

Cambridge International AS & A Level

MATHEMATICS**9709/51**

Paper 5 Probability & Statistics 1

May/June 2024**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **20** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

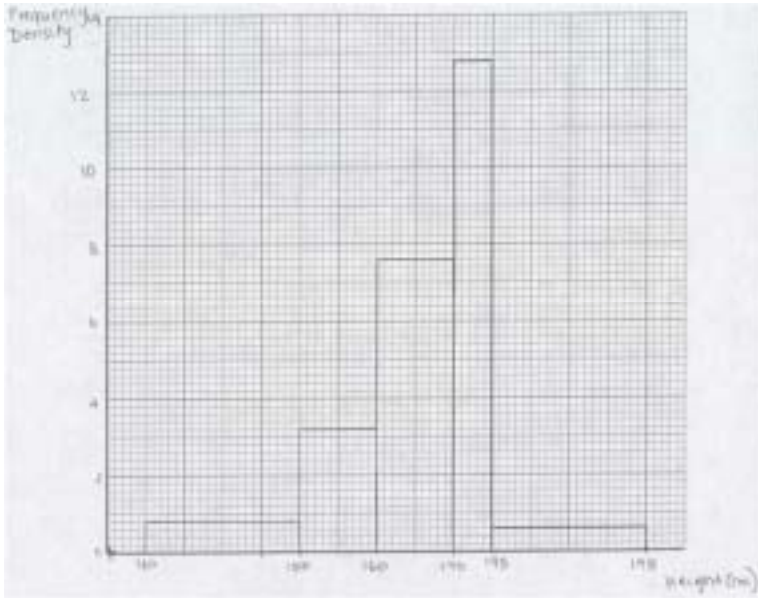
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Question	Answer	Marks	Guidance
1(a)	[For all 45 values Mean =] $\frac{439 + 470}{45} + 30$	M1	$\frac{439 + 470}{45}$ or $\frac{909}{45}$ seen.
	= 50.2	A1	If M0 awarded, SC B1 50.2 WWW.
	Alternative Method for Question 1(a)		
	[For all 45 values Mean =] $\frac{25 \times 30 + 470 + 20 \times 30 + 439}{45}$	(M1)	$\frac{1220 + 1039}{45}$ or $\frac{2259}{45}$ seen.
	= 50.2	(A1)	If M0 awarded, SC B1 50.2 WWW.
		2	
1(b)	For all 45 values $Sd^2 = \frac{12405 + 11346}{45} - \left(\frac{909}{45}\right)^2$	M1	$\frac{their(12405 + 11346)}{45}$ or $23751 - \left(\frac{their 909}{45}\right)^2$
	sd [= $\sqrt{119.76}$] = 10.9	A1	If M0 awarded, SC B1 10.9 WWW.
		2	

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Question	Answer	Marks	Guidance
2(a)	$P(23 < X < 35) = P\left(\frac{23-28}{3.3} < Z < \frac{35-28}{3.3}\right)$ [= $P(-1.515 < Z < 2.121)$]	M1	Using $\pm$ standardisation formula once with 23 or 35, 28 and 3.3, allow σ^2 , allow $\sqrt{\sigma}$, no continuity correction.
		A1	One fully correct $\pm$ standardisation formula.
	[= $\Phi(2.121) + \Phi(1.5151) - 1$] = $0.9830 + 0.9351 - 1$	M1	Appropriate area Φ , from final process, must be a probability.
	= 0.918	A1	AWRT
		4	
2(b)	$[P(X > 7.6) = P\left(Z > \frac{7.6-8.5}{\sigma}\right) = 0.75]$ $\frac{7.6-8.5}{\sigma} = -0.674$	B1	0.674 or – 0.674 seen. CAO as critical value.
		M1	Use of the $\pm$ standardisation formula with 7.6, 8.5, σ and a z-value (not 0.75, 0.25, 0.7734, 0.2266, 0.5987 nor $1 - z$ -value: 0.326, 0.5987). Condone use of $\frac{\pm 0.9}{\sigma}$.
	$\sigma = 1.34$	A1	$1.33 \leq \sigma \leq 1.34$
		3	

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Question	Answer	Marks	Guidance												
3(a)	<table><tr><td>cw</td><td>20</td><td>10</td><td>10</td><td>5</td><td>20</td></tr><tr><td>fd</td><td>0.8</td><td>3.2</td><td>7.6</td><td>12.8</td><td>0.6</td></tr></table>	cw	20	10	10	5	20	fd	0.8	3.2	7.6	12.8	0.6	M1	At least four frequency densities calculated $\frac{f}{cw}$ (e.g. $\frac{16}{20}$). Condone $\frac{f}{cw \pm 0.5}$ if unsimplified. Accept unsimplified, may be read from graph using <i>their</i> scale no lower than 1 cm = fd 2.
	cw	20	10	10	5	20									
	fd	0.8	3.2	7.6	12.8	0.6									
		A1	All bar heights correct on graph, not FT. Using their suitable linear scale with at least three values indicated, no lower than 1 cm = fd 2.												
B1		Bar ends at 150, 160, 170, 175, 195. Five bars drawn with a horizontal linear scale no lower than 1 cm = 10 cm, with at least three values indicated, $130 \leq \text{horizontal scale} \leq 195$.													
B1		Axes labelled frequency density (fd) height (h) and cm, OE, or an appropriate title. (Axes may be reversed)													
		4													
3(b)	[LQ:] $160 \leq h < 170$ [UQ:] $170 \leq h < 175$ $175 - 160 = 15$	M1	$170 \leq h < 175 - 160 \leq h < 170$ UQ and LQ classes seen.												
		A1	$175 - 160 = 15$												
			If M0 scored, SC B1 for $175 - 160 = 15$.												
		2													

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Question	Answer	Marks	Guidance
4(a)	Method 1		
	[Probability of 4 in 3 throws is] $1 - \left(\frac{3}{4}\right)^3 = \frac{37}{64}$	M1	$1 - (s)^3, s = \frac{3}{4} \text{ or } \frac{1}{4}.$
		A1	AG
	Method 2		
	[Probability of 4 in 3 throws is] $\frac{1}{4} + \frac{1}{4} \times \frac{3}{4} + \frac{1}{4} \times \left(\frac{3}{4}\right)^2 = \frac{37}{64}$	(M1)	$t + t(1-t) + t(1-t)^2, t = \frac{1}{4} \text{ or } \frac{3}{4}.$
		(A1)	AG
	Method 3		
	${}^3C_1 \times \frac{1}{4} \times \left(\frac{3}{4}\right)^2 + {}^3C_2 \times \left(\frac{1}{4}\right)^2 \times \frac{3}{4} + {}^3C_3 \times \left(\frac{1}{4}\right)^3 = \frac{37}{64}$	(M1)	
		(A1)	AG
		2	

