



Cambridge International AS & A Level

MATHEMATICS

9709/51

Paper 5 Probability & Statistics 1

May/June 2023

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\text{Var} = \left[\frac{\Sigma(x-q)^2}{50} - \left(\frac{\Sigma(x-q)}{50} \right)^2 \right] = \frac{14235}{50} - \left(\frac{700}{50} \right)^2$ $[= 284.7 - 196 = 88.7]$	M1	$\frac{14235}{a} - \left(\frac{700}{a} \right)^2$; where $a = 49, 50, 51$.
	$[\text{sd} = \sqrt{88.7} =] 9.42$	A1	9.4180677 rounded to at least 3SF.
		2	
1(b)	$\Sigma x - 50q = 700$ $[2865 - 50q = 700]$	M1	Forming equation with Σx , $50q$ and 700.
	$q = 43.3, 43\frac{3}{10}$	A1	If M0 scored, SC B1 for 43.3 WWW.
		2	

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Question	Answer	Marks	Guidance
2(a)	${}^6C_3 \times {}^8C_3$	M1	${}^6C_3 \times b$ or $c \times {}^8C_3$ seen. b, c integers ≥ 1 (1 may be implied).
	1120	A1	
		2	
2(b)	Method 1		
	0 brothers $[{}^3C_0] \times {}^{11}C_6$ 462	B1	${}^3C_x \times {}^{11}C_{6-x}$, with $x = 1$ or 2 seen.
	1 brother ${}^3C_1 \times {}^{11}C_5$ 1386	M1	Add values of 3 correct scenarios, (may be identified by the appropriate calculations) no incorrect/repeated scenarios, condone use of permutations.
	2 brothers ${}^3C_2 \times {}^{11}C_4$ 990		
	2838	A1	Only dependent on the M mark. SC B1 for the correct calculation or 2838 seen WWW.
	Method 2		
	${}^{14}C_6 - {}^{11}C_3$ 3003 – 165	B1	${}^{14}C_6 - d$, where d a positive integer.
		M1	$e - {}^{11}C_3$, where e is a positive integer > 165 .
	= 2838	A1	
		3	

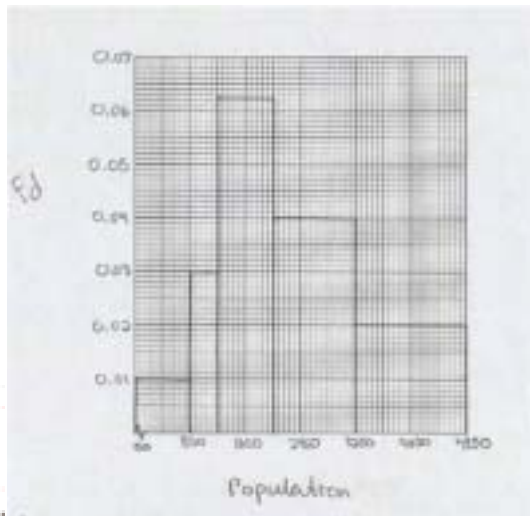
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Question	Answer	Marks	Guidance
3(a)	$\left[\frac{8!}{2!3!} = \right] 3360$	B1	
		1	
3(b)	$\frac{6!}{2!2!}$	M1	$\frac{6!}{2!f!}; f = 1, 2, 3.$
	180	A1	
		2	
3(c)	$\left[P(OOO CC) = \frac{P(OOO \cap CC)}{P(CC)} = \right]$ $\frac{5!}{7!}$ $\frac{1}{3!}$	M1	$\frac{5!}{g}$ g a positive integer, $g \neq 3360, 1.$ Condone numerator of $\frac{5!}{3360g}.$
		M1	$\frac{h}{7!}$ or $\frac{h}{8!}$, where h is a positive integer. $\frac{1}{3!}$ $\frac{1}{3!}$ Condone division by 3360 in denominator.
	$= \frac{120}{840}, \frac{1}{7}, 0.143$	A1	0.1428571... to at least 3SF. If M0 scored SC B1 for $\frac{1}{7}$ WWW.
		3	

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Question	Answer	Marks	Guidance
4(a)	$P\left(Z > \frac{20 - 14.6}{5.2}\right) = P(Z > 1.03846)$	M1	Use of $\pm$ standardisation formula with 20, 14.6 and 5.2 not σ^2 , not $\sqrt{\sigma}$, no continuity correction.
	$1 - 0.8504$	M1	Calculating the appropriate probability area (leading to their final answer).
	0.150	A1	0.1496, $0.149 < p \leq 0.15[0]$. Only dependent on the 2 nd M mark so M0M1A1 possible. SC B1 for $0.149 < p \leq 0.15[0]$ if M0M0A0 awarded.
	$[250 \times \text{their } 0.1496 =] 37, 38$	B1 FT	Strict FT <i>their</i> at least 4-figure probability seen anywhere (give BOD if they go on to use 0.150). Final answer must be positive integer, no approximation or rounding stated.
		4	
4(b)	$z_1 = \frac{14.5 - \mu}{\sigma} = -0.842$ $z_2 = \frac{18.5 - \mu}{\sigma} = -0.44$	B1	$-0.843 < z_1 < -0.841$ or $0.841 < z_1 < 0.843$.
		B1	$-0.441 < z_2 < -0.439$ or $0.439 < z_2 < 0.441$.
		M1	Use of the $\pm$ standardisation formula once with μ , σ and a z-value (not 0.20, 0.80, 0.67, 0.23, 0.5793, 0.7881, 0.7486, 0.591 or 1-z i.e. 0.158 etc.). Condone continuity correction ± 0.05 , not $\sigma^2, \sqrt{\sigma}$.
	Solve, obtaining values for μ and σ . $\mu = 22.9, \sigma = 9.95$	M1	Solve using the elimination method, substitution method or other appropriate approach to obtain values for both μ and σ .
		A1	AWRT 22.9, 9.95.
		5	

