

Cambridge International AS & A Level

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MATHEMATICS

9709/41

Paper 4 Mechanics

May/June 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

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- 1 Two particles P and Q , of masses m kg and 0.3 kg respectively, are at rest on a smooth horizontal plane. P is projected at a speed of 5 m s^{-1} directly towards Q . After P and Q collide, P moves with a speed of 2 m s^{-1} in the same direction as it was originally moving.

(a) Find, in terms of m , the speed of Q after the collision. [2]

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After this collision, Q moves directly towards a third particle R , of mass 0.6 kg, which is at rest on the plane. Q is brought to rest in the collision with R , and R begins to move with a speed of 1.5 m s^{-1} .

(b) Find the value of m . [2]

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2 A particle P of mass 0.4 kg is projected vertically upwards from horizontal ground with speed 10 m s^{-1} .

(a) Find the greatest height above the ground reached by P . [2]

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When P reaches the ground again, it bounces vertically upwards. At the first instant that it hits the ground, P loses 7.2 J of energy.

(b) Find the time between the first and second instants at which P hits the ground. [4]

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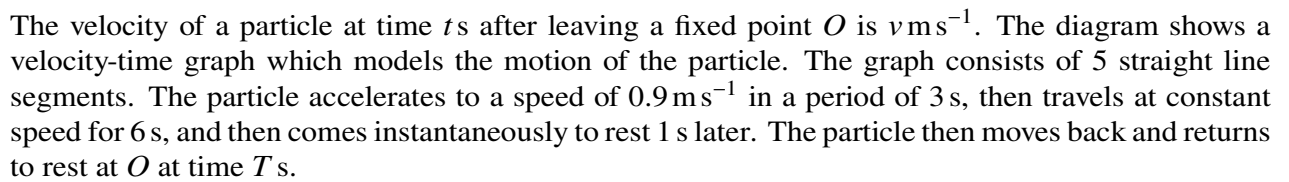
- 3** A particle moves in a straight line starting from rest. The displacement s m of the particle from a fixed point O on the line at time t s is given by

$$s = t^{\frac{5}{2}} - \frac{15}{4}t^{\frac{3}{2}} + 6.$$

Find the value of s when the particle is again at rest.

[4]

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- This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Given that $T = 12$, find the minimum velocity of the particle.

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- (c) Given instead that the greatest speed of the particle is 3 m s^{-1} , find the value of T and hence find the average speed of the particle for the whole of the motion.

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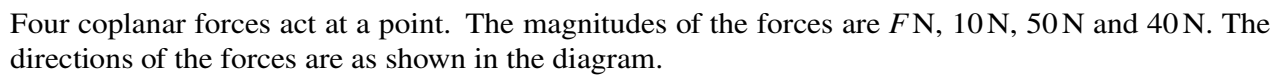
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- (b) Given instead that $F = 10\sqrt{2}$ and $\theta = 45$, find the direction and the exact magnitude the resultant force. [3]

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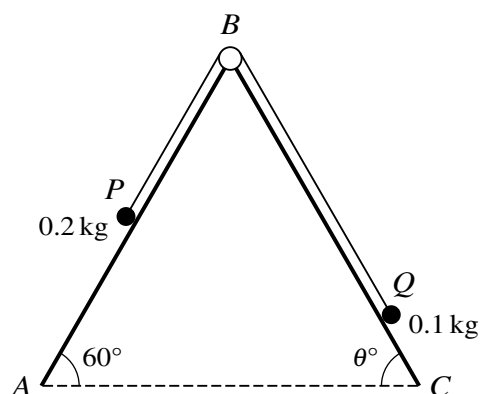
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Two particles P and Q , of masses 0.2 kg and 0.1 kg respectively, are attached to the ends of a light inextensible string. The string passes over a fixed smooth pulley at B which is attached to two inclined planes. Particle P lies on a smooth plane AB which is inclined at 60° to the horizontal. Particle Q lies on a plane BC which is inclined at an angle of θ° to the horizontal. The string is taut and the particles can move on lines of greatest slope of the two planes (see diagram).

- (a) It is given that $\theta = 60^\circ$, the plane BC is rough and the coefficient of friction between Q and the plane BC is 0.7. The particles are released from rest.

Determine whether the particles move.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** It is given instead that the plane BC is smooth. The particles are released from rest and in the subsequent motion the tension in the string is $(\sqrt{3} - 1)$ N.

Find the magnitude of the acceleration of P as it moves on the plane, and find the value of θ . [4]

[illegible]

- 7 A car of mass 1200 kg is travelling along a straight horizontal road. The power of the car's engine is constant and is equal to 16 kW. There is a constant resistance to motion of magnitude 500 N.