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Question	Answer	Marks	Guidance
4(b)	Method 1		
	$\left(1 - \frac{37}{64}\right)^4 = 0.0317$	B1FT	$(1 - \text{their (a)})^4$, accept unsimplified.
		1	
	Method 2		
	[Probability no 4s is] $\left(\frac{3}{4}\right)^6 \times \left(\frac{3}{4}\right)^6 = 0.0317$	(B1FT)	Accept unsimplified.
		1	
4(c)	X3 Y1 $\left(\frac{37}{64}\right)^3 \times \frac{37}{64} \times \left(\frac{27}{64}\right)^2 \times 3$ [= 0.059645]	B1	Correct probability for 1 identified scenario. Accept unsimplified.
	X2 Y0 $\left(\frac{37}{64}\right)^2 \times \frac{27}{64} \times \left(\frac{27}{64}\right)^3 \times 3$ [= 0.03176]	M1	Add values of 2 correct scenarios. Identification may be implied by correct unsimplified expressions (condone omission of $\times 3$). Values may not be probabilities.
	Probability = 0.0914	A1	If A0 scored, SC B1 for 0.0914 WWW.
		3	

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Question	Answer	Marks	Guidance
5(a)	Method 1		
	[P(5, 6, 7) =]	M1	One term ${}^7C_x (p)^x (1-p)^{7-x}$, with $0 < p < 1$, $x \neq 0$ or 7.
	${}^7C_5 0.7^5 0.3^2 + {}^7C_6 0.7^6 0.3^1 + 0.7^7$	A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	[= 0.31765 + 0.24706 + 0.08235]		
	= 0.647	B1	$0.647 \leq p < 0.6475$
	Method 2		
	[P(5, 6, 7) = 1 – P(0, 1, 2, 3, 4) =]	(M1)	One term ${}^7C_x (p)^x (1-p)^{7-x}$, with $0 < p < 1$, $x \neq 0$ or 7.
	$1 - \{0.3^7 + {}^7C_1 0.7^1 0.3^6 + {}^7C_2 0.7^2 0.3^5 + {}^7C_3 0.7^3 0.3^4 + {}^7C_4 0.7^4 0.3^3\}$	(A1)	Correct expression, accept unsimplified, no terms omitted leading to final answer. Condone omission of final bracket ‘}’. If other brackets omitted, allow recovery if $1 - 0.35294$ seen.
	= 0.647	(B1)	$0.647 \leq p < 0.6475$
		3	

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Question	Answer	Marks	Guidance
5(b)	Method 1		
	$[1 - P(0 \text{ white weeks}) =]$ $1 - (1 - 0.647)^3$	M1	$1 - p^3, 0 < p < 1, p = 1 - \text{their (a)},$ or correct.
	0.956	A1	
	Method 2		
	$[P(1, 2, 3 \text{ white weeks}) =]$ $3 \times 0.647 \times 0.353^2 + 3 \times 0.647^2 \times 0.353 + 0.647^3$	(M1)	$3 \times q \times (1 - q)^2 + 3 \times q^2 \times (1 - q) + q^3, q = \text{their (a)},$ or correct.
	0.956	(A1)	
		2	

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Question	Answer	Marks	Guidance
5(c)	[Mean = $60 \times 0.8 =$] 48 [Variance = $60 \times 0.8 \times 0.2 =$] 9.6	B1	48 and 9.6, $9\frac{3}{5}$, $\frac{48}{5}$ seen, allow unsimplified. May be seen in the standardisation formula ([$\sigma =$] $3.098 \leq \sigma \leq 3.1[0]$ implies correct variance). Incorrect notation penalised but values can be used as anticipated in remainder of question.
	$P(X < 47) = P\left(Z < \frac{46.5 - 48}{\sqrt{9.6}}\right)$	M1	Substituting <i>their</i> μ and σ into $\pm$ standardising formula (any number for 46.5), not <i>their</i> σ^2 or $\sqrt{\text{their } \sigma}$.
		M1	Use continuity correction 46.5 or 47.5 in <i>their</i> standardised formula. Note: $\frac{\pm 1.5}{\sqrt{9.6}}$ or $\frac{\pm 1.5}{3.098}$ seen gains M2 BOD.
	$[P(Z < -0.4841) = 1 - \Phi(0.4841)]$ 1 – 0.6858	M1	Appropriate area Φ , from final process, must be a probability. Expect final answer < 0.5. Note: appropriate final answer implies this M1.
	= 0.314	A1	$0.314 \leq p < 0.3145$
		5	

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Question	Answer					Marks	Guidance																		
6(a)	<table><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X = x)$</td><td>$\frac{24}{60}$</td><td>$\frac{26}{60}$</td><td>$\frac{9}{60}$</td><td>$\frac{1}{60}$</td></tr><tr><td></td><td>$\frac{2}{5}$</td><td>$\frac{13}{30}$</td><td>$\frac{3}{20}$</td><td>$\frac{1}{60}$</td></tr><tr><td></td><td>0.4</td><td>0.433</td><td>0.15</td><td>0.0167</td></tr></table>	x	0	1	2	3	$P(X = x)$	$\frac{24}{60}$	$\frac{26}{60}$	$\frac{9}{60}$	$\frac{1}{60}$		$\frac{2}{5}$	$\frac{13}{30}$	$\frac{3}{20}$	$\frac{1}{60}$		0.4	0.433	0.15	0.0167			B1	Table with correct X values and at least one probability. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted. Condone any additional X values if probability stated as 0.
	x	0	1	2	3																				
	$P(X = x)$	$\frac{24}{60}$	$\frac{26}{60}$	$\frac{9}{60}$	$\frac{1}{60}$																				
		$\frac{2}{5}$	$\frac{13}{30}$	$\frac{3}{20}$	$\frac{1}{60}$																				
		0.4	0.433	0.15	0.0167																				
			B1	$P(X = 1)$ or $P(X = 2)$ correct and identified, need not be in table, accept unsimplified.																					
			B1	Two more correct and identified probabilities, need not be in table, accept unsimplified.																					
			B1	4 correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3sf. SC B1 for four probabilities summing to 1 placed in a probability distribution table with the correct x values.																					
						4																			

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Question	Answer	Marks	Guidance
6(b)	$[P(Y=0)=] \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)^2;$ $[P(Y=5)=] \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p^2$	B1	Either $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)^2$, not $\frac{2}{5} \times (1-p)^2$; or $\frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p^2$, not $\frac{1}{60} \times p^2$.
	$\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times (1-p)^2 = 6 \times \frac{1}{3} \times \frac{1}{4} \times \frac{1}{5} \times p^2$ $[24(1-p)^2 = 6 \times p^2]$ $3p^2 - 8p + 4 = 0$	M1	Equating and forming a 3 term quadratic equation. <i>Their</i> $P(Y=0) = 6 \times \text{their } P(Y=5)$.
	$p = \frac{2}{3}$	A1	Not dependent on B1. A0 if $p = 2$ seen and not clearly rejected.
			SC B1 if $p = \frac{2}{3}$ obtained from a correct quadratic with more than three terms. If $p = 2$ seen and not clearly rejected, SC B0 .
		3	

