

Cambridge International AS & A Level

MATHEMATICS**9709/11**

Paper 1 Pure Mathematics 1

May/June 2024**MARK SCHEME**Maximum Mark: 75

Published

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This document consists of **18** printed pages.

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

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Question	Answer	Marks	Guidance
1(a)	$3(y-2)^2 - 27$ or $a = -2, b = -27$	B1 B1	
		2	
1(b)	$(x^2 - 2)^2 = 9$ leading to $x^2 - 2 = \pm 3$	M1	Must be x^2 unless substitution is clear.
	$x^2 = -1$ or $x^2 = 5$	M1	Allow omission of -1 if ± 3 seen.
	$x = \pm\sqrt{5}$	A1	B1 SC if M1M1 not awarded. Ignore $\pm i, i, -i, \sqrt{-1}$. Use of calculator with no working scores 0/3.
	Alternative method for Question 1(b)		
	$3x^4 - 12x^2 - 15 = 0$ leading to $3(x^2 - 5)(x^2 + 1) [= 0]$	(M1)	
	$x^2 = -1$ or $x^2 = 5$	(M1)	Allow omission of -1 if factors seen. Factorising or other valid method.
	$x = \pm\sqrt{5}$	(A1)	B1 SC if M1M1 not scored. Ignore $\pm i, i, -i, \sqrt{-1}$. Use of calculator with no working scores 0/3.
		3	

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Question	Answer	Marks	Guidance
2(a)	{Stretch} {factor 3} { in y-direction}	B2,1,0	2 out of 3 scores B1.
	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	B2,1,0	Accept shift.
	Alternative Method for Question 2(a)		
	{Translation} $\begin{pmatrix} \{0\} \\ \left\{-\frac{2}{3}\right\} \end{pmatrix}$	(B2,1,0)	2 out of 3 scores B1. Accept shift.
	{Stretch} {factor 3} { in y-direction}	(B2,1,0)	
		4	
2(b)	$[f(x)] = \{-3\sin x\} \{-2\}$	B1 B1	No marks awarded if extra terms seen.
		2	

Question	Answer	Marks	Guidance
3(a)	$20 \times 27 \times a^3 [=160]$	M1	Allow $6C3 \times 3^3 \times a^3 [=160]$. Accept $540a^3$ with no other working for M1.
	$[a] = \frac{2}{3}$	A1	Allow 0.667 AWRT. SC B1 is $a = \frac{2}{3}$ with no other working.
		2	

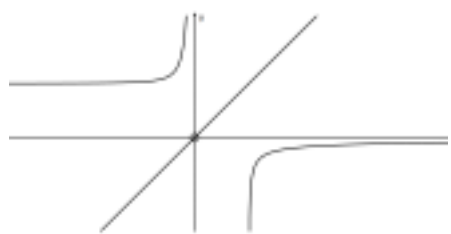
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Question	Answer	Marks	Guidance
3(b)	Coefficient of x^2 is $15 \times 81 \times \left(\text{their} \frac{2}{3}\right)^2 [= 540]$	B1 FT	May be in a list. $6C2$ and 3^4 must be evaluated but may be implied by later work. Condone 540 with no working.
	$160 \times 1 - 2 \times \text{their} 540$	M1	
	$= -920$	A1	Condone $-920x^3$.
		3	

Question	Answer	Marks	Guidance
4(a)	$[k] = 4.00063$	B1	CAO
		1	
4(b)	$[\text{Gradient } AE] = 6.3566$	B1	CAO
		1	
4(c)	Suggests that $[f(2)] = 6.25$	B1	CAO
		1	

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Question	Answer	Marks	Guidance
5(a)	$\frac{\sin^2 x - \cos x - 1}{1 + \cos x} = \frac{1 - \cos^2 x - \cos x - 1}{1 + \cos x}$ or $\frac{-\cos^2 x - \cos x}{1 + \cos x}$	M1	For use of $\sin^2 x + \cos^2 x = 1$. Allow use of s, c, t or omission of x throughout.
	$= \frac{-\cos x(1 + \cos x)}{1 + \cos x}$	M1	For factorising.
	$= -\cos x$	A1	
		3	
5(b)	$-\frac{1}{2}\cos x = \frac{1}{4} \Rightarrow x = \cos^{-1}\left(-\frac{1}{2}\right)$	M1	
	$x = 120^\circ$ or $x = 240^\circ$	A1	
		A1 FT	FT for 360 – <i>their</i> answer. A1 A0 if extra solution(s) in range. SC B1 if answer in radians for both $\frac{2\pi}{3}, \frac{4\pi}{3}$.
		3	

Question	Answer	Marks	Guidance
6(a)		B1	For curve in correct quadrant.
		B1	Fully correct including line $y = x$. Horizontal asymptote closer to x axis than vertical asymptote is to y axis.
		2	

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Question	Answer	Marks	Guidance
6(b)	$x = \frac{2}{y^2} + 4$ leading to $y^2(x-4)=2$ or $y^2 = \frac{2}{(x-4)}$	M1	Allow x and y swapped around.
	$y^2 = \frac{2}{(x-4)}$ leading to $y = [\pm]\sqrt{\frac{2}{x-4}}$ or $x = [\pm]\sqrt{\frac{2}{y-4}}$	M1	
	$[f^{-1}(x)] = -\sqrt{\frac{2}{x-4}}$	A1	
		3	
6(c)	$[x] = -2$	B1	
		1	
6(d)	Because f^{-1} is always negative and f is always positive or curves do not intersect	B1	Accept other correct answers e.g. ‘ f is only defined for positive values of x and f^{-1} is only defined for negative values of x ’ or ‘domains do not overlap’ or ‘the y values cannot be the same’ or ‘the x values cannot be the same’.
		1	

Question	Answer	Marks	Guidance
7(a)	Angle $\theta = \frac{\pi}{2} - \cos^{-1} \frac{10}{15}$ or $\sin^{-1} \frac{10}{15} = 0.7297$	B1	Condone working in degrees if converted to radians at the end. AG
		1	