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Question	Answer	Marks	Guidance												
5(a)	<table><tr><td>cw</td><td>800</td><td>400</td><td>800</td><td>1200</td><td>1600</td></tr><tr><td>fd</td><td>0.01</td><td>0.03</td><td>0.0625</td><td>0.04</td><td>0.02</td></tr></table>	cw	800	400	800	1200	1600	fd	0.01	0.03	0.0625	0.04	0.02	M1	At least 4 frequency densities calculated (F/cw , e.g. $\frac{8}{800} \left(\text{condone } \frac{8}{n}, 799 \leq n \leq 801 \right)$) Accept unsimplified, may be read from graph using <i>their</i> scale.
	cw	800	400	800	1200	1600									
	fd	0.01	0.03	0.0625	0.04	0.02									
		A1	All heights correct on graph.												
		B1	Bar ends at 50, 850, 1250, 2050, 3250, 4850 read at the axis with a horizontal linear scale with at least 3 values indicated. $50 \leq \text{horizontal scale} \leq 4850$.												
B1		Axes labelled frequency density (fd) and population (pop) OE, or in a title. Linear vertical scale, with at least 3 values indicated. Vertical axis must cover at least the range $0 \leq \text{vertical axis} \leq 0.0625$. Axes may be reversed.													
	4														
5(b)	2100 – 3200	B1	Accept 2050 – 3250 OE. Condone ‘4 th interval’.												
		1													
5(c)	3249 – 1250	M1	$2050 \leq UQ \leq 3250 - 1250 \leq LQ \leq 2050$.												
	1999	A1	Condone $3250 - 1250 = 2000$.												
		2													

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Question	Answer	Marks	Guidance												
6(a)	$[P(X = 3) =] \frac{3}{4} \times \left(\frac{1}{4}\right)^3 \times 4$	M1	$\frac{3}{4} \times \left(\frac{1}{4}\right)^3 \times q$; q a positive integer (1 may be implied).												
	$= \frac{3}{64}$	A1	AG.												
		2													
6(b)	<table border="1"><tr><td>x</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>$P(X = x)$</td><td>$\frac{81}{256}$</td><td>$\frac{27}{64}$</td><td>$\frac{27}{128}$</td><td>$\frac{3}{64}$</td><td>$\frac{1}{256}$</td></tr></table>	x	0	1	2	3	4	$P(X = x)$	$\frac{81}{256}$	$\frac{27}{64}$	$\frac{27}{128}$	$\frac{3}{64}$	$\frac{1}{256}$	B1	Either $P(1) = \frac{27}{64}, 0.421875$ or $P(2) = \frac{27}{128}, 0.2109375$ correct to at least 3SF. Condone not in table.
	x	0	1	2	3	4									
	$P(X = x)$	$\frac{81}{256}$	$\frac{27}{64}$	$\frac{27}{128}$	$\frac{3}{64}$	$\frac{1}{256}$									
	B1 FT	Both values in table. FT $P(1) + P(2) = \frac{81}{128}, 0.6328125$.													
	2														
6(c)	$[E(X) =] \left[0 \times \frac{81}{256} \right] + 1 \times \textit{their} \frac{27}{64} + 2 \times \textit{their} \frac{27}{128} + 3 \times \frac{12}{256} + 4 \times \frac{1}{256}$	M1	Correct method from <i>their</i> probability distribution table with at least 4 terms, $0 < \textit{their} P(x) < 1$, accept partially evaluated. $= 0 + \frac{27}{64} + \frac{54}{128} + \frac{36}{256} + \frac{4}{256}$												
	$= 1$	A1													
		2													

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Question	Answer	Marks	Guidance
6(d)	$\text{Mean} = 96 \times \frac{67}{256} = 25.125$ $\text{Var} = 96 \times \frac{67}{256} \times \frac{189}{256} = 18.549$	B1	25.125, $25\frac{1}{8}$ and 18.5493... to at least 3SF seen, allow unsimplified ($4.3068 \leq \sigma \leq 4.307$ implies correct variance).
	$P(X < 20) = P\left(Z < \frac{19.5 - 25.125}{\sqrt{18.549}}\right)$	M1	Substituting <i>their</i> μ and σ into $\pm$ standardisation formula (any number for 19.5). Condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 19.5 or 20.5 in <i>their</i> standardisation formula. Note: $\frac{\pm 5.625}{\sqrt{18.549}}$ seen gains M2 BOD.
	$[= P(Z < -1.306) = 1 - \Phi(1.306) =] 1 - 0.9042 =$	M1	Appropriate area Φ , from final process. Must be a probability.
	0.0958	A1	$0.0957 \leq p \leq 0.0958$. SC B1 for $0.0957 \leq p \leq 0.0958$ if B1M0M0M1 scored.
		5	

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Question	Answer	Marks	Guidance
7(a)	Method 1		
	$[P(X < 6) = P(X \leq 5) =] 1 - 0.8^5$	M1	$1 - 0.8^r, r = 5, 6.$
	$= 0.672$	A1	
	Method 2		
	$[P(X < 6) = P(X = 1) + P(X = 2) + P(X = 3) + P(X = 4) + P(X = 5) =]$ $\frac{1}{5} + \frac{4}{5} \times \frac{1}{5} + \left(\frac{4}{5}\right)^2 \times \frac{1}{5} + \left(\frac{4}{5}\right)^3 \times \frac{1}{5} + \left(\frac{4}{5}\right)^4 \times \frac{1}{5}$	M1	Condone an extra term $\left(\frac{4}{5}\right)^5 \times \frac{1}{5}$. First, last and one of the 3 middle terms implies M1.
	$= 0.672$	A1	
		2	