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Question	Answer	Marks	Guidance
7(a)	$\left[\frac{8!}{2!3!} \right] 3360$	B1	
		1	
7(b)	Number of arrangements with 2s at the end – number of arrangements with 2s at the end and the 4s together 2 _ _ _ _ _ 2 – 2 _ (444) _ _ 2		
	$\frac{6!}{3!} - 4!$	M1	$\frac{6!}{3!} \times r - s$, $r = 1, 2$ and s a positive integer (including 0).
		B1	4! Seen either alone or in $t - 4!$, t an integer value > 24 .
		M1	$\frac{6!}{3!} \times r - 4! \times u$, $r = 1, 2$ and $u = 1, 2$.
	= 96	A1	
		4	

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Question	Answer					Marks	Guidance
7(c)	Method 1						
	2s	4s	1,3,5			M1	One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.
	0	0	3	${}^3C_3$	1	A1	Two correct outcomes evaluated, accept unsimplified.
	0	1	2	${}^3C_1 \times {}^3C_2$	9	M1	Four correct scenarios added.
	1	0	2	${}^2C_1 \times {}^3C_2$	6		
	1	1	1	${}^2C_1 \times {}^3C_1 \times {}^3C_1$	18		
	[Total 34 ways]						
	[Total number of selections =] ${}^8C_3$ [= 56]					B1	Used as denominator of probability expression.
[Probability =] $\left[\frac{34}{{}^8C_3} \right] = \frac{17}{28}, 0.607$					A1		

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Question	Answer	Marks	Guidance	
7(c)	Method 2			
	Combinations of 3 numbers		(M1) One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.	
	1,2,3	${}^1C_1 \times {}^2C_1 \times {}^1C_1$	2	(A1) Five correct outcomes evaluated, accept unsimplified.
	1,2,4	${}^1C_1 \times {}^2C_1 \times {}^3C_1$	6	
	1,2,5	${}^1C_1 \times {}^2C_1 \times {}^1C_1$	2	(M1) Ten correct scenarios added.
	1,3,4	${}^1C_1 \times {}^1C_1 \times {}^3C_1$	3	
	1,3,5	${}^1C_1 \times {}^1C_1 \times {}^1C_1$	1	
	1,4,5	${}^1C_1 \times {}^3C_1 \times {}^1C_1$	3	
	2,3,4	${}^2C_1 \times {}^1C_1 \times {}^3C_1$	6	
	2,3,5	${}^2C_1 \times {}^1C_1 \times {}^1C_1$	2	
2,4,5	${}^2C_1 \times {}^3C_1 \times {}^1C_1$	6		
3,4,5	${}^1C_1 \times {}^3C_1 \times {}^1C_1$	3		
[Total 34 ways] [Total number of selections =] ${}^8C_3$ [= 56]		(B1)	${}^8C_3$ or 56 as denominator of probability expression.	
[Probability =] $\left[\frac{34}{{}^8C_3} \right] = \frac{17}{28}, 0.607$		(A1)		

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Question	Answer			Marks	Guidance
7(c)	Method 3				
	1,2,3	$\frac{1}{8} \times \frac{2}{7} \times \frac{1}{6} \times 3!$	$\frac{12}{336}$	(M1)	One correct calculation, unsimplified for an identified scenario containing 1, 2 and/or 1, 4.
	1,2,4	$\frac{1}{8} \times \frac{2}{7} \times \frac{3}{6} \times 3!$	$\frac{36}{336}$	(A1)	Five correct outcomes evaluated, accept unsimplified.
	1,2,5	$\frac{1}{8} \times \frac{2}{7} \times \frac{1}{6} \times 3!$	$\frac{12}{336}$	(M1)	Ten correct scenarios added.
	1,3,4	$\frac{1}{8} \times \frac{1}{7} \times \frac{3}{6} \times 3!$	$\frac{18}{336}$		
	1,3,5	$\frac{1}{8} \times \frac{1}{7} \times \frac{1}{6} \times 3!$	$\frac{6}{336}$		
	1,4,5	$\frac{1}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3!$	$\frac{18}{336}$		
	2,3,4	$\frac{2}{8} \times \frac{1}{7} \times \frac{3}{6} \times 3!$	$\frac{36}{336}$		
	2,3,5	$\frac{2}{8} \times \frac{1}{7} \times \frac{1}{6} \times 3!$	$\frac{12}{336}$		
	2,4,5	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3!$	$\frac{36}{336}$		
	3,4,5	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3!$	$\frac{18}{336}$		

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Question	Answer	Marks	Guidance																										
7(c)		(B1)	336 or 8×7×6 seen as a denominator.																										
	[Probability]= $\frac{17}{28}$, 0.607	(A1)																											
	Method 4																												
	<table><tr><td>444</td><td>$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$</td><td>$\frac{6}{336}$</td></tr><tr><td>445</td><td>$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{18}{336}$</td></tr><tr><td>443</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{18}{336}$</td></tr><tr><td>442</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{36}{336}$</td></tr><tr><td>441</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{18}{336}$</td></tr><tr><td>225</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{6}{336}$</td></tr><tr><td>224</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{18}{336}$</td></tr><tr><td>223</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{6}{336}$</td></tr><tr><td>221</td><td>$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$</td><td>$\frac{6}{336}$</td></tr></table>	444	$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$	$\frac{6}{336}$	445	$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$	443	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$	442	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{36}{336}$	441	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$	225	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{6}{336}$	224	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$	223	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{6}{336}$	221	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{6}{336}$	(M1)
444	$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6}$	$\frac{6}{336}$																											
445	$\frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$																											
443	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$																											
442	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{36}{336}$																											
441	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$																											
225	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{6}{336}$																											
224	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{18}{336}$																											
223	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{6}{336}$																											
221	$\frac{2}{8} \times \frac{3}{7} \times \frac{1}{6} \times 3$	$\frac{6}{336}$																											
7(c)		(A1)	Five correct probabilities evaluated, accept unsimplified.																										
		(M1)	Nine correct scenarios subtracted.																										

Question	Answer	Marks	Guidance
		(B1)	336 or $8 \times 7 \times 6$ seen as a denominator.
	[Probability] = $1 - \frac{132}{336}, \frac{204}{336}, 0.607$	(A1)	
		5	

Cambridge International AS & A Level

MATHEMATICS**9709/52**

Paper 5 Probability & Statistics 1

May/June 2024**MARK SCHEME**Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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PUBLISHED**Mathematics-Specific Marking Principles**

