >. Nb if "+" root taken, answer will be ≤2, but could be ≤ 2 if returning to the original function.

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