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Question	Answer	Marks	Guidance
7(b)	Method 1		
	$[1 - P(0, 1, 2)]$ $= 1 - ({}^{12}C_0 (0.8)^{12} + {}^{12}C_1 (0.2)(0.8)^{11} + {}^{12}C_2 (0.2)^2 (0.8)^{10})$ $[= 1 - (0.06872 + 0.20615 + 0.28347)]$	M1	One term ${}^{12}C_x (p)^x (1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer. Correct unsimplified expression or better.
	= 0.442	B1	$0.411 < p \leq 0.442$ WWW.
	Method 2		
	$[P(3,4,5,6,7,8,9,10,11,12) =]$ ${}^{12}C_3 (0.2)^3 (0.8)^9 + {}^{12}C_4 (0.2)^4 (0.8)^8 + \dots + {}^{12}C_{11} (0.2)^{11} (0.8)^1 + {}^{12}C_{12} (0.2)^{12}$ $[= 0.23622 + 0.13288 + \dots + 1.966 \times 10^{-7} + 4.096 \times 10^{-9}]$	M1	One term ${}^{12}C_x (p)^x (1-p)^{12-x}$, $0 < p < 1$, $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, leading to final answer. Accept first, last and 8 of the middle terms.
	=0.442	B1	$0.411 < p \leq 0.442$.
		3	

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Question	Answer	Marks	Guidance
7(c)	$(0.2)^5 \times 5!$	M1	$(0.2)^5 \times s$, s a positive integer. 1 may be implied.
		M1	$t \times 5!$ where $0 < t < 1$.
	$= 0.0384, \frac{24}{625}$	A1	
	Alternative Method for Question 7(c)		
	$\frac{{}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]}{({}^5C_1)^5}$	M1	$({}^5C_1)^5$ or 5^5 as denominator.
		M1	${}^5C_1 \times {}^4C_1 \times {}^3C_1 \times {}^2C_1 \times [{}^1C_1]$ or $5!$ as numerator.
	$= 0.0384, \frac{24}{625}$	A1	
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Question	Answer	Marks	Guidance								
1(a)	$[3k + 3k + 8k = 1, so] k = \frac{1}{14}$	B1									
	<table><tr><td>x</td><td>-2</td><td>2</td><td>3</td></tr><tr><td>$P(x)$</td><td>$\frac{3}{14}, 0.214$</td><td>$\frac{3}{14}, 0.214$</td><td>$\frac{8}{14}, 0.571$</td></tr></table>	x	-2	2	3	$P(x)$	$\frac{3}{14}, 0.214$	$\frac{3}{14}, 0.214$	$\frac{8}{14}, 0.571$	B1 FT	Table with correct values of x , and at least one correct probability linked with outcome. FT <i>their k</i> . Condone any additional X values if probability stated as 0.
	x	-2	2	3							
	$P(x)$	$\frac{3}{14}, 0.214$	$\frac{3}{14}, 0.214$	$\frac{8}{14}, 0.571$							
			B1 FT	The outcomes in the table must be -2 , 2 and 3 . 2 further correct probabilities in table or 3 correct probabilities not in table linked to outcomes, or 3 correct FT probabilities in table using <i>their k</i> , or 3 incorrect probabilities summing to 1 in table if k not stated.							
			If k not calculated, SC B1 for the below. <table><tr><td>x</td><td>-2</td><td>2</td><td>3</td></tr><tr><td>$P(x)$</td><td>$3k$</td><td>$3k$</td><td>$8k$</td></tr></table>	x	-2	2	3	$P(x)$	$3k$	$3k$	$8k$
x	-2	2	3								
$P(x)$	$3k$	$3k$	$8k$								
		3									

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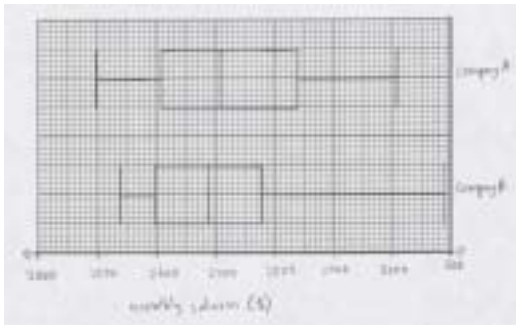
Question	Answer	Marks	Guidance
1(b)	$\left[E(X) = -2 \times \frac{3}{14} + 2 \times \frac{3}{14} + 3 \times \frac{8}{14} = \right.$ $\left. -\frac{6}{14} + \frac{6}{14} + \frac{24}{14} \right]$	M1	Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 3 probabilities summing to $0.999 \leq \text{total} \leq 1$ ($0 < p < 1$) or in terms of k .
	$\left[\text{Var}(X) = (-2)^2 \times \frac{3}{14} + 2^2 \times \frac{3}{14} + 3^2 \times \frac{8}{14} - \left(\text{their } E(X) \right)^2 = \right]$ $4 \times \frac{3}{14} + 4 \times \frac{3}{14} + 9 \times \frac{8}{14} - \left(\text{their } \frac{12}{7} \right)^2$ $\left[\frac{12 + 12 + 72}{14} - \left(\text{their } \frac{12}{7} \right)^2 \right]$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1, or in terms of k with an expression no more evaluated than shown.
	$E(X) = \frac{12}{7}, 1.71, 1\frac{5}{7}$ $\text{Var}(X) = \frac{192}{49}, 3.92, 3\frac{45}{49}$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance (condone E , V , μ , σ^2 , etc.). If A0 earned, SC B1 for identified correct final answers.
		3	

Question	Answer	Marks	Guidance
2(a)	$[P(\text{no rain}) = 0.6 \times (0.8)^3 =] 0.3072, \frac{192}{625}$	B1	Exact value required
		1	