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- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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Question	Answer	Marks	Guidance
1(a)	$[(0.7)^6 \times 0.3] = 0.0353$	B1	$\frac{352947}{10000000}$ or 0.03529... to at least 3sf.
		1	
1(b)	Method 1		
	$[P(X < 6) =] 1 - 0.7^5$	M1	$1 - 0.7^d, d = 5, 6.$
	$= 0.832$	A1	Accept 0.83193 to at least 3sf. If M0 scored, SC B1 for 0.8319[3].
	Method 2		
	$[P(X < 6) =]$ $0.3 + (0.3)(0.7) + (0.3)(0.7)^2 + (0.3)(0.7)^3 + (0.3)(0.7)^4$	(M1)	$0.3 + (0.3)(0.7) + (0.3)(0.7)^2 + (0.3)(0.7)^3 + (0.3)(0.7)^4 \left[+ (0.3)(0.7)^5 \right]$
	$= 0.832$	(A1)	Accept 0.83193 to at least 3sf. If M0 scored, SC B1 for 0.8319[3].
		2	
1(c)	$(0.7)^8 \times (0.3)^2 \times {}^9C_1$ or $(0.7)^8 \times (0.3) \times {}^9C_1 \times (0.3)$	M1	$(0.7)^8 \times (0.3)^2 \times k, k$ a positive integer, 1 may be implied. No addition/subtraction/additional terms.
	$= 0.0467$	A1	
		2	

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Question	Answer	Marks	Guidance
2(a)	Method 1		
	$P(HR) + P(TR) + P(TBR)$ $\frac{1}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{4}{9} + \frac{2}{3} \times \frac{5}{9} \times \frac{4}{8}$ or $P(HR) + (P(TRR) + P(TRB)) + P(TBR)$ $\frac{1}{3} \times \frac{4}{9} + \left(\frac{2}{3} \times \frac{4}{9} \times \frac{3}{8} + \frac{2}{3} \times \frac{4}{9} \times \frac{5}{8} \right) + \frac{2}{3} \times \frac{5}{9} \times \frac{4}{8}$	B1	Two of the calculations for P(HR), P(TBR), either P(TR) or P(TRR) + P(TRB) unsimplified, ignore any identification. Condone $\frac{4}{8} = \frac{1}{2}$ in the unsimplified calculation. Condone use of tree diagram to show calculation if values correct at end.
		M1	Values of all correct identified scenarios added. Correct branches may be identified on the tree diagram.
	$\left[= \frac{4}{27} + \frac{8}{27} + \frac{5}{27} \right] = \frac{17}{27}$	A1	0.6296..., 0.630 If M0 scored SC B1 for acceptable answers, WWW.
	Method 2		
	$1 - P(HB) - P(TBB) = 1 - \left(\frac{1}{3} \times \frac{5}{9} + \frac{2}{3} \times \frac{5}{9} \times \frac{4}{8} \right) = \left[1 - \frac{5}{27} - \frac{5}{27} \right]$	(B1)	One calculation of P(HB), P(TBB), unsimplified, ignore any identification. 1 – probability must be seen. Condone use of tree diagram to show calculation if values correct at end.
		(M1)	1 – values of two correct identified scenarios subtracted. Correct branches may be identified on the tree diagram.
	$= \frac{17}{27}$	(A1)	0.6296..., 0.630 If M0 scored SC B1 for acceptable answers, WWW.

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Question	Answer	Marks	Guidance
2(a)	Method 3		
	$P(HR) + P(T, (1 - \text{no } R)) =$ $\frac{1}{3} \times \frac{4}{9} + \frac{2}{3} \left(1 - \left(\frac{5}{9} \times \frac{4}{8} \right) \right) =$ $\left[\frac{4}{27} + \frac{2}{3} \left(1 - \frac{20}{27} \right) \right]$	(B1)	Calculation for $P(T, (1 - \text{no } R))$ seen unsimplified. Condone use of tree diagram to show calculation if values correct at end.
		(M1)	Values of two correct identified scenarios added. Correct branches may be identified on the tree diagram.
	$= \frac{17}{27}$	(A1)	0.6296..., 0.630 If M0 scored SC B1 for acceptable answers, WWW.
		3	

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Question	Answer	Marks	Guidance
2(b)	Method 1		
	$\left[P(\text{head} \mid \text{no reds}) = \frac{P(\text{head} \cap \text{no reds})}{P(\text{no reds})} \right]$ $= \frac{\frac{1}{3} \times \frac{5}{9}}{1 - \frac{17}{27}} = \frac{\frac{5}{27}}{\frac{10}{27}} =$	M1	$\frac{d}{1 - \text{their}(\mathbf{a})}$ or $\frac{d}{1 - \frac{17}{27}}$ or $\frac{d}{\frac{10}{27}}, 0 < d < 1.$ Condone $\frac{10}{27} = 0.3704$ or more accurate.
	$= \frac{1}{2}$	A1	OE Condone 0.499[9...].
	Method 2		
	$\left[P(\text{head} \mid \text{no reds}) = \frac{P(\text{head} \cap \text{blue})}{P(\text{HB}) + P(\text{TBB})} \right]$ $= \frac{\frac{1}{3} \times \frac{5}{9}}{\frac{1}{3} \times \frac{5}{9} + \frac{2}{3} \times \frac{5}{9} \times \frac{4}{8}} = \frac{\frac{5}{27}}{\frac{10}{27}} =$	(M1)	$\frac{d}{\frac{1}{3} \times \frac{5}{9} + \frac{2}{3} \times \frac{5}{9} \times \frac{4}{8}}$ or $\frac{d}{\frac{10}{27}}, 0 < d < 1.$ Condone $\frac{10}{27} = 0.3704$ or more accurate.
	$= \frac{1}{2}$	(A1)	OE Condone 0.499[9...].
		2	

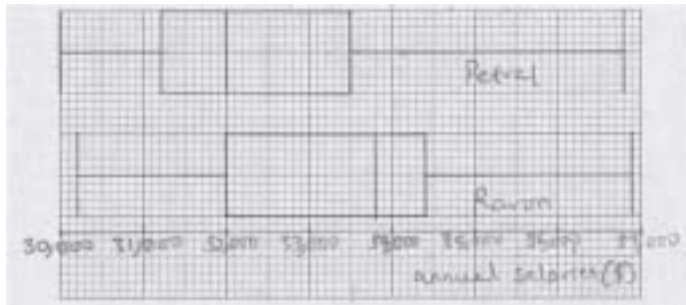
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Question	Answer	Marks	Guidance
3(a)	$P(X > 184) = P\left(Z > \frac{184 - 131}{54}\right) [= P(Z > 0.9815)]$	M1	Use of $\pm$ standardisation formula with 184, 131 and 54, no continuity correction. Condone use of σ^2 , $\sqrt{\sigma}$.
	$1 - 0.837 [= 0.163]$	M1	Calculating the appropriate probability area (leading to their final answer).
	Percentage $[= 0.163 \times 100] = 16.3$	A1	AWRT
		3	
3(b)	$[P(X < w) = P(Z < \frac{w - 131}{54}) = 0.2]$	B1	$-0.842 \leq z < -0.8415$ or $0.8415 < z \leq 0.842$ seen.
	$\frac{w - 131}{54} = -0.842$	M1	Use of the $\pm$ standardisation formula with 131, 54, w and a z -value (not 0.2, 0.8, 0.158, 0.508[0], 0.492[0], 0.7881, 0.2119, 0.5593, 0.4407).
	$w = 85.5$	A1	$85.5 \leq p \leq 85.6$ Signs must be consistent to create a positive answer.
		3	