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Question	Answer	Marks	Guidance
7(b)	$BC = \sqrt{15^2 - 10^2} [= 11.18... \text{ or } 5\sqrt{5}]$	B1	
	$\text{Arc } AB = 15 \times 0.7297 [= 10.9455]$	B1	
	$\text{Perimeter} = \text{their } BC + \text{their arc } AB + 25 + 5\pi$	M1	
	$\text{Perimeter} = 62.8$	A1	AWRT
	$\text{Area sector } AOB = \frac{1}{2} \times 15^2 \times 0.7297 [= 82.09]$	B1	
	$\text{Area} = \frac{1}{2} \times 10 \times \text{their } BC + \text{their sector } AOB + \frac{\pi}{4} \times 10^2$	M1	
	$\text{Area} = 217$	A1	AWRT
		7	

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Question	Answer	Marks	Guidance
8(a)	$2(4p-1)=25+13-p$	*M1	
	$p = \frac{40}{9}$	A1	
	$d = \left(4 \times \frac{40}{9} - 1\right) - 25 \left[= -\frac{74}{9} \right]$	DM1	Using <i>their p</i> to find <i>d</i> .
	10 th term = $25 + 9d = -49$	A1	
	Alternative Method for first 3 marks of Question 8(a)		
	$d = 4p - 26$, $d = 14 - 5p$, $p + 2d = -12$ Any two	(*M1)	Allow unsimplified or equivalent.
	Solving simultaneously to find p or d	(DM1)	
	$\left[p = \frac{40}{9} \right], d = -\frac{74}{9}$	(A1)	
		4	

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Question	Answer	Marks	Guidance
8(b)	$(4q-1)^2 = 25(13-q) \Rightarrow 16q^2 + 17q - 324 [=0]$	M1	
	$(q-4)(16q+81)[=0]$ leading to $[\Rightarrow q=4]$	M1	Solve 3 term quadratic with real solutions.
	$[r =]\frac{3}{5}$	A1	Ignore $\frac{-17}{20}$.
	Sum to infinity $= \frac{25}{1 - \frac{3}{5}} = \frac{125}{2}$	A1	Ignore extra solution. SC B1 if no method shown for solving quadratic.
	Alternative Method for Question 8(b)		
	$25r = 4q - 1, 25r^2 = 13 - q$ leading to $100r^2 + 25r - 51 = 0$	(M1)	
	$(5r-3)(20r+17)=0$	(M1)	Solve 3 term quadratic with real solutions.
	$r = \frac{3}{5}$	(A1)	Ignore $\frac{-17}{20}$.
	Sum to infinity $= \frac{25}{1 - \frac{3}{5}} = \frac{125}{2}$	(A1)	Ignore extra solution. SC B1 if no method shown for solving quadratic.
		4	

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Question	Answer	Marks	Guidance
9	Volume of cylinder = $\pi \times 1^2 \times \frac{7}{5} = \frac{7}{5}\pi$	B1	May be done using $\int_1^{2.4} 1$. This would be the only mark available if candidate integrates y .
	Volume under curve = $[\pi] \int \frac{1}{(5x-4)^{\frac{2}{3}}} dx$	M1	No further marks available if $\int y$.
	$= [\pi] \left\{ \frac{3}{5} \right\} \left\{ (5x-4)^{\frac{1}{3}} \right\}$	B1 B1	Calculator used for integration scores no further marks.
	$= [\pi] \frac{3}{5} \left(8^{\frac{1}{3}} - 1 \right) \left[= \frac{3}{5}\pi \right]$	M1	Uses limits 1, 2.4 in an integral of y^2 .
	Volume = $\frac{7}{5}\pi - \frac{3}{5}\pi = \frac{4}{5}\pi$	A1	SC B1 if the only error is not showing substitution.
		6	

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Question	Answer	Marks	Guidance
10	$(x-3)^2 + y^2 = 18$ $y = mx - 9$ leading to $(x-3)^2 + (mx-9)^2 = 18$	M1	Finding equation of tangent and substituting into circle equation. Must be $mx-9$.
	$x^2 - 6x + 9 + m^2x^2 - 18mx + 81 = 18$ leading to $(m^2 + 1)x^2 - (6 + 18m)x + 72 = 0$	M1	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant. m cannot be numeric.
	$(6 + 18m)^2 - 4(m^2 + 1) \times 72 = 0$	*M1	Use of $b^2 - 4ac$. Not in quadratic formula. m cannot be numeric, c must be numeric.
	$36m^2 + 216m - 252 = 0$ [leading to $m^2 + 6m - 7 = 0$]	DM1	Simplifies to 3 term quadratic.
	$m = 1$ or $m = -7$	A1	Condone no method for solving quadratic shown.
	$m = 1$ leading to $2x^2 - 24x + 72 = 0$ leading to $x = 6$	DM1	Must be correct x for <i>their</i> quadratic.
	$m = -7$ leading to $50x^2 + 120x + 72 = 0$ leading to $x = -\frac{6}{5}$	DM1	Must be correct x for <i>their</i> quadratic.
	$(6, -3), \left(-\frac{6}{5}, -\frac{3}{5}\right)$	A1	

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Question	Answer	Marks	Guidance
10	Alternative Method 1 for first 4 marks of Question 10		
	$\frac{ 3m-1(0)-9 }{\sqrt{m^2+1}}$	(M1)	Use of the formula for the length of a perpendicular from a point to a line.
	$\frac{ 3m-1(0)-9 }{\sqrt{m^2+1}} = \sqrt{18}$	(M1)	Equates length of a perpendicular from a point to a line to the radius.
	$(3m-9)^2 = 18(m^2+1)$	(M1)	Squares and clears the fraction.
	$9m^2 - 54m + 81 = 0$ [leading to $m^2 + 6m - 7 = 0$]	(M1)	
	Alternative Method 2 for first 3 marks of Question 10		
	$(3-x)(9+6x-x^2)^{-1/2} = m$	(M1)	OE Differentiates implicitly or otherwise and equates $\frac{dy}{dx}$ to m .
	$(1+m^2)x^2 - 6(1+m^2)x + 9(1-m^2) = 0$	(M1)	Brackets expanded and all terms collected on one side of the equation. May be implied in the discriminant.
	$36(1+m^2)^2 - 4(1+m^2) \times 9(1-m^2) = 0$	(M1)	Use of $b^2 - 4ac$.
		8	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2}$	B1	
	$\frac{dy}{dx} = -\frac{12}{x^4} + \frac{3}{x^2} = 0$ leading to $3x^4 - 12x^2 = 0$ or $-12 + 3x^2 = 0$	M1	Set = 0 or uses <, ≤ and simplifies. Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
	$3x^2(x^2 - 4) = 0$ leading to $x = \pm 2$ only	A1	SC B1 for $x = \pm 2$ if M0 scored.
	$-2 < x < 0$ and $0 < x < 2$ or $(-2, 0)$ and $(0, 2)$ or $-2 < x < 2$ and $x \neq 0$	B1FT	Allow and/or.
		B1FT	Allow $-2 \leq x < 0$ and / or $0 < x \leq 2$ but only B1B0 if 0 included in either or both. Allow $[-2, 0)$ and $(0, 2]$. Allow B1B0 for $-2 < x < 2$ or $(-2, 2)$. Must be from $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.
		5	B marks only available if $\frac{dy}{dx} = \frac{A}{x^4} + \frac{B}{x^2}$.