(a) Find the acceleration of the car at an instant when its speed is 20 m s^{-1} . [3]

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(b) Assuming that the power and the resistance forces remain unchanged, find the steady speed at which the car can travel. [2]

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The car comes to the bottom of a straight hill of length 316 m, inclined at an angle to the horizontal of $\sin^{-1}(\frac{1}{60})$. The power remains constant at 16 kW, but the magnitude of the resistance force is no longer constant and changes such that the work done against the resistance force in ascending the hill is 128 400 J. The time taken to ascend the hill is 15 s.

- (c) Given that the car is travelling at a speed of 20 m s^{-1} at the bottom of the hill, find its speed at the top of the hill. [6]

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Cambridge International AS & A Level

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MATHEMATICS

9709/42

Paper 4 Mechanics

May/June 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- Where a numerical value for the acceleration due to gravity (g) is needed, use 10 m s^{-2} .

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 1** A particle of mass 1.6 kg is dropped from a height of 9 m above horizontal ground. The speed of the particle at the instant before hitting the ground is 12 m s^{-1} .

Find the work done against air resistance.

[3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 Two particles A and B , of masses 3.2 kg and 2.4 kg respectively, lie on a smooth horizontal table. A moves towards B with a speed of $v\text{ ms}^{-1}$ and collides with B , which is moving towards A with a speed of 6 ms^{-1} . In the collision the two particles come to rest.

(a) Find the value of v .

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(b) Find the loss of kinetic energy of the system due to the collision.

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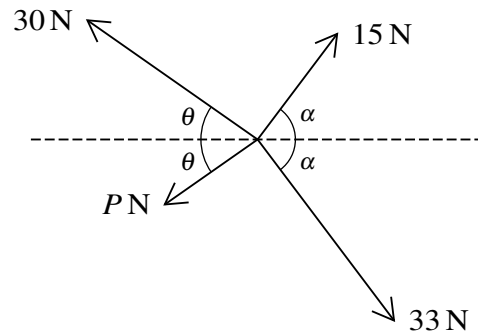
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Coplanar forces of magnitudes 30 N, 15 N, 33 N and P N act at a point in the directions shown in the diagram, where $\tan \alpha = \frac{4}{3}$. The system is in equilibrium.

- (a) Show that $\left(\frac{14.4}{30-P}\right)^2 + \left(\frac{28.8}{P+30}\right)^2 = 1$. [4]

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- (b) Verify that $P = 6$ satisfies this equation and find the value of θ . [2]

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4 An athlete of mass 84 kg is running along a straight road.

- (a) Initially the road is horizontal and he runs at a constant speed of 3 m s^{-1} . The athlete produces a constant power of 60 W .

Find the resistive force which acts on the athlete. [1]

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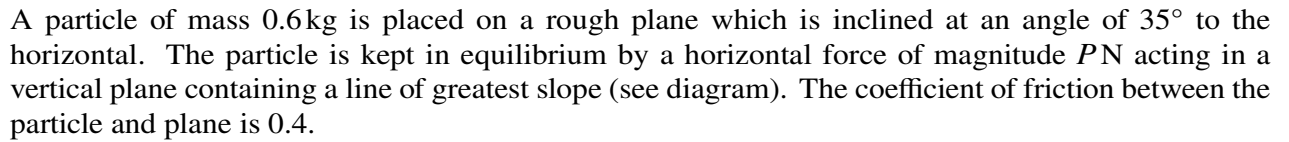
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- (b) The athlete then runs up a 150 m section of the road which is inclined at 0.8° to the horizontal. The speed of the athlete at the start of this section of road is 3 m s^{-1} and he now produces a constant driving force of 24 N. The total resistive force which acts on the athlete along this section of road has constant magnitude 13 N.

Use an energy method to find the speed of the athlete at the end of the 150 m section of road. [6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, which is otherwise blank white space.



[6]

[illegible]

- 6** A particle P starts at rest and moves in a straight line from a point O . At time t s after leaving O , the velocity of P , $v \text{ m s}^{-1}$, is given by $v = bt + ct^{\frac{3}{2}}$, where b and c are constants. P has velocity 8 m s^{-1} when $t = 4$ and has velocity 13.5 m s^{-1} when $t = 9$.