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Question	Answer	Marks	Guidance
4(a)	Median = 32000	B1	Clearly identified, e.g. Q2, med. Accept 32 k.
	[UQ = 33500, LQ = 31200] [IQR =] 33500 – 31200	M1	$33300 \leq UQ \leq 33700 - 31100 \leq LQ \leq 31200$ Implied if both quartile values are stated and an appropriate IQR is calculated accurately.
	= 2300	A1	WWW Ignore \$ signs. If M0 scored, SC B1 for 2300 WWW. If key ignored consistently: B0 Median = 320 SC M1 $325 \leq UQ \leq 335 - 311 \leq LQ \leq 312$ SC A1 23.
		3	

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Question	Answer	Marks	Guidance												
4(b)	Box-and-whisker plot on provided grid	B1	All five key values for R plotted accurately in standard format using a linear scale with at least three linear values. Labelled R . Condone whiskers through box or at corners of boxes or extending $\frac{1}{2}$ square beyond limit. Scale no less than 1 cm = \$1000. Daylight rule applied to vertical lines of box.												
		B1FT	All five key values for P , FT from (a), plotted accurately in standard format using a linear scale with at least three linear values. Labelled P . Condone whiskers through box or at corners of boxes or extending $\frac{1}{2}$ square beyond limit. Scale no less than 1 cm = \$1000. Daylight rule applied to vertical lines of box.												
	<table border="1"><tr><td>R</td><td>30 200</td><td>32 000</td><td>33 800</td><td>34 400</td><td>36 900</td></tr><tr><td>P</td><td>30 000</td><td>31 200</td><td>32 000</td><td>33 500</td><td>36 800</td></tr></table>	R	30 200	32 000	33 800	34 400	36 900	P	30 000	31 200	32 000	33 500	36 800	B1	Whiskers not through box (condone $\frac{1}{2}$ square in box) for either, not drawn at corners of boxes. single linear scale for the diagram and labelled 'salaries' (OE) and \$. If only one plot attempted, SC B1 for meeting all the requirements above.
	R	30 200	32 000	33 800	34 400	36 900									
P	30 000	31 200	32 000	33 500	36 800										
		3													
4(c)	Median because there is an extreme value (\$36 800)	B1	Do not accept 'values'. Must identify median and reference either the extreme value (anomaly, outlier, 36 800) or the skew in context (e.g. concentrated in lower values, positive skew).												
		1													

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Question	Answer	Marks	Guidance												
5(a)	$[\$7 =] [\$]5 + [\$]2$ $[\text{Probability} =] \frac{1}{5} \times \frac{2}{4} \times 2 = \frac{1}{5} = 0.2$ Or $[\text{Probability} =] 0.2 \times 0.5 \times 2 = 0.2$	B1	AG Must include [\$7], 5, 2 and link the probabilities to the appropriate value $\frac{{}^1C_1 \times {}^2C_1}{{}^5C_2} = 0.2.$ $\frac{1}{5} \times \frac{2}{4} + \frac{2}{5} \times \frac{1}{4}$, not $\frac{1}{5} \times \frac{2}{4} + \frac{1}{5} \times \frac{2}{4}$ unless 5 and 2 and 2 and 5 seen in solution. If all possibilities identified (e.g. outcome table), must be clearly labelled and terms fulfilling the condition identified.												
		1													
5(b)	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>6</td><td>7</td></tr><tr><td>$P(X=x)$</td><td>0.1</td><td>0.4</td><td>0.1</td><td>0.2</td><td>0.2</td></tr></table>	x	2	3	4	6	7	$P(X=x)$	0.1	0.4	0.1	0.2	0.2	B1	Table with correct x values and at least one further non-zero probability correct. Condone extra x values if probability stated as 0.
	x	2	3	4	6	7									
	$P(X=x)$	0.1	0.4	0.1	0.2	0.2									
		B1	Two more correct non-zero probabilities linked with correct outcomes. Accept probabilities not in table if clearly identified.												
	B1	All five probabilities correct. Accept probabilities not in table if clearly identified. SC B1 for four further non-zero probabilities adding to 0.8 if B1 max scored.													
		3													

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Question	Answer	Marks	Guidance
5(c)	$[E(X) = 0.1 \times 2 + 0.4 \times 3 + 0.1 \times 4 + 0.2 \times 6 + 0.2 \times 7]$ 0.2 + 1.2 + 0.4 + 1.2 + 1.4 [= 4.4]	M1	Accept unsimplified expression. May be calculated in the variance, FT <i>their</i> table with at least 5 probabilities, $0 < p < 1$, that sum to 1. FT acceptable at the bold partially evaluated stage.
	$[\text{Var}(X) = 0.1 \times 2^2 + 0.4 \times 3^2 + 0.1 \times 4^2 + 0.2 \times 6^2$ $+ 0.2 \times 7^2 - 4.4^2$ $0.1 \times 4 + 0.4 \times 9 + 0.1 \times 16 + 0.2 \times 36 + 0.2 \times 49 - 4.4^2$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with at least 4 probabilities, $0 < p < 1$, that may not sum to 1. FT acceptable at the bold partially evaluated stage. Note: if table is correct, $22.6 - (4.4^2 \text{ or } 19.36)$ implies this M1.
	= 3.24	A1	CAO $\frac{81}{25}, 3\frac{6}{25}$ scores A0. Only dependent upon previous M1 (M0 M1 A1 possible). If M0 M0 scored, SC B1 for 3.24 WWW.
		3	

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Question	Answer	Marks	Guidance
6(a)	Mean [=110 × 0.25] = 27.5 Variance [=110 × 0.25 × 0.75] = 20.625, $\frac{165}{8}$	B1	27.5 and 20.625 (CAO) seen, allow unsimplified. May be in standardisation formula (4.541475... to at least 4sf or $\sqrt{\frac{165}{8}}$ or $\frac{\sqrt{330}}{4}$ implies correct variance). Penalise incorrect identification, condone no identification.
	$P(X < 22) = P\left(Z < \frac{21.5 - 27.5}{\sqrt{20.625}}\right)$	M1	Substituting <i>their</i> 27.5 and <i>their</i> 20.625 into the $\pm$ standardising formula (any number for 21.5), not σ^2 , not $\sqrt{\sigma}$.
		M1	Using continuity correction 21.5 or 22.5 in <i>their</i> standardisation formula.
	$[P(Z < -1.3212) = 1 - \Phi(1.3212)]$ $1 - 0.9068 =$	M1	Appropriate probability area, from final process, must be a probability. May be implied by a sketch of the required probability area. Expect final answer < 0.5.
	0.0932	A1	$0.0932 \leq p < 0.09325$ If either M1 M1 not awarded for standardisation and/or M1 not awarded for finding probability area, SC B1 $0.0932 \leq p < 0.09325$ WWW.
		5	