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Question	Answer	Marks	Guidance
11(b)	[At $x = 1$] $y = 3$ and $m \tan = -9$	*M1	Using <i>their</i> $\frac{dy}{dx}$.
	$m \text{ norm} = -\frac{1}{-9} = \frac{1}{9}$	DM1	
	Equation of normal is $y - 3 = \frac{1}{9}(x - 1)$ [leading to $y = \frac{1}{9}x + \frac{26}{9}$]	A1	
	At $x = -1, y = 1, m = -9$	M1	
	Equation of tangent is $y - 1 = -9(x + 1)$ [leading to $y = -9x - 8$]	A1	
	Meet when $\frac{1}{9}x + \frac{26}{9} = -9x - 8$ [leading to $x = -1.19512\dots, \frac{-49}{41}$]	M1	Equates <i>their</i> tangent and <i>their</i> normal.
	Area = $\frac{1}{2} \times \text{their } 1.19512\dots \times \text{their} \left(\frac{26}{9} + 8 \right)$	M1	If $\int y_2 - y_1$ is used integration must be correct and substitution shown.
	6.51	A1	AWRT Accept fraction wrt 6.51
		8	

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1	$240[x^2]$ or $80a^2[x^2]$	B1	May be seen in an expansion.
	$240 = 12 \times 80a^2$	M1	<i>Their</i> 240 equated to $12 \times \textit{their } 80a^2$ which must contain a^2 .
	0.5	A1	OE Condone ± 0.5
		3	

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Question	Answer	Marks	Guidance
2	Stretch factor 4 in y-direction/parallel to the y axis/vertically.	B1	Allow use of SF in place of factor. Allow in/on/along the y axis or 'the x axis is invariant.'
	Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or 3 parallel to the x axis or in the x direction, allow horizontally. $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$ or -8 parallel to the y axis or in the y direction, allow vertically.	B2	Condone 'Shift'. These translations can be combined as $\begin{pmatrix} 3 \\ -8 \end{pmatrix}$, this counts as 2 elements. Give priority to a correct vector over any incorrect wording. B2 for all 3 B1 for 2 out of 3
	Two translations, one in each direction, and a stretch only.	M1	Condone inaccurate terminology, such as up, down, left and right, if the intention is clear.
	Correct order of operations. The stretch which must be in the in the y direction must come before the translation in the y direction.	A1	Condone inaccurate terminology if the intention is clear but numerical values must be correct.

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Question	Answer	Marks	Guidance
2	Alternative Method for Question 2		
	Translation $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$ or 3 parallel to the x axis or in the x direction, allow horizontally. $\begin{pmatrix} 0 \\ -2 \end{pmatrix}$ or -2 parallel to the y axis or in the y direction, allow vertically.	(B2)	Condone ‘Shift’. These translations can be combined as $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$, this counts as 2 elements. Give priority to a correct vector over any incorrect wording. B2 for all 3. B1 for 2 out of 3.
	Stretch factor 4 in y -direction/parallel to the y axis/vertically.	(B1)	Allow use of SF in place of factor. Allow in/on/along the y axis or “the x axis is invariant.”
	Two translations, one in each direction, and a stretch only.	(M1)	Condone inaccurate terminology, such as transform, move, up, down, left and right, if the intention is clear.
	Correct order of operations. The stretch which must be in the in the y direction must come after the translation in the y direction.	(A1)	Condone inaccurate terminology if the intention is clear but numerical values must be correct.
		5	

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Question	Answer	Marks	Guidance
3(a)	$7 \frac{\sin \theta}{\cos \theta} \div \cos \theta + 12 [= 0] \left[\text{leading to } 7 \frac{\sin \theta}{\cos \theta} + 12 \cos \theta = 0 \right]$	M1*	OE Use of $\tan \theta = \frac{\sin \theta}{\cos \theta}$.
	$7 \sin \theta + 12(1 - \sin^2 \theta) [= 0]$	DM1	Use of $s^2 + c^2 = 1$.
	$\Rightarrow 12 \sin^2 \theta - 7 \sin \theta - 12 = 0$	A1	AG, WWW Condone use of s, c and t and/or omission of θ throughout working but the A1 is for cao.
		3	
3(b)	$[12 \sin^2 \theta - 7 \sin \theta - 12 = 0 \text{ leading to}] (4 \sin \theta + 3)(3 \sin \theta - 4)$	M1	
	$\sin \theta = -\frac{3}{4} \left[\text{or } \frac{4}{3} \right]$	B1	OE, WWW Can be implied by a correct value for $\sin^{-1}\left(-\frac{3}{4}\right)$ e.g. -48.6° .
	$[\theta =] 228.6^\circ, 311.4^\circ$	B1	AWRT, WWW No others in the range $0^\circ \leq \theta \leq 360^\circ$. Ignore any answers outside this range. Condone $229^\circ, 311^\circ$.
		3	

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Question	Answer	Marks	Guidance
4(a)	$\left[f^{-1}(x) = \right] (x+1)^2$	B1	ISW Condone 'y = '.
		1	
4(b)	$0 < g(x) \leq \frac{1}{2}$ or $g(x) > 0$ and $g(x) \leq \frac{1}{2}$ or $\left(0, \frac{1}{2} \right]$	B1	Do not allow $g(x) > 0, g(x) \leq \frac{1}{2}$. Do not allow $g(x) > 0$ or $g(x) \leq \frac{1}{2}$. Condone g or y in place of $g(x)$.
	g^{-1} does not exist because it is one to many or g^{-1} does not exist because it is not one to one. Or g^{-1} does not exist because g is not one to one or g^{-1} does not exist because g is many to one or g^{-1} does not exist because g fails the horizontal line test.	B1	g^{-1} can be replaced by 'It' throughout. A correct statement followed by any further incorrect explanation can be awarded B1.
		2	