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Question	Answer	Marks	Guidance
2(b)	$0.6 \times 0.8 \times 0.2$	M1	$a \times b \times c$ where $a, b = 0.6, 0.8, c = 0.2, 0.4, 0.7$. Condone including Wednesday with both 0.3 and 0.7 used.
	$= 0.096[0], \frac{12}{125}$	A1	
		2	
2(c)	$P(\text{RDDD}) = 0.4 \times 0.3 \times 0.8 \times 0.8 = 0.0768, \frac{48}{625}$ $P(\text{DRDD}) = 0.6 \times 0.2 \times 0.3 \times 0.8 = 0.0288, \frac{18}{625}$ $P(\text{DDRD}) = 0.6 \times 0.8 \times 0.2 \times 0.3 = 0.0288, \frac{18}{625}$ $P(\text{DDDR}) = 0.6 \times 0.8 \times 0.8 \times 0.2 = 0.0768, \frac{48}{625}$	B1	Correct probability for one clearly identified outcome evaluated accept unsimplified. A correct unsimplified expression is not sufficient.
		M1	Add 4 probability values, $0 < p < 1$, for appropriate identified scenarios. Accept unsimplified. Ways of identifying scenarios for this mark: Stating the days. All the unsimplified probability calculations exactly as stated in the mark scheme. Identifying the correct branches on a tree diagram and linking with the values. No repeated scenarios. No incorrect scenarios.
	$0.2112, \frac{132}{625}$	A1	Accept 0.211 If 0/3 scored SC B1 for $0.2112, \frac{132}{625}$.
		3	

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Question	Answer	Marks	Guidance
3(a)	Median = 2710	B1	Must be identified, condone Q2. Ignore units throughout.
	2840 – 2610	M1	$2820 \leq UQ \leq 2850 - 2600 \leq LQ \leq 2620$.
	230	A1	www If M0 scored SC B1 for 230 www. If key ignored consistently: B0 Median = 271 SC M1 $282 \leq UQ \leq 285 - 260 \leq LQ \leq 262$ SC A1 23.
		3	
3(b)	Box-and-whisker plot on provided grid. 	B1	All 5 key values for <i>B</i> plotted accurately in standard format using a linear scale with 3 identified values. Labelled <i>B</i> . Scale at least 1 cm = \$100.
	B: 2540 2600 2690 2780 3090 A: 2500 2610 2710 2840 3010	B1FT	All 5 key values for <i>A</i> , FT from (a), plotted accurately in standard format using a linear scale with 3 identified values. Labelled <i>A</i> . Scale at least 1cm = \$100
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale for the diagram and labelled 'salaries' (oe) and \$.
		3	

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Question	Answer	Marks	Guidance
3(c)	Examples: Mean less appropriate than median because of extreme value for company <i>B</i> [at \$3090]. No, extreme value in company B. No, \$3090 is an anomaly.	B1	Must refer to company B, may be implied by appropriate use of \$3090. Must include an indication that the mean is not appropriate. No contradictory statements can be present, e.g. acceptable comment with ‘but mean could be used for company A’. Condone reference to \$309.
		1	

Question	Answer	Marks	Guidance
4(a)	$[P(X = 4) = (0.8)^3 (0.2) =] 0.1024, \frac{64}{625}$	B1	Condone 0.102 .
		1	

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Question	Answer	Marks	Guidance
4(b)	$[P(X < 6) =] 1 - 0.8^5$	M1	$1 - 0.8^d$, $d = 5, 6$.
	$= 0.672, \frac{2101}{3125}$	A1	0.67232 to at least 3SF. If M0 awarded, SC B1 for $\frac{2101}{3125}$ or 0.67232 only.
	Alternative Method for Question 4(b)		
	$[P(X < 6) =] \left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right)$	M1	If answer correct, condone omission of 2 from 3 middle terms. Allow M1 for $\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^2\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^3\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^4\left(\frac{1}{5}\right) + \left(\frac{4}{5}\right)^5\left(\frac{1}{5}\right)$
	$= 0.672, \frac{2101}{3125}$	A1	0.67232 to at least 3SF. If M0 awarded, SC B1 for $\frac{2101}{3125}$ or 0.67232 only.
		2	

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Question	Answer	Marks	Guidance																																			
4(c)	$\left[P(X > 0 X \neq 2) = \frac{P(X > 0 \cap X \neq 2)}{P(X \neq 2)} = \right]$ $= \frac{\frac{14}{25}}{\frac{19}{25}}$ $= \frac{14}{19}, 0.737$	M1	$[P(X > 0 \cap X \neq 2) =] \frac{14}{25}$, 0.56[0] seen as numerator or denominator of conditional probability fraction.																																			
		M1	$[P(X \neq 2) =] \frac{19}{25}$, 0.76[0] seen as denominator of conditional probability fraction.																																			
		A1	Final answer = $\frac{14}{19}$, 0.7368421... to at least 3SF. If A0, SC B1 for correct final answer www.																																			
	Alternative Method for Question 4(c)																																					
	<table><tr><td></td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr><tr><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td><td>3</td></tr><tr><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td><td>2</td></tr><tr><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td><td>1</td></tr><tr><td>5</td><td>4</td><td>3</td><td>2</td><td>1</td><td>0</td></tr></table> $\left[P(X > 0 X \neq 2) = \frac{\text{Number of outcome}(X > 0 \cap X \neq 2)}{\text{Number of outcomes } X \neq 2} = \right]$ $\frac{14}{19}, 0.737$		1	2	3	4	5	1	0	1	2	3	4	2	1	0	1	2	3	3	2	1	0	1	2	4	3	2	1	0	1	5	4	3	2	1	0	M1
	1	2	3	4	5																																	
1	0	1	2	3	4																																	
2	1	0	1	2	3																																	
3	2	1	0	1	2																																	
4	3	2	1	0	1																																	
5	4	3	2	1	0																																	
M1	$[\text{Number of outcome}(X \neq 2) =] 19$ seen as denominator of conditional probability fraction.																																					
A1	Final answer = $\frac{14}{19}$, 0.7368421... to at least 3SF.																																					
	3																																					