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Question	Answer	Marks	Guidance
6(b)	Method 1		
	$[1 - P(8, 9, 10) =]$	M1	One term ${}^{10}C_x (p)^x (1-p)^{10-x}$, $0 < p < 1$, $x \neq 0$ or 10.
	$1 - ({}^{10}C_8 0.85^8 0.15^2 + {}^{10}C_9 0.85^9 0.15^1 + 0.85^{10})$	A1	Correct unsimplified expression. Condone omission of last bracket only.
	$[= 1 - (0.275897 + 0.347425 + 0.196874)]$		
	$= 0.180$	B1	$0.1795 < p \leq 0.180$
	Method 2		
	$[P(0, 1, 2, 3, 4, 5, 6, 7) =]$	(M1)	One term ${}^{10}C_x (p)^x (1-p)^{10-x}$, $0 < p < 1$, $x \neq 0$ or 10.
	$0.15^{10} + {}^{10}C_1 0.85 \times 0.15^9 + \dots + {}^{10}C_7 0.85^7 0.15^3$	(A1)	Correct unsimplified expression.
	$= 0.180$	(B1)	$0.1795 < p \leq 0.180$
6(c)	$0.25 \times 0.6 \times 0.15 \times 6$	M1	$0.25 \times 0.6 \times 0.15 \times k$, k an integer > 1 .
	$0.135, \frac{27}{200}$	A1	
		2	

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Question	Answer	Marks	Guidance
7(a)	$\left[\frac{10!}{2!4!} \right] 75600$	B1	
		1	
7(b)	$4! \times 3!$	M1	4! SOI in all terms leading to final answer. Allow 24 if $4! = 24$ is seen.
		M1	Ignoring any values used to justify 4!. Either 3! SOI in expression leading to final answer, or at least 6 distinct scenarios identified and added in expression leading to final answer. Condone 3 distinct scenarios $\times 2$. Ignore repeated scenarios.
		A1	$4! \times 3!$ Fully correct unsimplified expression leading to final answer.
	144	B1	WWW
		4	

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Question	Answer	Marks	Guidance
7(c)	Method 1: If denominator is from 7(a), no denominator or incorrect denominator		
	[Numerator = Number of required arrangements =]	B1	$\frac{5!}{2!}$ seen (arrangements of consonants).
	$\frac{5!}{2!} \times \frac{5!}{4!} \times 2$ [= 600]	B1	$\frac{5!}{4!}$ seen (arrangements of vowels).
		M1	$\frac{5!}{r} \times \frac{5!}{s} \times 2$, $r = 1$ or 2 , $s = 1, 4, 4!$ or 24 .
	[Probability =] $\frac{\text{their } 600}{\text{their } 75600}$	M1	$\frac{\text{their } 600}{\text{their (a)}}$ or $\frac{\text{their } 600}{75600}$.
	$= \frac{1}{126}$, 0.00794	A1	Accept $\frac{600}{75600}$ OE.
	Method 2: If denominator 10!		
	[Numerator = Number of required arrangements =]	(B1)	5! seen (arrangements of consonants).
	$5! \times 5! \times 2$ [= 28800]	(B1)	A second 5! seen (arrangements of vowels).
		(M1)	$5! \times 5! \times k$, $k = 1$ or 2 .
	[Probability =] $\frac{\text{their } 28800}{10!}$	(M1)	
	$= \frac{1}{126}$, 0.00794	(A1)	Accept $\frac{600}{75600}$ OE.

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Question	Answer	Marks	Guidance
7(c)	Method 3: Using probabilities		
	$\frac{5}{10} \times \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5} \times \frac{2}{4} \times \frac{2}{3} \times \frac{1}{2} \times \frac{1}{1} \times 2$	(B1)	$\frac{5}{a} \times \frac{4}{b} \times \frac{3}{c} \times \frac{2}{d} \times \frac{1}{e}$ seen. $10 \geq a > b > c > d > e \geq 1$ (arrangements of consonants).
		(B1)	A second $\frac{5}{f} \times \frac{4}{g} \times \frac{3}{h} \times \frac{2}{i} \times \frac{1}{j}$ seen. $10 \geq f > g > h > i > j \geq 1$ (arrangements of vowels).
		(M1)	$\frac{5}{a} \times \frac{4}{b} \times \frac{3}{c} \times \frac{2}{d} \times \frac{1}{e} \times \frac{5}{f} \times \frac{4}{g} \times \frac{3}{h} \times \frac{2}{i} \times \frac{1}{j} \times k$ $k = 1 \text{ or } 2.$
		(M1)	$\frac{\text{their } 5 \times 4 \times 3 \times 2 \times 1 \times 5 \times 4 \times 3 \times 2 \times 1}{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}.$
	$= \frac{1}{126}, 0.00794$	(A1)	Accept $\frac{28800}{362800}$ OE.
		5	

Cambridge International AS & A Level

MATHEMATICS**9709/53**

Paper 5 Probability & Statistics 1

May/June 2024**MARK SCHEME**Maximum Mark: 50

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- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer						Marks	Guidance																		
1(a)	<table><tr><td>x</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td></tr><tr><td>$P(X = x)$</td><td>$\frac{1}{36}$</td><td>$\frac{4}{36}$</td><td>$\frac{10}{36}$</td><td>$\frac{12}{36}$</td><td>$\frac{9}{36}$</td></tr><tr><td></td><td>$\frac{1}{36}$</td><td>$\frac{1}{9}$</td><td>$\frac{5}{18}$</td><td>$\frac{1}{3}$</td><td>$\frac{1}{4}$</td></tr></table>						x	2	3	4	5	6	$P(X = x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$		$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$	B1	Table with correct X values and at least one correct probability associated with the correct X value. Values need not be in order, lines may not be drawn, may be vertical, X and $P(X)$ may be omitted. Condone any additional X values if probability stated as 0.
	x	2	3	4	5	6																				
	$P(X = x)$	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$																				
		$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$																				
							B1	Three other probabilities associated with correct x values, need not be in table, accept unsimplified.																		
Decimal equivalent 3sf: 0.0278, 0.111, 0.278, 0.333, 0.25						B1	Five correct probabilities linked with correct outcomes, may not be in table. Decimals correct to at least 3sf. SC B1 for five probabilities summing to 1 placed in a probability distribution table with the correct x values.																			
						3																				