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Question	Answer	Marks	Guidance
4(c)	$f\left(\frac{25}{16}\right) = \frac{1}{4}$	B1	SOI
	$\frac{1}{(\sqrt{x}-1)^2+2} = \frac{1}{4}$	M1	Equating $\frac{1}{(\sqrt{x}-1)^2+2}$, or <i>their</i> ‘simplified’ version, to <i>their</i> $f\left(\frac{25}{16}\right)$.
	$\left[(\sqrt{x}-1)^2 + 2 = 4 \text{ leading to } \right] \quad \sqrt{x}-1 = \sqrt{2} \text{ leading to } x = (1 \pm \sqrt{2})^2$ Or $\left[x - 2\sqrt{x} + 1 + 2 = 4 \text{ leading to } \right] \quad x - 2\sqrt{x} - 1 = 0 \text{ leading to } x = (1 \pm \sqrt{2})^2$ Or $\left[x - 1 = 2\sqrt{x} \text{ leading to } \right] \quad x^2 - 6x + 1 = 0 \text{ leading to } x = \frac{6 \pm \sqrt{36-4}}{2}$	A1	Simplification as far as $x = \dots$ Allow just + in the results because – can be disregarded at this stage. Can be implied by the final answer. Note: $x = 1 \pm \sqrt{2}$ scores A0.
	$3 + 2\sqrt{2}$	A1	Must discount the solution $3 - 2\sqrt{2}$.
		4	

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Question	Answer	Marks	Guidance
5(a)	$[d =] \sin \theta - \tan \theta$	M1	For subtraction of <i>their</i> first term from <i>their</i> second term. Condone incorrect evaluation before subtraction.
	$[d =] \frac{\sqrt{2}}{2} - 1$	A1	OE Sight of -0.29 AWRD can be awarded M1A1.
	$[S_{40} =] \frac{40}{2}(2 \times \tan \theta + 39(\sin \theta - \tan \theta))$	M1	Use of a correct formula for S_{40} . Condone use of <i>their</i> , clearly identified, incorrect values for a and d for this mark.
	$390\sqrt{2} - 740$ or $\frac{780}{\sqrt{2}} - 740$	A1	ISW If A0 then sight of -188 AWRD, -188.5 or -189 should be awarded M1A1M1A0.
		4	
5(b)(i)	$r = \frac{\sin \theta}{\tan \theta} \quad [= \cos \theta]$	B1	Condone omission of θ .
	$S_{\infty} = \frac{\tan \theta}{1 - \cos \theta}$ or $\frac{\sin \theta}{\cos \theta - \cos^2 \theta}$ or $\frac{\tan^2 \theta}{\tan \theta - \sin \theta}$	B1	ISW Do not allow fractions within fractions nor omission of θ .
		2	

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Question	Answer	Marks	Guidance
5(b)(ii)	$a = \sqrt{3}(1.73..) \text{ and } r = \frac{1}{2}$	B1	OE, SOI.
	$[S_{10}] \sqrt{3} \left(\frac{1 - \left(\frac{1}{2}\right)^{10}}{1 - \frac{1}{2}} \right)$	M1	This mark can be awarded for a correct formula with <i>their</i> values for a and r or $a = \tan \theta$ and $r = \frac{\sin \theta}{\tan \theta}$ or $\cos \theta$. Condone $\frac{1}{2}^{10}$.
	$= 3.46$	A1	AWRT Condone $\frac{1023\sqrt{3}}{512}$.
		3	Note: S ₉ gives the same answer but scores B1M0A0.

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Question	Answer	Marks	Guidance
6(a)	$\frac{dy}{dx} = 2 - \frac{1}{2} \times 8x^{-\frac{1}{2}}$	B1	
	$2 - 4x^{-\frac{1}{2}} = 0$	M1	Equating <i>their</i> two term $\frac{dy}{dx}$, with at least one term correct, to 0.
	[A is] (4, -8) or $x = 4, y = -8$	A1	
	[B is] (16, 0) or $x = 16, y = 0$	B1	
		4	Note: Correct answers without use of $\frac{dy}{dx}$ can be awarded 4/4.

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Question	Answer	Marks	Guidance
6(b)	$[\pm] \frac{2x^2}{2} - \frac{8}{3}x^{\frac{3}{2}} \quad [+C]$	B1	Seen correct in unsimplified form or better.
	$[\pm] \frac{x^2 - 32x}{3} \text{ or } \frac{(2x - 32)^2}{12} \quad [+C]$	B1	Seen correct in unsimplified form or better.
	Attempt to integrate, defined by at least one correct power in each expression, and then subtract.	M1	Multiplying by 3 before integration scores M0.
	$\left\{ \left(16^2 - \frac{8}{3} \cdot 16^{\frac{3}{2}} \right) - \left(4^2 - \frac{8}{3} \cdot 4^{\frac{3}{2}} \right) \right\} \quad [-] \quad \left\{ \left(\frac{16^2 - 32 \times 16}{3} \right) - \left(\frac{4^2 - 32 \times 4}{3} \right) \right\}$	M1	Use of <i>their</i> x values, > 0 , from (a) as limits in <i>their</i> integrated expressions. Allow, for correct limits, sight of $\pm \left(\left\{ \left(-\frac{256}{3} \right) - \left(-\frac{80}{3} \right) \right\} [-] \left\{ \left(-\frac{256}{3} \right) - \left(-\frac{112}{3} \right) \right\} \right).$ If incorrect limits are used, then clear substitution must be seen.

