

# Cambridge International AS & A Level

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## MATHEMATICS

9709/51

Paper 5 Probability &amp; Statistics 1

May/June 2023

**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

- 1** A summary of 50 values of  $x$  gives

$$\Sigma(x - q) = 700, \quad \Sigma(x - q)^2 = 14\,235,$$

where  $q$  is a constant.

- (a)** Find the standard deviation of these values of  $x$ . [2]

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- (b)** Given that  $\Sigma x = 2865$ , find the value of  $q$ . [2]

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- 2 (a) Find the number of ways in which a committee of 6 people can be chosen from 6 men and 8 women if it must include 3 men and 3 women. [2]

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A different committee of 6 people is to be chosen from 6 men and 8 women. Three of the 6 men are brothers.

- (b) Find the number of ways in which this committee can be chosen if there are no restrictions on the numbers of men and women, but it must include no more than two of the brothers. [3]

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- 3 (a) Find the number of different arrangements of the 8 letters in the word COCOONED. [1]

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- (b) Find the number of different arrangements of the 8 letters in the word COCOONED in which the first letter is O and the last letter is N. [2]

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- (c) Find the probability that a randomly chosen arrangement of the 8 letters in the word COCOONED has all three Os together given that the two Cs are next to each other. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 A mathematical puzzle is given to a large number of students. The times taken to complete the puzzle are normally distributed with mean 14.6 minutes and standard deviation 5.2 minutes.
- (a) In a random sample of 250 of the students, how many would you expect to have taken more than 20 minutes to complete the puzzle? [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

All the students are given a second puzzle to complete. Their times, in minutes, are normally distributed with mean  $\mu$  and standard deviation  $\sigma$ . It is found that 20% of the students have times less than 14.5 minutes and 67% of the students have times greater than 18.5 minutes.

- (b) Find the value of  $\mu$  and the value of  $\sigma$ . [5]

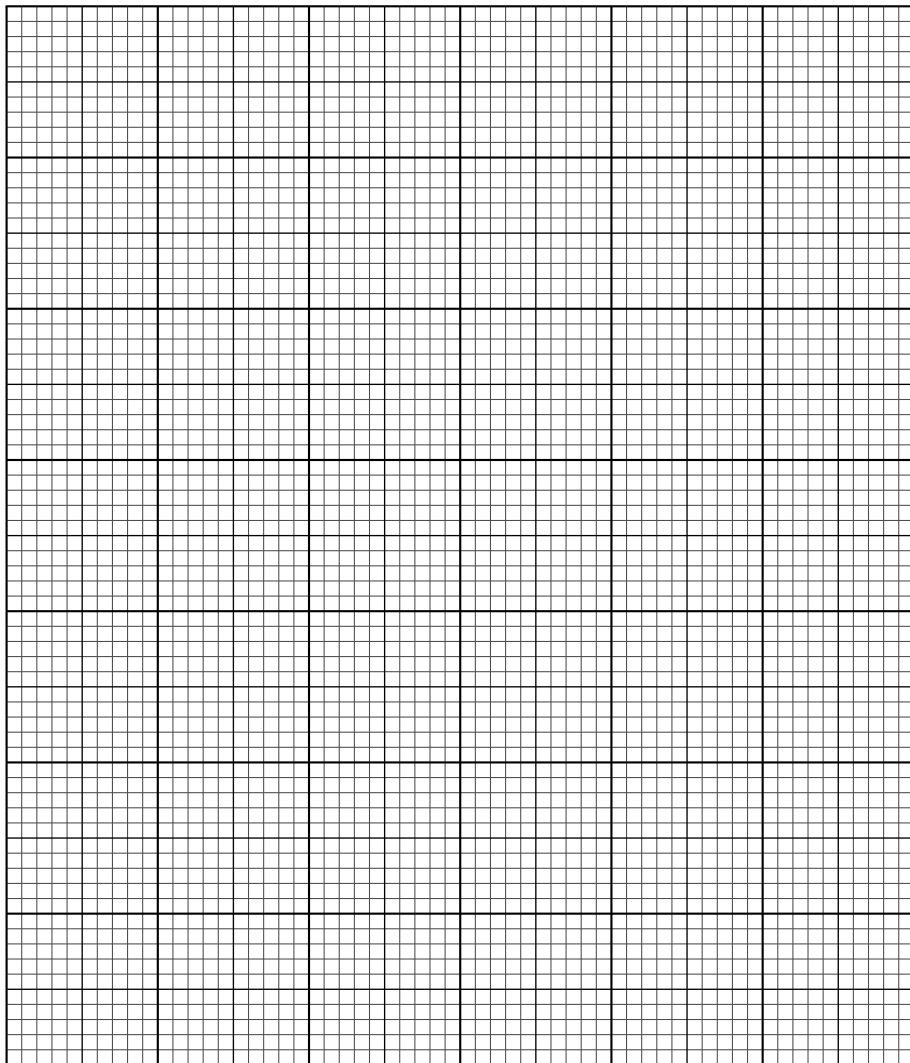
This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 5 The populations of 150 villages in the UK, to the nearest hundred, are summarised in the table.