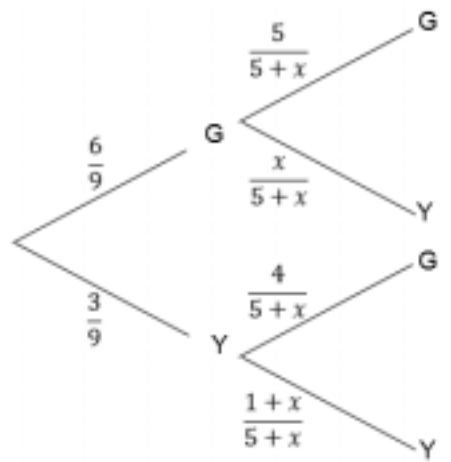
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Question	Answer	Marks	Guidance
1(b)	$E(X) = \frac{1 \times 2 + 4 \times 3 + 10 \times 4 + 12 \times 5 + 9 \times 6}{36} =$ $\frac{2}{36} + \frac{12}{36} + \frac{40}{36} + \frac{60}{36} + \frac{54}{36} \left[= \frac{14}{3} \text{ or } 4.67 \right]$	M1	Accept unsimplified expression or sum of fractions seen. May be calculated in variance. FT <i>their</i> table with five probabilities summing to $0.999 \leq \text{total} \leq 1$ ($0 < p < 1$).
	$\text{Var}(X) = \frac{1 \times 2^2 + 4 \times 3^2 + 10 \times 4^2 + 12 \times 5^2 + 9 \times 6^2}{36} - \left(\frac{14}{3} \right)^2$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 4 or more probabilities. ($0 < p < 1$) which need not sum to 1. Note: If table is correct, then $\left(\frac{824}{36} \text{ or } \frac{206}{9} \text{ or } 22.89 \right) - \left(\frac{196}{9} \text{ or } 21.78 \text{ or } \left(\frac{14}{3} \right)^2 \right)$ implies M1.
	$\left[= \frac{824}{36} - \frac{196}{9} = 22.89 - 21.78 \right] = \frac{10}{9}$	A1	$1\frac{1}{9}$, 1.11[1], 1.1
		3	
1(c)	$P(X \text{ even} \mid X > 3) = \frac{\frac{10}{36} + \frac{9}{36}}{\frac{31}{36}}$	M1	$\frac{\text{their } P(4) + \text{their } P(6)}{\text{their } P(4) + P(5) + P(6)}$, all probabilities ($0 < p < 1$). If sample space seen in any part of the question, then M1 $\frac{\text{their } 19}{\text{their } 31}$.
	$= \frac{19}{31}$	A1	0.613
		2	

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Question	Answer	Marks	Guidance
2(a)	$P\left(Z > \frac{1.93 - 1.64}{0.25}\right) = P(Z > 1.16)$	M1	Using $\pm$ standardisation formula with 1.93, 1.64 and 0.25 substituted, not σ^2 , not $\sqrt{\sigma}$, no continuity correction.
	$1 - 0.8770$	M1	Appropriate area Φ resulting from a standardisation, from final process, must be probability. Note: the appropriate probability answer implies this M1.
	0.123	A1	$0.123 \leq p < 0.12303$ If M0 M0, SC B1 if no standardisation shown.
		3	
2(b)	$\frac{1.56 - \mu}{\sigma} = -0.44$	B1	$-0.441 < z_1 < -0.439$ or $0.439 < z_1 < 0.441$ seen.
	$\frac{1.86 - \mu}{\sigma} = 0.674$	B1	$z_2 = 0.674$ or $z_2 = -0.674$ seen, CAO, critical value.
		M1	Use of the $\pm$ standardisation formula once with μ, σ equating to a z-value (not 0.33, 0.67, 0.25, 0.75, 0.6293, 0.5987, 0.7486, 0.7734, $(1 - 0.44)$, $(1 - 0.674)$). Condone continuity correct ± 0.005 , not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ	M1	Solve two equations in μ and σ using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ .
	$\mu = 1.68, \sigma = 0.269$	A1	AWRT $\mu = 1.68, \sigma = 0.269$. If one or both of the M marks have not been awarded, SC B1 for both correct.
		5	

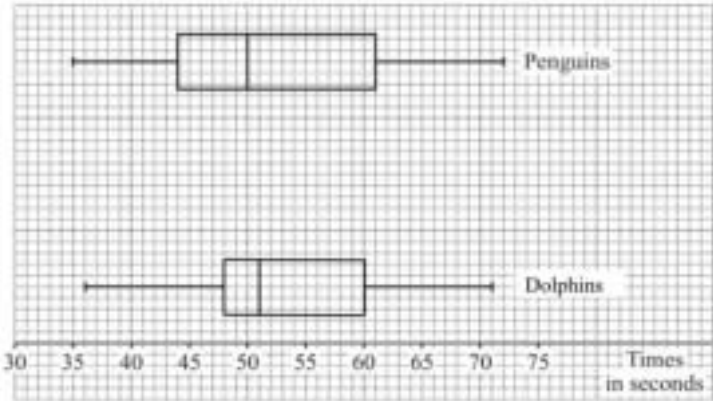
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Question	Answer	Marks	Guidance
3(a)	Box A Box B 	B1	Correct structure and probabilities for Box A branches.
		B1	Completely correct structure and one correct probability for a Box B branch including label for G or Y.
		B1	Completely correct structure and second correct probability on a Box B branch including label G or Y.
		B1	Completely correct structure and remaining two probabilities correct on Box B branches, including labels for G or Y. SC B1 if correct shape diagram but only four correct algebraic probs for GG, GY, YG and YY.
		4	
3(b)	$P(\text{same colour}) = \frac{6}{9} \times \frac{5}{5+x} + \frac{3}{9} \times \frac{1+x}{5+x}$	M1	$P(GG) + P(YY) =$ $\left(\frac{6}{9} \text{ or } \frac{3}{9}\right) \times \text{their} \left(\frac{5}{5+x}\right) + \left(\frac{3}{9} \text{ or } \frac{6}{9}\right) \times \text{their} \left(\frac{1+x}{5+x}\right)$
	$\frac{6}{9} \times \frac{5}{5+x} + \frac{3}{9} \times \frac{1+x}{5+x} = \frac{8}{15} \text{ and arrange as a linear equation}$	M1	$15(x+11) = 24(x+5)$ OE Accept sum of their products equated to $\frac{8}{15}$ and rearranged to form a linear equation.
	Solve: $x = 5$	A1	
		3	

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Question	Answer					Marks	Guidance						
4(a)	Penguins			Dolphins		B1	Correct stem, ignore extra values (not in reverse, not split). If a split stem-and-leaf plot is used (i.e. stem values are repeated), the remaining B marks are available.						
	9	5	3	6									
	8	5	5	4	2	4	1	3	8	9	9	B1	Correct Penguins labelled on left, leaves in order from right to left and lined up vertically (less than halfway to next column), no commas or other punctuation.
	9	8	6	0	5	0	1	4	6	6	B1	Correct Dolphins labelled on same diagram, leaves in order and lined up vertically (less than halfway to next column), no commas or other punctuation.	
	8	6	1	6	0	1	4						
	2		7	1									
	Key: 2 4 1 means 42 seconds for Penguins and 41 seconds for Dolphins						If the correct data for Penguins & Dolphins is transposed, treat as a single error in Penguins and condone in Dolphins.						
						B1	Correct key for their diagram, need both clubs labelled and ‘sec’ or ‘s’ stated at least once here, or in leaf headings or title. If two separate diagrams drawn: SC B1 if both keys meet these criteria.						
						4							

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Question	Answer	Marks	Guidance
4(b)			
	For Dolphins, median is 51	B1	Plotted on box.
	LQ = 48, UQ = 60	B1	Plotted on box.
	Correct end points for whiskers and diagram labelled Dolphins	B1	Correct end points of whiskers (36 and 71). Whiskers not through box, not drawn at corners of boxes, diagram labelled.
		3	
4(c)	Dolphins have more consistent times than Penguins or Penguins are faster (have faster times) than Dolphins	B1	Reason given in context. Can be reference to either the central tendency or spread.
		1	