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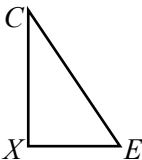
Question	Answer	Marks	Guidance
6(b)	Alternative Method 1 for first 4 marks of Question 6(b)		
	$\left[[\pm] \int 2x - 8x^{\frac{1}{2}} dx = \right] x^2 - \frac{8}{\frac{3}{2}} x^{\frac{3}{2}} [+C]$	(B1)	Seen correct in unsimplified form or better.
	[Area of triangle =] 48	(B1)	
	Attempt to integrate, defined by at least one correct power, and then subtract <i>their</i> triangle area.	(M1)	
	$\left\{ \left(16^2 - \frac{8}{\frac{3}{2}} \cdot 16^{\frac{3}{2}} \right) - \left(4^2 - \frac{8}{\frac{3}{2}} \cdot 4^{\frac{3}{2}} \right) \right\}$	(M1)	Use of <i>their</i> x values, > 0 , from (a) as limits in <i>their</i> integrated expression. Allow sight of $\pm \left(\left(-\frac{256}{3} \right) - \left(-\frac{80}{3} \right) \right)$. If incorrect limits are used, then clear substitution must be seen.

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Question	Answer	Marks	Guidance
6(b)	Alternative Method 2 for first 4 marks of Question 6(b)		
	Subtract and then integrate, defined by at least two correct powers. Condone functions being the wrong way round.	(M1)	If terms in x have not been combined use the first scheme.
	$[\pm] \left(\frac{4}{3 \times 2} x^2 - \frac{8}{\frac{3}{2}} x^{\frac{3}{2}} + \frac{32x}{3} \right)$	(B2,1,0)	B2 for 3 correct terms, B1 for any 2 correct terms.
	$[\pm] \left(\left(\frac{4}{3 \times 2} \times 16^2 - \frac{8}{\frac{3}{2}} \times 16^{\frac{3}{2}} + \frac{32 \times 16}{3} \right) - \left(\frac{4}{3 \times 2} \times 4^2 - \frac{8}{\frac{3}{2}} \times 4^{\frac{3}{2}} + \frac{32 \times 4}{3} \right) \right)$	(M1)	Use of <i>their</i> x values, >0 , from (a) as limits in <i>their</i> integrated expression. Allow sight of $\pm \left(0 - \frac{32}{3} \right)$. If incorrect limits are used, then clear substitution must be seen.
	$\frac{32}{3}$, $10\frac{2}{3}$ or 10.7	(B1)	AWRT Allow $-\frac{32}{3}$ or $-\frac{32}{3}$ changed to $+\frac{32}{3}$ for this mark.
		(5)	Condone the inclusion of π for the first 4 marks but use of $\int y^2$ scores a maximum of B1 for the triangle.

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Question	Answer	Marks	Guidance
7(a)	$(x-6)^2 + (2a-x+a)^2 = 18$	M1*	Replacing y with $2a-x$ in the circle equation, condone incorrect expansion before substitution.
	$2x^2 - 12x - 6ax + 9a^2 + 36 - 18 [= 0]$	A1	All terms collected on one side of the equation. May be implied by the discriminant.
	$(12+6a)^2 - 4 \times 2 \times (9a^2 + 18) [= 0]$	DM1	Correct use of “ $b^2 - 4ac$ ” from <i>their</i> 3 term quadratic equation in x , with an x term of the form $(m+na)x$ with both m and $n \neq 0$.
	$-36a^2 + 144a [+0 = 0]$	A1	
	$a = 0, a = 4$	A1	
		5	
7(b)	[Centre is] $(6, -4)$ or [Point of intersection is] $(9, -1)$	B1	
	[Gradient of diameter] $= 1$	B1	
	$y + 4 = x - 6$ or $y + 1 = x - 9$ [leading to $y = x - 10$]	B1FT	FT on <i>their</i> point of intersection or <i>their</i> centre with an x co-ordinate of ± 6 and gradient $= 1$.
		3	

Question	Answer	Marks	Guidance
8(a)(i)	 $X\hat{C}E = \frac{\pi}{6}$ $C\hat{E}X = \frac{\pi}{3}$		
	$\frac{XE}{0.4} = \sin \frac{\pi}{6}$ or $\frac{XE}{0.4} = \cos \frac{\pi}{3}$ [$XE = 0.2$]	M1	A correct trig expression involving XE . Do not condone a mixture of degrees and radians.
	Length $EF = 2 + 2 \times 0.2 = 2.4$	A1	AG
		2	

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Question	Answer	Marks	Guidance
8(a)(ii)	$[CX =] 0.4 \cos \frac{\pi}{6} \text{ or } 0.4 \sin \frac{\pi}{3} \text{ or } \sqrt{0.4^2 - 0.2^2}$	B1	OE, SOI Expect $\frac{\sqrt{3}}{5}$ or 0.3464.
	$[\text{Sector} =] \frac{1}{2} \times (0.4)^2 \times \frac{\pi}{3}$	B1	SOI Expect 0.0838 or $\frac{2\pi}{75}$. Allow use of $\frac{60}{360} \pi (0.4)^2$.
	Either Area of <i>their</i> (rectangle + two triangles + two sectors) Or Area of <i>their</i> (trapezium + two sectors)	M1	Either implied by a correct answer or areas clearly labelled. Expect $0.6928 + 0.06928 + 0.1676$ or $\frac{2\sqrt{3}}{5} + \frac{\sqrt{3}}{25} + \frac{4\pi}{75}$. Or $0.7621 + 0.1676$ or $\frac{11\sqrt{3}}{25} + \frac{4\pi}{75}$.
	0.930	A1	AWRT Condone $\frac{11\sqrt{3}}{25} + \frac{4\pi}{75}$.
		4	

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Question	Answer	Marks	Guidance
8(b)	[Length $AD =$] $2 + 2r$	B1	Must be seen alone or part of a list and not part of a product.
	[Arc length $=$] $r \times \frac{\pi}{3}$	B1	May be implied by $r \times \frac{\pi}{3} \times 2$. Must be seen alone or part of a list.
	[EF $=$] $2 + 2r \sin \frac{\pi}{6}$ or $2 + 2r \cos \frac{\pi}{3}$ or $2 + r$	B1	Must be seen alone or part of a list and not part of a product.
	$[4 + 3r + \frac{2\pi r}{3} = 6 \text{ leading to }] 0.393$	B1	AWRT Condone $\frac{6}{2\pi + 9}$. NB: Using $EF = 2.4$ gives 0.391.
		4	