| Population         | 100 – 800 | 900 – 1200 | 1300 – 2000 | 2100 – 3200 | 3300 – 4800 |
|--------------------|-----------|------------|-------------|-------------|-------------|
| Number of villages | 8         | 12         | 50          | 48          | 32          |

- (a) Draw a histogram to represent this information.

[4]



- (b) Write down the class interval which contains the median for this information. [1]

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- (c) Find the greatest possible value of the interquartile range for the populations of the 150 villages. [2]

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- 6** Eli has four fair 4-sided dice with sides labelled 1, 2, 3, 4. He throws all four dice at the same time. The random variable  $X$  denotes the number of 2s obtained.

(a) Show that  $P(X = 3) = \frac{3}{64}$ . [2]

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(b) Complete the following probability distribution table for  $X$ . [2]

| $x$        | 0                | 1 | 2 | 3              | 4               |
|------------|------------------|---|---|----------------|-----------------|
| $P(X = x)$ | $\frac{81}{256}$ |   |   | $\frac{3}{64}$ | $\frac{1}{256}$ |

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(c) Find  $E(X)$ .

[2]

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Eli throws the four dice at the same time on 96 occasions.

(d) Use an approximation to find the probability that he obtains at least two 2s on fewer than 20 of these occasions. [5]

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- 7 A children's wildlife magazine is published every Monday. For the next 12 weeks it will include a model animal as a free gift. There are five different models: tiger, leopard, rhinoceros, elephant and buffalo, each with the same probability of being included in the magazine.

Sahim buys one copy of the magazine every Monday.

- (a) Find the probability that the first time that the free gift is an elephant is before the 6th Monday. [2]

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- (b) Find the probability that Sahim will get more than two leopards in the 12 magazines. [3]

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- (c) Find the probability that after 5 weeks Sahim has exactly one of each animal. [3]

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## MATHEMATICS

**9709/52**

## Paper 5 Probability & Statistics 1

May/June 2023

**1 hour 15 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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## INFORMATION

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- The number of marks for each question or part question is shown in brackets [ ].

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- 1** The random variable  $X$  takes the values  $-2$ ,  $2$  and  $3$ . It is given that

$$P(X = x) = k(x^2 - 1),$$

where  $k$  is a constant.

- (a)** Draw up the probability distribution table for  $X$ , giving the probabilities as numerical fractions. [3]

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- (b)** Find  $E(X)$  and  $\text{Var}(X)$ . [3]

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- 2 A sports event is taking place for 4 days, beginning on Sunday. The probability that it will rain on Sunday is 0.4. On any subsequent day, the probability that it will rain is 0.7 if it rained on the previous day and 0.2 if it did not rain on the previous day.

(a) Find the probability that it does **not** rain on any of the 4 days of the event. [1]

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(b) Find the probability that the first day on which it rains during the event is Tuesday. [2]

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- (c) Find the probability that it rains on exactly one of the 4 days of the event.

[3]

[illegible]

- 3** The following back-to-back stem-and-leaf diagram represents the monthly salaries, in dollars, of 27 employees at each of two companies, *A* and *B*.

| Company A |   |   |   |   |   |   |    | Company B |   |   |   |   |   |   |   |
|-----------|---|---|---|---|---|---|----|-----------|---|---|---|---|---|---|---|
| 9         |   | 5 | 4 | 1 | 1 | 0 | 25 | 4         | 4 | 5 | 6 | 6 | 7 |   |   |
|           | 9 | 8 | 7 | 2 | 1 | 0 | 26 | 0         | 1 | 3 | 5 | 5 | 7 | 9 | 9 |
|           |   | 8 | 6 | 4 | 2 | 1 | 0  | 27        | 1 | 3 | 4 | 6 | 6 | 8 | 8 |
|           |   |   | 6 | 5 | 4 | 2 | 0  | 28        | 0 | 1 | 2 | 2 | 2 |   |   |
|           |   |   |   |   | 9 | 8 | 5  | 29        |   |   |   |   |   |   |   |
|           |   |   |   |   |   | 1 | 30 | 9         |   |   |   |   |   |   |   |