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Question	Answer	Marks	Guidance
4(d)	$[P(X > 2) = 1 - P(0, 1, 2) \text{ with } p = \frac{6}{25}]$ $1 - ({}^9C_0 \left(\frac{19}{25}\right)^9 + {}^9C_1 \left(\frac{6}{25}\right)^1 \left(\frac{19}{25}\right)^8 + {}^9C_2 \left(\frac{6}{25}\right)^2 \left(\frac{19}{25}\right)^7)$ $[1 - (0.08459 + 0.2404 + 0.3037)]$	M1	One term ${}^9C_x (p)^x (1-p)^{9-x}$, $0 < p < 1$, $0 < x < 9$.
		A1	$1 - ({}^9C_0 (1-p)^9 + {}^9C_1 (p)^1 (1-p)^8 + {}^9C_2 (p)^2 (1-p)^7)$, $0 < p < 1$. Correct expression from <i>their</i> p , accept unsimplified, no terms omitted leading to final answer. Condone omission of last bracket only.
	0.371	B1	$0.371 \leq p < 0.3715$.
	Alternative Method for Question 4(d)		
	$[P(X > 2) = P(3, 4, 5, 6, 7, 8, 9) \text{ with } p = \frac{6}{25}]$ ${}^9C_3 \left(\frac{6}{25}\right)^3 \left(\frac{19}{25}\right)^6 + {}^9C_4 \left(\frac{6}{25}\right)^4 \left(\frac{19}{25}\right)^5 + \dots + {}^9C_8 \left(\frac{6}{25}\right)^8 \left(\frac{19}{25}\right)^1 + {}^9C_9 \left(\frac{6}{25}\right)^9$ $[0.2238 + 0.1060 + \dots + 7.529 \times 10^{-5} + 2.642 \times 10^{-6}]$	M1	One term ${}^9C_x (p)^x (1-p)^{9-x}$, $0 < p < 1$, $0 < x < 9$.
		A1	${}^9C_3 (p)^3 (1-p)^6 + {}^9C_4 (p)^4 (1-p)^5 + \dots + {}^9C_8 (p)^8 (1-p)^1 + {}^9C_9 (p)^9$, $0 < p < 1$. Correct expression from <i>their</i> p , accept unsimplified, no terms omitted leading to final answer.
	0.371	B1	$0.371 \leq p < 0.3715$.
		3	

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Question	Answer	Marks	Guidance
5(a)	$[P(15.4 < X < 16.8) =] P\left(\frac{15.4 - 16.5}{0.6} < Z < \frac{16.8 - 16.5}{0.6}\right)$ $[= P(-1.833 < Z < 0.5)]$	M1	Use of $\pm$ standardisation formula once with 16.5, 0.6 and either 15.4 or 16.8 substituted.
	$[= \Phi(0.5) + \Phi(1.833) - 1 =]$ $0.6915 + 0.9666 - 1$	M1	Calculating the appropriate probability area (leading to their final answer, expect > 0.5). $0.6915 - (1 - 0.9666)$ or $(0.6915 - 0.5) + (0.9666 - 0.5)$ OE are alternatives.
	$= 0.658$	A1	$0.658 \leq p < 0.6585$. If A0 scored, SC B1 for $0.658 \leq p < 0.6585$.
	[Expected number =] 0.6581×150 $= 98, 99$	B1 FT	FT <i>their</i> 4SF (or better) probability from a normal calculation. Must be a positive single integer answer. No approximation notation.
		4	
5(b)	$\left[P\left(Z > \frac{17.1 - 18.4}{\sigma} \right) = 0.72 \right]$ $\frac{17.1 - 18.4}{\sigma} = -0.583$	B1	$0.5825 < z \leq 0.583$ or $-0.583 \leq z < -0.5825$ seen.
		M1	Use of the $\pm$ standardisation formula with 17.1, 18.4, σ and a z -value (not 0.28, 0.72, 0.4175, 0.2358, 0.7642, 0.6103, 0.3897, ...). Condone continuity correct ± 0.05 , not $\sigma^2, \sqrt{\sigma}$.
	$\sigma = 2.23$	A1	AWRT
		3	

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Question	Answer	Marks	Guidance
5(c)	[Mean = $120 \times 0.72 =$] 86.4 [Var = $120 \times 0.72 \times 0.28 =$] 24.192	B1	86.4, $84\frac{2}{5}$ and $24\frac{24}{125}$, 24.192 to at least 3SF seen, allow unsimplified. May be seen in standardisation formula. ($4.918 \leq \sigma \leq 4.919$ implies correct variance) Incorrect notation is penalised.
	$P(X < 80) = P\left(Z < \frac{79.5 - 86.4}{\sqrt{24.192}}\right)$	M1	Substituting <i>their</i> mean (not 18.4) and <i>their positive</i> 4.9185 into $\pm$ standardisation formula (any number for 79.5), condone <i>their</i> 4.918^2 and $\sqrt{\text{their } 4.918}$.
		M1	Using continuity correction 79.5 or 80.5 in <i>their</i> standardisation formula.
	$[P(Z < -1.4029) = 1 - \Phi(1.403)]$ 1 – 0.9196	M1	Appropriate area Φ , from final process, must be a probability. Expect final answer < 0.5 . Note: correct final answer implies this M1.
	0.0804	A1	$0.0803 \leq p \leq 0.0804$
		5	