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Question	Answer	Marks	Guidance
9(a)	$6(2x-3)^2 - 6x < 0$ or $= 0$	B1*	Condone ≤ 0 . If $6(2x-3)^2$ only or $6(2x-3)^2 - 6$ is used, do not treat as a MR.
	$24x^2 - 78x + 54$ or $4x^2 - 13x + 9$ or $(x-1)(4x-9)$ OR $6(2x-3)^2 < 6x$ leading to $(2x-3) < \sqrt{x}$ leading to $2x - \sqrt{x} - 3$	M1	Expanding brackets and collecting terms to arrive at a three term quadratic, only condone sign errors.
	$[x=]1, \frac{9}{4}$	B1	
	$1 < x < \frac{9}{4}$ or $x > 1$ and $x < \frac{9}{4}$ or $\left(1, \frac{9}{4}\right)$	DB1FT	OE Condone consistent use of $\leq$ and $\geq$ or $[]$. Do not allow $x > 1$ or $x < \frac{9}{4}$ nor $x > 1, x < \frac{9}{4}$. FT on <i>their</i> values coming from a correct initial statement.
		4	

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Question	Answer	Marks	Guidance
9(b)	$\left[f(x) = \right] \left\{ \frac{6}{3 \times 2} (2x-3)^3 \right\} \left\{ -\frac{6}{2} x^2 \right\} \quad [+C]$	B1 B1	B1 for each {Correct integral}.
	$-1 = (-1)^3 - 3 \times 1^2 + C$	M1	$f(x) = -1$ equated to <i>their</i> integrated expression, defined by two terms with at least one correct power + C, with $x = 1$.
	$\left[f(x) = \right] (2x-3)^3 - 3x^2 + 3$	A1	CAO Only condone $C = 3$ as final answer if coefficients have been simplified earlier. Do not ISW if the result is of the form $y = mx + c$.
	Alternative method for Question 9(b)		
	$\left[f'(x) = 24x^2 - 78x + 54 \text{ leading to } \right] \left[f(x) = \right] 8x^3 - 39x^2 + 54x [+C]$	(B2,1,0)	B2 completely correct, B1 any two correct terms.
	$-1 = 8 - 39 + 54 + C$	(M1)	$f(x) = -1$ equated to <i>their</i> integrated expression, defined by three terms with at least one correct power + C, with $x = 1$.
	$\left[f(x) = \right] 8x^3 - 39x^2 + 54x - 24$	(A1)	Only condone $C = -24$ as final answer if coefficients have been simplified earlier. Do not ISW if the result is of the form $y = mx + c$.
		4	

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Question	Answer	Marks	Guidance
10(a)	$[x =] - 2$	B1	
	$\frac{dy}{dx} = k(5 - 2x)^{\frac{1}{2}} \quad \left[= -2 \times \frac{3}{2}(5 - 2x)^{\frac{1}{2}} \right]$	M1*	OE Differentiating to get $k(5 - 2x)^{\frac{1}{2}}$ only.
	$\left[\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx} \text{ leading to } -9 = \pm 5 \times \frac{dt}{dx} \right]$	DM1	Correct statement linking <i>their</i> numerical expression for $\frac{dy}{dx}$ with $\frac{dt}{dx}$ and ± 5 .
	$\frac{5}{9}$ or 0.556 =	A1	AWRT
		4	

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Question	Answer	Marks	Guidance
10(b)	$k(5-2x)^{\frac{1}{2}} = -3$	M1	Equating <i>their</i> $\frac{dy}{dx}$ of the form $k(5-2x)^{\frac{1}{2}}$ to -3 .
	[B is] (2,6)	A1	
	Gradient $AB = m_1 = \frac{32-6}{-2-2}$, gradient of perpendicular $= -\frac{1}{m_1} = \frac{4}{26}$	M1*	For A , y must be 32. Clear use of $\frac{\text{difference in y co-ordinates}}{\text{difference in x co-ordinates}}$ for points A and B , condone inconsistent order, and using $m_1 m_2 = -1$. If incorrect values or another complete method used, then working must be clear.
	Mid point is $\left(\frac{2-2}{2}, \frac{6+32}{2}\right) = (0, 19)$	M1*	Finding the midpoint of AB using A and B . If incorrect values used then all working must be clear. For A , y must be 32.
	$y - 19 = \frac{2}{13}(x - 0)$	DM1	Finding the equation of the perpendicular bisector using <i>their</i> midpoint and <i>their</i> perpendicular gradient.
	$2x - 13y + 247 = 0$ or $\pm$ integer multiples of this.	A1	
		6	

Cambridge International AS & A Level

MATHEMATICS**9709/13**

Paper 1 Pure Mathematics 1

May/June 2024**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Correct second term $30x$ in expansion of $(1+3x)^{10}$	B1	WWW, may be implied later.
	Correct third term $+405x^2$	B1	Ignore subsequent terms, may be implied later.
	Multiply $(2-5x)$ by <i>their</i> $30x+405x^2$ to obtain two x^2 terms only	M1	Expect $-150x^2, 810x^2$.
	Coefficient is 660	A1	Must be clearly identified. Allow final answer $660x^2$.
		4	

Question	Answer	Marks	Guidance
2(a)	State $(\frac{5}{3}\pi, 0)$ for point A	B1	Or exact equivalent. Allow $x = \frac{5}{3}\pi$ or exact equivalent.
	$x = \frac{19}{6}\pi$ for point B	B1	Or exact equivalent. May be implied in coordinate or vector form.
	$y = -k$ for point B	B1	May be implied in coordinate or vector form.
		3	
2(b)	Solve at least as far as $\sin^{-1} 3t = k\pi$ with correct value for $\cos^{-1}(\frac{1}{2}\sqrt{2})$	M1	Allow use of $\pi = 3.14\dots$ Allow $\sin^{-1} 3t = 30$.
	$\sin^{-1} 3t = \frac{1}{6}\pi$ and hence $t = \frac{1}{6}$	A1	Or exact equivalent. Can use degrees if consistent.
		2	