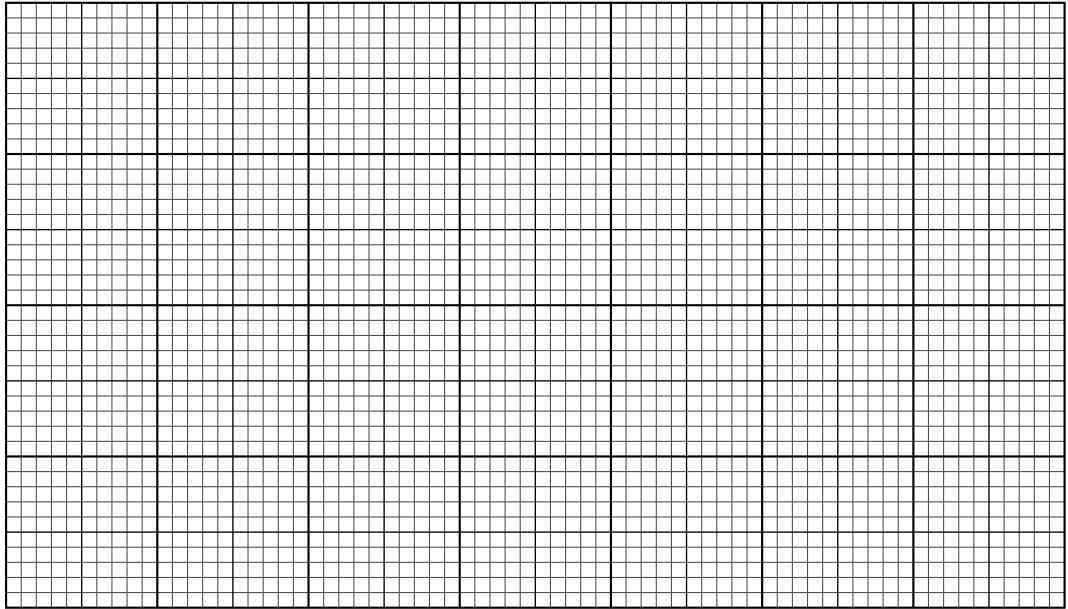
Key: 1 | 27 | 6 means \$2710 for company *A* and \$2760 for company *B*

- (a) Find the median and the interquartile range of the monthly salaries of employees in company A. [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

The lower quartile, median and upper quartile for company *B* are \$2600, \$2690 and \$2780 respectively.

- (b) Draw two box-and-whisker plots in a single diagram to represent the information for the salaries of employees at companies *A* and *B*. [3]



- (c) Comment on whether the mean would be a more appropriate measure than the median for comparing the given information for the two companies. [1]

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- 4 A fair 5-sided spinner has sides labelled 1, 2, 3, 4, 5. The spinner is spun repeatedly until a 2 is obtained on the side on which the spinner lands. The random variable  $X$  denotes the number of spins required.

(a) Find  $P(X = 4)$ . [1]

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(b) Find  $P(X < 6)$ . [2]

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Two fair 5-sided spinners, each with sides labelled 1, 2, 3, 4, 5, are spun at the same time. If the numbers obtained are equal, the score is 0. Otherwise, the score is the higher number minus the lower number.

(c) Find the probability that the score is greater than 0 given that the score is **not** equal to 2. [3]

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The two spinners are spun at the same time repeatedly .

- (d) For 9 randomly chosen spins of the two spinners, find the probability that the score is greater than 2 on at least 3 occasions. [3]

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- 5 The lengths of Western bluebirds are normally distributed with mean 16.5 cm and standard deviation 0.6 cm.

A random sample of 150 of these birds is selected.

- (a) How many of these 150 birds would you expect to have length between 15.4 cm and 16.8 cm? [4]

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The lengths of Eastern bluebirds are normally distributed with mean 18.4 cm and standard deviation  $\sigma$  cm. It is known that 72% of Eastern bluebirds have length greater than 17.1 cm.