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Question	Answer	Marks	Guidance
6(a)	$S + 4C + 2R \quad {}^6C_1 \times {}^8C_4 \times {}^{11}C_2 [= 6 \times 70 \times 55] = 23\,100$ $S + 5C + 1R \quad {}^6C_1 \times {}^8C_5 \times {}^{11}C_1 [= 6 \times 56 \times 11] = 3696$ $S + 6C [+ 0R] \quad {}^6C_1 \times {}^8C_6 [\times {}^{11}C_0][= 6 \times 28] = 168$	M1	${}^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ seen.
		B1	Correct outcome/value for 1 identified scenario, accept unsimplified, www.
		M1	Add values of 3 correct scenarios. No incorrect scenarios, no repeated scenarios. Condone ${}^6C_e \times {}^8C_f \times {}^{11}C_g$, with $e + f + g = 7$ to identify S, C, R.
	[Total =] 26964	A1	cao
		4	
6(b)	$2! \times 3! \times 4! \times 6$	M1	$2! \times 3! \times 4! \times k$, k an integer > 0 . 1 can be implied.
	=1728	A1	If A0 scored SC B1 for 1728 www.
		2	

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Question	Answer	Marks	Guidance
6(c)	Method 1		
	$6! \times 7 \times 6 \times 5$	M1	$6! \times k$, k an integer > 0 . 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 7 \times n \times r$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$; $1 \leq n, r \leq 6$, $n \neq r$.
		M1	$\frac{m!}{a! \times b!} \times 7 \times 6 \times 5$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	A1	Condone 151 000. If A0 scored SC B1 for 151 200 www.
	Method 2		
	$6! \times {}^7P_3$	M1	$6! \times k$, k an integer > 0 . 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_q$, or $\frac{m!}{a! \times b!} \times {}^7C_q \times q!$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$; $1 \leq q \leq 6$.
		M1	$\frac{m!}{a! \times b!} \times {}^7P_3$, or $\frac{m!}{a! \times b!} \times {}^7C_3 \times 3!$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	A1	Condone 151 000. If A0 scored SC B1 for 151 200 www.

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Question	Answer	Marks	Guidance
6(c)	Method 3		
	$6! \times 35 \times 3!$	M1	$6! \times k$, k an integer > 0 . 1 can be implied.
		M1	$\frac{m!}{a! \times b!} \times 35 \times q!$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$; $1 \leq q \leq 3$.
		M1	$\frac{m!}{a! \times b!} \times 35 \times 6$; $6 \leq m \leq 9$; $a = 1, 2$; $b = 1, 4$.
	151 200	A1	Condone 151 000. If A0 scored SC B1 for 151 200 www.
	Method 4		
	$9! - 7!3! - {}^3P_2 \times 6! \times 7 \times 6$ Or $9! - 7!3! - 3! \times 7! \times 6$ [= 362 880 – 30 240 – 181 440]	M1	$9! - 7!r! - q$, r an integer > 1 , q an integer ≤ 0 . 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - q$; $s = 8, 9$; $a = 1, 2$; $b = 1, 3$; $c = 1, 4$; q an integer ≥ 0 . 0 and 1 may be implied.
		M1	$\frac{s!}{a! \times b! \times c!} - 7!3! - {}^3P_2 \times 6! \times 6 \times 7$, $6 \leq s \leq 9$, or $\frac{s!}{a! \times b! \times c!} - 7!3! - 3! \times 7! \times 6$, $6 \leq s \leq 9$. $a = 1, 2$ $b = 1, 3$ $c = 1, 4$. 1 may be implied.
		A1	Condone 151 000. If A0 scored SC B1 for 151 200 www.
	151 200		
		4	



Cambridge International AS & A Level

MATHEMATICS

9709/53

Paper 5 Probability & Statistics 1

May/June 2023

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	$\left[P(HH) = \frac{1}{4} \right] [E(X) =] 4$	B1	
		1	
1(b)	$\left[P(X = 5) = \left(\frac{3}{4}\right)^4 \left(\frac{1}{4}\right) = \right] 0.0791$	B1	$\frac{81}{1024}$
		1	
1(c)	$[P(X < 7) =] 1 - \left(\frac{3}{4}\right)^6$ or $\frac{1}{4} + \frac{3}{4} \times \frac{1}{4} + \frac{3^2}{4} \times \frac{1}{4} + \dots + \frac{3^5}{4} \times \frac{1}{4}$	M1	$1 - p^n, 0 < p < 1, n = 6, 7$ or $p + p(1 - p) + p(1 - p)^2 + \dots + p(1 - p)^n$, where $n = 4, 5$.
	$= \frac{3367}{4096}, 0.822$	A1	Accept 0.82202148... to at least 3SF.
		2	

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Question	Answer	Marks	Guidance
2	Mean = $120 \times 0.4 = 48$ Var = $120 \times 0.4 \times 0.6 = 28.8$	B1	48 and $28\frac{4}{5}$, 28.8 seen, allow unsimplified. ($5.366 \leq \sigma \leq 5.367$ or $\frac{12\sqrt{5}}{5}$ implies correct variance).
	$P(36 \leq X \leq 54) = P\left(\frac{35.5 - 48}{\sqrt{28.8}} < Z < \frac{54.5 - 48}{\sqrt{28.8}}\right)$	M1	Substituting <i>their</i> μ and σ into one $\pm$ standardisation formula (any number for 35.5 or 54.5), condone σ^2 and $\sqrt{\sigma}$.
		M1	Using continuity correction 35.5, 36.5 or 53.5, 54.5 once in <i>their</i> standardisation formula. Note: $\frac{\pm 12.5}{\sqrt{28.8}}$ or $\frac{\pm 6.5}{\sqrt{28.8}}$ seen gains M2 BOD.
	$[= P(-2.3292 < Z < 1.211) =] 0.8871 + 0.9900 - 1$	M1	Appropriate area Φ , from final process. Must be a probability. Expect final answer > 0.5 . Note: correct final answer implies this M1.
	$= 0.877$	A1	$0.877 \leq p < 0.8772$.
		5	