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Question	Answer	Marks	Guidance
5(a)	$\left[(0.35)^4 \times 0.65 = \right] 0.00975$	B1	AWRT
		1	
5(b)	$(0.35)^3 \times (0.65)^2 \times 4$	M1	$(0.35)^3 \times (0.65)^2 \times k$, where k is an integer. $1 \leq k \leq 5$ no + or –
	$= 0.0725$	A1	
		2	
5(c)	Method 1		
	$[1 - P(5, 6, 7) =]$ $1 - ({}^7C_5 0.65^5 0.35^2 + {}^7C_6 0.65^6 0.35^1 + 0.65^7)$ $[= 1 - (0.29848 + 0.18478 + 0.049022)]$	M1	One term ${}^7C_x (p)^x (1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer
	$= 0.468$	B1	AWRT
	Method 2		
	$[P(0, 1, 2, 3, 4) =]$ $0.35^7 + {}^7C_1 0.65^1 0.35^6 + {}^7C_2 0.65^2 0.35^5 + {}^7C_3 0.65^3 0.35^4 + {}^7C_4 0.65^4 0.35^3$ $[0.00064 + 0.00836 + 0.04660 + 0.14424 + 0.26787]$	(M1)	One term ${}^7C_x (p)^x (1-p)^{7-x}$, $0 < p < 1, 0 < x < 7$.
		(A1)	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	$= 0.468$	(B1)	AWRT
		3	

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Question	Answer	Marks	Guidance
5(d)	[Mean =] $84 \times 0.65 = 54.6$ [Var =] $84 \times 0.65 \times 0.35 = 19.11$	B1	54.6 and 19.11 seen, allow unsimplified. May be seen in the standardisation formula $([\sigma =] 4.371 \text{ or } \frac{7\sqrt{39}}{10} \text{ implies correct variance}).$ Incorrect notation is penalised, but condone use of values in standardisation formula.
	$P(X > 50) = P\left(Z > \frac{50.5 - 54.6}{\sqrt{19.11}}\right)$	M1	Substituting <i>their</i> μ and <i>their</i> positive σ into the $\pm$ standardising formula (any number), not <i>their</i> σ^2 , or $\sqrt{\text{their } \sigma}$.
		M1	Use continuity correction 49.5 or 50.5 in <i>their</i> standardisation formula. Note: $\frac{\pm 4.1}{\sqrt{19.11}}$ or $\pm \frac{4.1}{4.371}$ seen gains M2 BOD.
	$P(Z > -0.9379) = \Phi(0.9379)$	M1	Appropriate area Φ , from final process, must be a probability.
	0.826	A1	$0.8255 < p \leq 0.826$
		5	

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Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!3!} \right] 30240$	B1	
		1	
6(b)	Method 1: Number of arrangements with E at each end – Number of arrangements with E at each end and the three Rs together		
	$\frac{7!}{3!} - 5! =$	B1	$\frac{7!}{3!} - e$, ${}^7P_4 - e$, e a positive integer.
		M1	$f - \frac{5!}{r!}$, $f > 120$, $r = 1, 2$
	720	A1	If no marks scored SC B1 for $840 - 120 = 720$.
	Method 2: Number of arrangements with E at each end and no Rs together + No of arrangements with E at each end and two Rs together		
	${}^5C_3 \times 4! + {}^4C_1 \times 5!$ or $\frac{{}^5P_3}{3!} \times 4! + {}^5P_2 \times 4!$	(B1)	One of ${}^5C_3 \times 4!$, $\frac{{}^5P_3}{3!} \times 4!$, ${}^4C_1 \times 5!$ or ${}^5P_2 \times 4!$ seen.
		(M1)	$a \times 4! + b \times 5!$ where a and b are integers between 1 and 10 inclusive, or $c \times 4! + d \times 4!$ where c and d are integers between 1 and 20 inclusive.
	$240 + 480 = 720$	(A1)	
		3	

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Question	Answer	Marks	Guidance
6(c)	Method 1		
	Group of 5 $3 \text{ Rs } 2 \text{ Es} = 1$ $3 \text{ Rs } 1 \text{ E} = {}^2\text{C}_1 \times {}^4\text{C}_1 = 8$ $3 \text{ Rs } 0 \text{ Es} = {}^4\text{C}_2 = 6$ Group of 4 $3 \text{ Rs } 1 \text{ E} = {}^2\text{C}_1 = 2$ $3 \text{ Rs } 0 \text{ Es} = {}^4\text{C}_1 = 4$	B1	Correct no of ways for two correct identified scenarios other than three Rs two Es.
	[Total =] 21	M1	No of ways for five correct identified scenarios added or correct.
	[Number of ways of splitting into the two groups =] ${}^9\text{C}_5 (= 126)$ seen as a denominator	M1	Accept evaluated, accept ${}^9\text{C}_4$.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	A1	

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Question	Answer	Marks	Guidance
6(c)	Method 2		
	3Rs in Group of 5 = ${}^6C_2$ = 15	(B1)	One correct case evaluated accurately and linked with correct scenario.
	3Rs in Group of 4 = ${}^6C_1$ = 6		
	[Total =] 21	(M1)	No of ways for two correct scenarios added or correct.
	[Number of ways of splitting into the two groups =] ${}^9C_5$ (= 126) seen as a denominator	(M1)	Accept evaluated, accept ${}^9C_4$.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	(A1)	
	Method 3: Considering the possible positions of R within the groups		
	3Rs in Group of 5 $\frac{5}{9} \times \frac{4}{8} \times \frac{3}{7} = \frac{15}{126}$	(B1)	For one correct product unsimplified and linked with correct scenario.
	3Rs in Group of 4 $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} = \frac{6}{126}$	(M1)	For second correct product.
	$\frac{15}{126} + \frac{6}{126}$	(M1)	For adding probabilities of two correct scenarios or correct.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	(A1)	

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Question	Answer	Marks	Guidance
6(c)	Method 4: Probability method		
	Group of 5 2Es $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{2}{6} \times \frac{1}{5} \times \frac{5!}{3!2!} = \frac{1}{126}$	(B1)	Two correct probabilities linked with correct scenarios, accept unsimplified.
	1E $\frac{4}{9} \times \frac{3}{8} \times \frac{2}{7} \times \frac{1}{6} \times \frac{2}{5} \times \frac{5!}{3!} = \frac{8}{126}$ 0E $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{4}{6} \times \frac{3}{5} \times \frac{5!}{3!2!} = \frac{6}{126}$	(M1)	Four probabilities with denominators including a factor of $9 \times 8 \times 7 \times 6 \times n$, where n is 1 or 5.
	Group of 4 $\frac{3}{9} \times \frac{2}{8} \times \frac{1}{7} \times \frac{6}{6} \times \frac{4!}{3!} = \frac{6}{126}$		
	$\frac{1+8+6+6}{126}$	(M1)	Probabilities of four correct scenarios added or correct.
	Probability = $\frac{21}{126} = \frac{1}{6}$ (0.167)	(A1)	
		4	