- (b) Find the value of  $\sigma$ . [3]

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(c) Use an approximation to find the probability that fewer than 80 of these 120 bluebirds have length greater than 17.1 cm. [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 6** In a group of 25 people there are 6 swimmers, 8 cyclists and 11 runners. Each person competes in only one of these sports. A team of 7 people is selected from these 25 people to take part in a competition.
- (a)** Find the number of different ways in which the team of 7 can be selected if it consists of exactly 1 swimmer, at least 4 cyclists and at most 2 runners. [4]

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For another competition, a team of 9 people consists of 2 swimmers, 3 cyclists and 4 runners. The team members stand in a line for a photograph.

- (b)** How many different arrangements are there of the 9 people if the swimmers stand together, the cyclists stand together and the runners stand together? [2]

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[illegible]



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Paper 5 Probability &amp; Statistics 1

May/June 2023

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## INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **12** pages.

- 1** Two fair coins are thrown at the same time repeatedly until a pair of heads is obtained. The number of throws taken is denoted by the random variable  $X$ .

**(a)** State the value of  $E(X)$ . [1]

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**(b)** Find the probability that exactly 5 throws are required to obtain a pair of heads. [1]

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**(c)** Find the probability that fewer than 7 throws are required to obtain a pair of heads. [2]

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- 2 Anil is a candidate in an election. He received 40% of the votes. A random sample of 120 voters is chosen.

Use an approximation to find the probability that, of the 120 voters, between 36 and 54 inclusive voted for Anil. [5]

[illegible]

- 3 The random variable  $X$  takes the values 1, 2, 3, 4. It is given that  $P(X = x) = kx(x + a)$ , where  $k$  and  $a$  are constants.

- (a)** Given that  $P(X = 4) = 3P(X = 2)$ , find the value of  $a$  and the value of  $k$ . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

- (b) Draw up the probability distribution table for  $X$ , giving the probabilities as numerical fractions. [1]

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- (c) Given that  $E(X) = 3.2$ , find  $\text{Var}(X)$ . [2]

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- 4 The times taken, in minutes, to complete a cycle race by 19 cyclists from each of two clubs, the Cheetahs and the Panthers, are represented in the following back-to-back stem-and-leaf diagram.

| Cheetahs |   |   |   |   |    | Panthers    |  |  |  |  |  |
|----------|---|---|---|---|----|-------------|--|--|--|--|--|
| 9 8      |   |   |   |   | 7  | 4           |  |  |  |  |  |
| 8        | 7 | 3 | 2 | 0 | 8  | 6 8         |  |  |  |  |  |
| 9 8 7    |   |   |   |   | 9  | 1 7 8 9 9   |  |  |  |  |  |
| 6        | 5 | 3 | 3 | 1 | 10 | 2 3 4 4 5 6 |  |  |  |  |  |
| 9 8 2    |   |   |   |   | 11 | 1 2 8       |  |  |  |  |  |
| 4        |   |   |   |   | 12 | 0 6         |  |  |  |  |  |

Key: 7 | 9 | 1 means 97 minutes for Cheetahs and 91 minutes for Panthers

- (a) Find the median and the interquartile range of the times of the Cheetahs. [3]

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The median and interquartile range for the Panthers are 103 minutes and 14 minutes respectively.

- (b) Make two comparisons between the times taken by the Cheetahs and the times taken by the Panthers. [2]

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Another cyclist, Kenny, from the Cheetahs also took part in the race. The mean time taken by the 20 cyclists from the Cheetahs was 99 minutes.

- (c) Find the time taken by Kenny to complete the race. [3]

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- 5 Jasmine throws two ordinary fair 6-sided dice at the same time and notes the numbers on the uppermost faces. The events  $A$  and  $B$  are defined as follows.

$A$ : The sum of the two numbers is less than 6.

$B$ : The difference between the two numbers is at most 2.

- (a) Determine whether or not the events  $A$  and  $B$  are independent. [4]

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- (b) Find  $P(B \mid A')$ . [3]

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- 6** The mass of grapes sold per day by a large shop can be modelled by a normal distribution with mean 28 kg. On 10% of days less than 16 kg of grapes are sold.

(a) Find the standard deviation of the mass of grapes sold per day. [3]

[illegible]

The mass of grapes sold on any day is independent of the mass sold on any other day.

**(b)** 12 days are chosen at random.

Find the probability that less than 16 kg of grapes are sold on more than 2 of these 12 days. [3]

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- [illegible]

- 7 (a) Find the number of different arrangements of the 10 letters in the word CASABLANCA in which the two Cs are **not** together. [3]

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- (b) Find the number of different arrangements of the 10 letters in the word CASABLANCA which have an A at the beginning, an A at the end and exactly 3 letters between the 2 Cs. [3]

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- [illegible]

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