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Question	Answer	Marks	Guidance																				
3(a)	$[P(X = 4) = 3P(X = 2)]$ $4k(4 + a) = 3 \times 2k(2 + a)$ $16k + 4ak = 12k + 6ak$	M1	Using $P(X = 4) = 3P(X = 2)$ to form an equation in a and k .																				
	$a = 2$	A1	If M0 scored, SC B1 for $a = 2$ www.																				
	$3k + 8k + 15k + 24k = 1$	M1	Using sum of probabilities = 1 to form an equation in k : $k(1 + a) + 2k(2 + a) + 3k(3 + a) + 4k(4 + a) = 1$.																				
	$k = \frac{1}{50}$	A1	If M0 scored, SC B1 for $k = \frac{1}{50}$ www.																				
		4																					
3(b)	<table border="1"><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$\frac{3}{50}, 0.06$</td><td>$\frac{8}{50}, 0.16$</td><td>$\frac{15}{50}, 0.3$</td><td>$\frac{24}{50}, 0.48$</td></tr></table>	X	1	2	3	4	P(X)	$\frac{3}{50}, 0.06$	$\frac{8}{50}, 0.16$	$\frac{15}{50}, 0.3$	$\frac{24}{50}, 0.48$	B1 FT	<table border="1"><tr><td>X</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>P(X)</td><td>$k(1 + a)$</td><td>$2k(2 + a)$</td><td>$3k(3 + a)$</td><td>$4k(4 + a)$</td></tr></table> $0 < p < 1$ for all outcomes, must be numerical.	X	1	2	3	4	P(X)	$k(1 + a)$	$2k(2 + a)$	$3k(3 + a)$	$4k(4 + a)$
	X	1	2	3	4																		
P(X)	$\frac{3}{50}, 0.06$	$\frac{8}{50}, 0.16$	$\frac{15}{50}, 0.3$	$\frac{24}{50}, 0.48$																			
X	1	2	3	4																			
P(X)	$k(1 + a)$	$2k(2 + a)$	$3k(3 + a)$	$4k(4 + a)$																			
		1																					
3(c)	$\text{Var}(X) = \frac{3}{50} \times 1 + \frac{8}{50} \times 2^2 + \frac{15}{50} \times 3^2 + \frac{24}{50} \times 4^2 - 3.2^2$	M1	Correct formula for variance method from their probability distribution table, $0 \leq \text{their } P(x) \leq 1$. Accept $\frac{3 + 32 + 135 + 384}{50} - \frac{256}{25}$.																				
	$[= 11.08 - 3.2^2 =] 0.84[0], \frac{21}{25}$	A1	If M0 score SC B1 for 0.84 www.																				
		2																					

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Question	Answer	Marks	Guidance
4(a)	Median = 99 [minutes]	B1	
	[IQR =] 106 – 83	M1	$105 \leq \text{UQ} \leq 112 - 82 \leq \text{LQ} \leq 87$.
	23 [minutes]	A1	www. If M0 scored SC B1 for 23 www.
		3	
4(b)	The times for the Cheetahs are faster than the times for the Panthers	B1	Correct statement comparing central tendency in context.
	The times for the Cheetahs are more spread than the times for the Panthers	B1	Correct statement comparing range/IQR in context.
		2	
4(c)	[Total time including Kenny = $99 \times 20 =$] 1980	B1	Accept unsimplified.
	[Kenny's time =] 1980 – 1862	M1	For <i>their</i> 1980 – <i>their</i> 1862.
	= 118 [minutes]	A1	Accept 1 hour 58 mins.
	Alternative Method for Question 4(c)		
	$\frac{1862 + \text{their Kenny's time}}{20} = 99$	B1	$\frac{1862 + \text{their Kenny's time}}{20} = 99$ seen.
	[Kenny's time = $99 \times 20 - 1862$]	M1	For <i>their</i> $99 \times 20 - \text{their } 1862$.
	= 118 [minutes]	A1	Accept 1 hour 58 mins.
		3	

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Question	Answer	Marks	Guidance
5(a)	$P(A) = \frac{10}{36}$ $P(B) = \frac{24}{36}$	B1	Accept $P(A) = \frac{10}{36}, \frac{5}{18}, 0.278$ and $P(B) = \frac{24}{36}, \frac{2}{3}, 0.667$.
	$P(A \cap B) = \frac{8}{36}$	B1	
	$\frac{10}{36} \times \frac{24}{36}$	M1	<i>Their</i> $P(A) \times \text{their } P(B)$ seen numerically, $0 \leq \text{their } P(A), P(B) \leq 1$.
	$= \frac{5}{27}, 0.185 \left[\neq \frac{8}{36} \right]$ Events are not independent	A1 FT	Multiplication evaluated correctly and compared with intersection that is not a product of multiplication, conclusion stated, notation $P(A)$, $P(B)$ and $P(A \cap B)$ used.
		4	

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Question	Answer	Marks	Guidance
5(b)	$\left[P(B A') = \frac{P(B \cap A')}{P(A')} = \right]$ $\frac{\frac{16}{36}}{\left(1 - \frac{10}{36}\right)}$	M1	$\left[P(B \cap A') = \right] \frac{16}{36}, 0.4444$ or <i>their</i> P(B) – <i>their</i> P(A ∩ B) seen as numerator or denominator of conditional probability fraction.
		M1	$\left[P(A') = \right] \left(1 - \frac{10}{36}\right), \frac{26}{36}, 0.7222$ or $1 - \text{their } P(A)$ seen as denominator of conditional probability fraction.
	$= \frac{8}{13}$	A1	Final answer $\frac{16}{26}, \frac{8}{13}, 0.6153846$ to at least 3SF.
	Alternative Method for Question 5(b): Direct from outcome tables		
	$\left[P(B A') = \frac{\text{Number of outcomes}(B \cap A')}{\text{Number of outcomes}(A')} = \right]$ $\frac{16}{26}$	M1	$\left[\text{Number of outcomes}(B \cap A') = \right] 16$ seen as numerator or denominator of conditional probability fraction.
		M1	$\left[\text{Number of outcomes}(A') = \right] 26$ seen as denominator of conditional probability fraction.
		A1	Final answer $\frac{16}{26}, \frac{8}{13}, 0.6153846$ to at least 3SF.
		3	