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Question	Answer	Marks	Guidance
3(a)	State $2r + r\theta = 65$ and $\frac{1}{2}r^2\theta = 225$	B1	
	Form a 3-term quadratic or cubic in r or θ or $r\theta$ from correct arc and sector formula	*M1	Condone sign errors.
	Solve <i>their</i> 3 term quadratic or cubic to obtain values of r or θ	DM1	Expect $2r^2 - 65r + 450 = (2r - 45)(r - 10)$ or $18\theta^2 - 97\theta + 72 = (9\theta - 8)(2\theta - 9)$.
	$r = 10$ and $\theta = 4.5$ ignore $r = 22.5$ and $\theta = \frac{8}{9}$, do not ignore $r = 0$	A1	B1 SC if no quadratic or cubic solution. If $r = 0$ included A0 or B0 SC.
		4	
3(b)	Use correct formula for area of triangle with clear use of angle being $2\pi - \text{their } \theta$	M1	Expect 1.783 or 102.2° , <i>their</i> θ must be reflex.
	48.9	A1	AWRT, WWW or a second answer. Or greater accuracy; condone absence of units.
		2	

Question	Answer	Marks	Guidance
4(a)	Use identity $\tan \theta = \frac{\sin \theta}{\cos \theta}$	M1	
	Use identity $\cos^2 \theta = 1 - \sin^2 \theta$	M1	
	$\pm(5\sin^2 \theta + 7\sin \theta - 6 = 0)$	A1	
		3	

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Question	Answer	Marks	Guidance
4(b)	Attempt solution of <i>their</i> 3 term equation and correct process to find at least 1 value of $\sin x$ or $\sin 2x$ or $\sin \theta$	M1	Expect $(5s - 3)(s + 2) = 0$, $s = 3/5$.
	$x = 18.4$	A1	Or greater accuracy. B1 SC if no solution to the quadratic.
	$x = 71.6$ or $(90 - \text{their } 18.4)$ or greater accuracy; and no other solutions for $0^\circ < x < 180^\circ$	A1FT	WWW B1 SC FT if no solution to the quadratic. B1 SC both correct in radians, 0.322, 1.25.
		3	

Question	Answer	Marks	Guidance
5(a)	Differentiate to obtain $4x + \frac{1}{2}x^{-2}$	B1	OE Condone '+c'.
	Equate first derivative to zero and solve $4x + \frac{K}{x^2} = 0$ as far as $x^3 = k$, K and k non-zero	M1	Not given if '+c' used.
	$x = -\frac{1}{2}$ and $y = \frac{9}{2}$	A1	OE B1 SC if no visible solution of the cubic.
		3	

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Question	Answer	Marks	Guidance
5(b)	Differentiate <i>their</i> first derivative, substitute <i>their</i> x value. Substitution may be implied by a correct inequality or correct value,	M1	Must differentiate one term correctly. Expect $4 - x^{-3} = 12$ at $x = \frac{-1}{2}$ Alternative: substitute values of x into $\frac{dy}{dx}$. One value $x < -\frac{1}{2}$ and one value $-\frac{1}{2} < x < 0$.
	conclude minimum	A1	Following correct work only
		2	
5(c)	State increasing ...	B1	
	... with clear reference to first derivative always being positive [for $x > 0$]	B1	Dependent on first derivative being correct. It is not sufficient to substitute values of x .
		2	

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Question	Answer	Marks	Guidance
6(a)	Integrate to obtain form $k(5x-3)^{-1}$	*M1	OE
	$4(5x-3)^{-1}$	A1	Or unsimplified equivalent. Condone absence of $\dots + c$ so far.
	Substitute $x = \frac{4}{5}$ and $y = -3$ to attempt value of c	DM1	DM0 for substituting $\left(-3, \frac{4}{5}\right)$.
	$y = 4(5x-3)^{-1} - 7$ allow $f(x)$ or $f = 4(5x-3)^{-1} - 7$	A1	OE Condone $c = -7$ as the final answer providing $y =$ or $f(x) = \frac{4}{(5x-3)} + c$ OE is seen earlier. Attempts to write equation in $y = mx + c$ form scores A0. Do not ISW. Gains max 3/4.
		4	
6(b)	Carry out stretch by replacing x by $2x$ in <i>their</i> equation	M1	Award if given as the second transformation. Do not ignore sign errors.
	Carry out translation by replacing x by $x-2$ and y by $y-10$	M1	OE Award if given as the first transformation. Do not ignore sign errors.
	$y = \frac{4}{10x-23} + 3$	A1	Or similarly simplified equivalent, WWW.
		3	

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Question	Answer	Marks	Guidance
7(a)	$5(3 + 9d) = 127.5$	B1	OE
	$d = 2.5$	B1	
		2	
7(b)	Attempt to find either the first term or the last term in the set by considering $1.5 + 2.5(n - 1) > 25$ or $1.5 + 2.5(n - 1) < 100$ or equivalent equations	M1	Using <i>their d</i> . May be implied by correct answers.
	State or imply that 11th term or 26.5 is the first in the set	A1	
	State or imply that 40th term or 99 is the last in the set	A1	
	<u>Either use</u> $S_{40} - S_{10}$ <u>Or use</u> $\frac{1}{2}n(a + l)$ with correct results for <i>their d</i> <u>Or use</u> $\frac{1}{2}n[2a + (n - 1)d]$ with correct results for <i>their d</i>	DM1	<i>Their</i> 40 and 10 from correct working with <i>their d</i> . Correct values 30, 26.5 and 99 respectively. Correct values 30, 26.5 and 2.5 respectively.
	Obtain 1882.5	A1	OE
		5	

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Question	Answer	Marks	Guidance
8	Substitute $x = 0$ and attempt solution of 3-term quadratic equation in y	M1	If $y = 0$ used can score a maximum of M0 A0 B1 M1 A0 A1FT DM1 A0, i.e. 4/8.
	-5 and 3	A1	B1 SC if no working to solve the quadratic.
	State or imply centre of circle is $(3, -1)$	B1	Condone errors which don't affect finding centre. May be implied by the correct final y coordinate.
	Attempt gradient of AC or BC	*M1	
	$-\frac{4}{3}$ or $\frac{4}{3}$	A1	
	State or imply gradient of tangent is $\frac{3}{4}$ or $-\frac{3}{4}$	A1FT	Following <i>their</i> gradient of radius. Only FT when previous 2 marks are M1 A0.
	<u>Either</u> solve simultaneous equations (of 2 tangent equations) to find x - coordinate <u>Or</u> Substitute y -value of centre into either tangent equation	DM1	
	$x = -\frac{16}{3}, y = -1$	A1	
	Alternative Method 1: for the 4th and 5th marks		
	Rearrange and differentiate the circle equation or differentiate implicitly	(M1)	Replaces the second M1.
	$\frac{dy}{dx} = \frac{3-x}{y+1}$ or $\frac{dy}{dx} = \frac{3-x}{(25-(x-3)^2)^{\frac{1}{2}}}$	(A1)	Replaces the second A1.