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Question	Answer	Marks	Guidance
6(a)	$\left[P(X < 16) = P\left(Z < \frac{16-28}{\sigma}\right) = 0.1 \right]$ $\frac{16-28}{\sigma} = -1.282$	B1	± 1.282 seen, cao – critical value.
		M1	Use of the $\pm$ standardisation formula with 16, 28, σ and a z -value (not 0.1, 0.9, 0.282, 0.5398, 0.8159) equated to a z -value. Condone continuity correct ± 0.5 , not $\sigma^2, \sqrt{\sigma}$. Condone $\pm \frac{12}{\sigma} = -1.282$.
	$\sigma = 9.36$	A1	
		3	
6(b)	$[1 - P(0, 1, 2) =] 1 - ({}^{12}C_0(0.1)^0(0.9)^{12} + {}^{12}C_1(0.1)^1(0.9)^{11} + {}^{12}C_2(0.1)^2(0.9)^{10})$ $[1 - (0.2824 + 0.3766 + 0.2301)]$	M1	One term ${}^{12}C_x (p)^x (1-p)^{12-x}$, $0 < p < 1$. $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	B1	0.1108699... rounded to at least 3SF.
	Alternative Method for Question 6(b)		
	$P(3, 4, 5, 6, 7, 8, 9, 10, 11, 12) = {}^{12}C_3(0.1)^3(0.9)^9 + {}^{12}C_4(0.1)^4(0.9)^8 + \dots + {}^{12}C_{11}(0.1)^{11}(0.9)^1 + {}^{12}C_{12}(0.1)^{12}(0.9)^0$ $[0.08523 + 0.02131 + \dots + 1.08 \times 10^{-10} + 1 \times 10^{-12}]$	M1	One term ${}^{12}C_x (p)^x (1-p)^{12-x}$, $0 < p < 1$. $x \neq 0, 1, 2$.
		A1	Correct expression, accept unsimplified, no terms omitted leading to final answer.
	0.111	B1	0.1108699... rounded to at least 3SF.
		3	

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Question	Answer	Marks	Guidance
6(c)	$[P(-1.3 < Z < 1.3)$ $= 2 \Phi(1.3) - 1]$ $= 2 \times 0.9032 - 1$	B1	Identifying at least one of -1.3 or 1.3 as the appropriate z -values.
		M1	Calculating the appropriate probability area from 2 symmetrical z -values (leading to their final answer, expect > 0.5).
	$= 0.806, \frac{504}{625}$	A1	$0.8064, 0.806 \leq p < 0.8065$.
	[In 365 days 0.8064×365] $= 294$ or 295	B1 FT	Strict FT <i>their</i> at least 4-figure probability (not z -value). Final answer must be positive integer, no approximation or rounding stated.
		4	

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Question	Answer	Marks	Guidance
7(a)	Method 1: Total number of arrangements – number of arrangements with Cs together		
	$\frac{10!}{2!4!} - \frac{9!}{4!}$ [75600-15120]	M1	$\frac{10!}{a!b!} - c$, $a \neq b$, $a = 1, 2$, $b = 1, 4$, with c being a positive integer.
		M1	$d - \frac{e!}{4!}$, $e = 8, 9, 10$, with d being a positive integer.
	= 60480	A1	Exact value only. SC B1 for final answer 60480 www.
	Method 2: Arrangements ${}^8P_4 \times {}^9C_2$		
	$\frac{8!}{4!} \times \frac{9 \times 8}{2}$	M1	$\frac{8!}{4!} \times f$ seen, with f being a positive integer.
		M1	$g \times \frac{9 \times 8}{h}$, with g being a positive integer, $h = 1, 2$. $g \times {}^9C_2$ and $g \times {}^9P_2$ are acceptable.
	= 60480	A1	Exact value only. SC B1 for final answer 60480 www.
		3	

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Question	Answer	Marks	Guidance
7(b)	$\frac{6!}{2!} \times 4$	M1	$\frac{6!}{2!} \times s$, with s being a positive integer.
		M1	$\frac{t!}{r!} \times 4$, $r = 1, 2, 3$ and $t = 8, 7, 6$.
	1440	A1	
	Alternative Method for Question 7(b)		
	$\frac{4 \times {}^6P_3 \times 3!}{2!}$	M1	$\frac{{}^6P_3}{2!} \times k$, with k being a positive integer.
		M1	$4 \times 3! \times \frac{{}^6P_m}{n!}$, $m = 2, 3$ and $n = 1, 2, 3$.
	1440	A1	
		3	

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Question	Answer	Marks	Guidance
7(c)	Scenarios AA _ _ _ ${}^5C_3 = 10$ AAA _ _ ${}^5C_2 = 10$ AAAA _ ${}^5C_1 = 5$	B1	Correct number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www.
		M1	Add 3 values for 2, 3 and 4 As, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	A1	
	Alternative Method 2 for Question 7(c)		
	Scenarios: AAC _ _ ${}^4C_2 = 6$ AA _ _ _ ${}^4C_3 = 4$ AAAC _ ${}^4C_1 = 4$ AAA _ _ ${}^4C_2 = 6$ AAAAC 1 AAAA _ 4	B1	Correct total number of ways for identified scenarios of 2 or 3 As, accept unsimplified, www (e.g., both values for AAC^^ and AA^^^ shown would be fine for 2As).
		M1	Add 6 values of appropriate scenarios only, no additional, incorrect or repeated scenarios. Accept unsimplified.
	25	A1	
		3	