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Question	Answer	Marks	Guidance
8	Alternative Method 2: for the last 5 marks		
	$\widehat{ACP} = \widehat{MAP} = \tan^{-1} \frac{4}{3}$ or identifying similar triangles PMA and AMC	(M1A1)	C is the circle centre, P is intersection of the two tangents, M is intersection of PC and the y -axis.
	$\tan MAP = \frac{PM}{4}, \frac{4}{3} = \frac{PM}{4}, PM = \frac{16}{3}$ or use of similar triangles	(M1A1)	
	P is $\left(\frac{-16}{3}, -1\right)$	(A1)	
	Alternative Method 3: for the last 5 marks		
	Pythagoras on triangle PAC , $PC^2 = PA^2 + AC^2$,	(M1)	Identifies the required 3 sides and sets up formula.
	$PC^2 = (PM + 3)^2$, $PA^2 = PM^2 + 4^2$, $AC = \text{radius} = 5$	(A1)	Finds each side with two in terms of PM OE.
	$(PM + 3)^2 = PM^2 + 4^2 + 5^2$ leads to $6PM = 32$, $PM = \frac{16}{3}$	(M1A1)	Sets up and solves equation.
	P is $\left(\frac{-16}{3}, -1\right)$	(A1)	
		8	

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Question	Answer	Marks	Guidance
9(a)	Differentiate to obtain form $kx^2(2x^3 + 10)^{-\frac{1}{2}}$	M1	OE
	$3x^2(2x^3 + 10)^{-\frac{1}{2}}$	A1	Or unsimplified equivalent.
	Substitute $x = 3$ in first derivative and evaluate to find gradient	*M1	Expect $\frac{27}{8}$. Allow if first derivative of forms $k(2x^3 + 10)^{\frac{-1}{2}}$, $kx(2x^3 + 10)^{\frac{-1}{2}}$ or $kx^2(2x^3 + 10)^{\frac{-1}{2}}$.
	Attempt equation of tangent at (3,8) with numerical gradient	DM1	Use of gradient of the normal is DM0.
	$[\pm](27x - 8y - 17) = 0$ or integer multiples	A1	
		5	
9(b)	State or imply volume is $\pi \int (2x^3 + 10) \, dx$	B1	Implied if π appears only at the end. Do not allow an unsimplified: $\pi \int \left((2x^3 + 10)^{1/2} \right)^2$.
	Integrate to obtain $k_1x^4 + k_2x$ and evaluate using limits 1 and 3	M1	Where $k_1k_2 \neq 0$.
	60π	A1	OE Allow from a correct integral and sight of limits. Allow numerical answers in the range 188-189.
		3	

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Question	Answer	Marks	Guidance
10(a)	Substitute to obtain equation $9r^4 + 14r^2 - 8 = 0$	B1	OE
	Attempt solution of quadratic equation in r^2 to obtain at least one value of r or r^2	M1	Expect $(9r^2 - 4)(r^2 + 2)$.
	$r = \frac{2}{3}$ only	A1	SC B1 answer without working.
		3	
10(b)	Substitute $a = 2$ and <i>their</i> r in correct formula and attempt to evaluate	M1	Expect $\frac{2\left(1 - \left(\frac{2}{3}\right)^{20}\right)}{\left(1 - \frac{2}{3}\right)}$ or $\frac{2\left(\left(\frac{2}{3}\right)^{20} - 1\right)}{\left(\frac{2}{3} - 1\right)}$.
	5.998	A1	AWRT and no other value.
		2	
10(c)	Identify $a_2 = \frac{4}{3}$ and common ratio as $\frac{8}{27}$.	B1 FT	Following <i>their</i> r provided $ r < 1$. May be implied in the sum to infinity. Allow $\left(\frac{2}{3}\right)^3$.
	Substitute <i>their</i> new a and r in correct formula for sum to infinity and evaluate	M1	$ r < 1$ otherwise M0.
	$\frac{36}{19}$	A1	OE Accept 1.89 or better from 1.894736.....
		3	

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Question	Answer	Marks	Guidance
11(a)	Express $f(x)$ as: $a - (x - 3)^2$ or $a - (3 - x)^2$ where $a = \pm 19$ or ± 1	M1	OE If the form $-f(x) = (x^2 - 6x - 10)$ is used the form must be returned to $f(x) = \dots$ Completed square form must give $-x^2$. Answers must come from completion of the square (not calculus or graphs).
	$19 - (3 - x)^2$ or $19 - (x - 3)^2$	A1	OE
	$f(x) \leq 19$ or $y \leq 19$ with $\leq$, not $<$ or $-\infty < f(x) \leq 19$ or $-\infty \leq f(x) \leq 19$ or $(-\infty, 19]$ or $[-\infty, 19]$	A1 FT	Using <i>their</i> constant following the award of M1. SC B1 answer only or answer from a method not involving completion of the square.
		3	

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Question	Answer	Marks	Guidance
11(b)	$g^{-1}(x) = \frac{1}{4}(x - k)$	B1	
	$g^{-1}f(x) = \frac{1}{4}(10 + 6x - x^2 - k) = 4x + k$	M1	OE May use <i>their</i> completed square form for $f(x)$.
	Simplify the quadratic equation obtained from $g^{-1}f(x) = g(x)$ provided k is present and apply $b^2 - 4ac = 0$ to this quadratic equation	*M1	Expect $x^2 + 10x - 10 + 5k = 0$.
	Obtain $100 - 4(5k - 10) = 0$ and hence $k = 7$	A1	
	Use <i>their</i> k to form and solve a quadratic in x	DM1	Allow if <i>their</i> quadratic has two solutions.
	$(-5, -13)$ only	A1	SC B1 if no method seen.
	Alternative Method for first 4 marks		
	State $f(x) = gg(x)$	(B1)	
	$gg(x) = 16x + 5k$	(M1)	
	Apply $b^2 - 4ac = 0$ to quadratic equation obtained from $f(x) = gg(x)$	(*M1)	Provided k is present.
	$100 - 4(5k - 10) = 0$ and hence $k = 7$	(A1)	
		6	