(a) Show that $b = 3$ and $c = -0.5$. [1]

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(b) Find the acceleration of P when $t = 1$. [2]

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(c) Find the positive value of t when P is at instantaneous rest and find the distance of P from O at this instant. [5]

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- (d) Find the speed of P at the instant it returns to O . [3]

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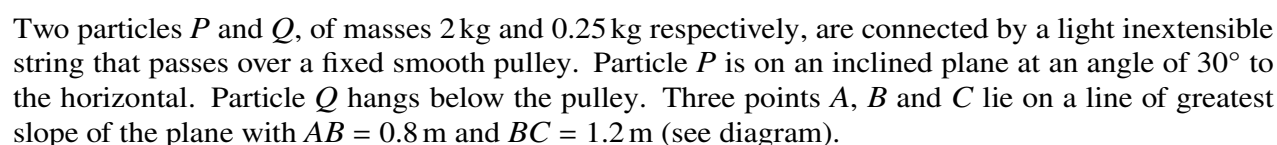
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(a) (i) Find the acceleration of P between A and B . [4]

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(ii) Hence, find the speed of P at C .

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(b) Find the time taken for P to travel from A to C .

[4]

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MATHEMATICS

9709/43

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May/June 2023

1 hour 15 minutes

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INFORMATION

- The total mark for this paper is 50.
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This document has **12** pages.

- 1 Two particles P and Q , of masses 0.1 kg and 0.4 kg respectively, are free to move on a smooth horizontal plane. Particle P is projected with speed 4 m s^{-1} towards Q which is stationary. After P and Q collide, the speeds of P and Q are equal.

Find the two possible values of the speed of P after the collision.

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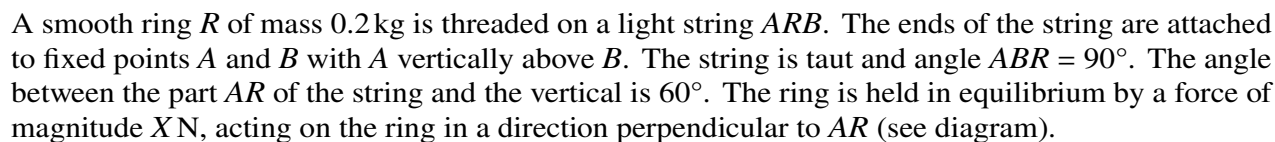
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- 2 A car of mass 1500 kg is towing a trailer of mass m kg along a straight horizontal road. The car and the trailer are connected by a tow-bar which is horizontal, light and rigid. There is a resistance force of F N on the car and a resistance force of 200 N on the trailer. The driving force of the car's engine is 3200 N, the acceleration of the car is 1.25 m s^{-2} and the tension in the tow-bar is 300 N.

Find the value of m and the value of F .

[4]

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[5]

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- 4** A lorry of mass $15\,000\text{ kg}$ moves on a straight horizontal road in the direction from A to B . It passes A and B with speeds 20 m s^{-1} and 25 m s^{-1} respectively. The power of the lorry's engine is constant and there is a constant resistance to motion of magnitude 6000 N . The acceleration of the lorry at B is 0.5 times the acceleration of the lorry at A .

- (a)** Show that the power of the lorry's engine is 200 kW , and hence find the acceleration of the lorry when it is travelling at 20 m s^{-1} . [5]

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The lorry begins to ascend a straight hill inclined at 1° to the horizontal. It is given that the power of the lorry's engine and the resistance force do not change.

- (b)** Find the steady speed up the hill that the lorry could maintain. [2]

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- 5** A particle starts from rest from a point O and moves in a straight line. The acceleration of the particle at time t s after leaving O is $a \text{ m s}^{-2}$, where $a = kt^{\frac{1}{2}}$ for $0 \leq t \leq 9$ and where k is a constant. The velocity of the particle at $t = 9$ is 1.8 m s^{-1} .

(a) Show that $k = 0.1$.

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For $t > 9$, the velocity $v \text{ m s}^{-1}$ of the particle is given by $v = 0.2(t - 9)^2 + 1.8$.

- (b)** Show that the distance travelled in the first 9 seconds is one tenth of the distance travelled between $t = 9$ and $t = 18$.

[4]

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- (c) Find the greatest acceleration of the particle during the first 10 seconds of its motion. [3]

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- 6** An elevator is pulled vertically upwards by a cable. The elevator accelerates at 0.4 m s^{-2} for 5 s, then travels at constant speed for 25 s. The elevator then decelerates at 0.2 m s^{-2} until it comes to rest.
- (a) Find the greatest speed of the elevator and hence draw a velocity-time graph for the motion of the elevator. [3]

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- (b) Find the total distance travelled by the elevator. [2]

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The mass of the elevator is 1200 kg and there is a crate of mass m kg resting on the floor of the elevator.

- (c) Given that the tension in the cable when the elevator is decelerating is 12 250 N, find the value of m . [3]

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- (d) Find the greatest magnitude of the force exerted on the crate by the floor of the elevator, and state its direction. [3]

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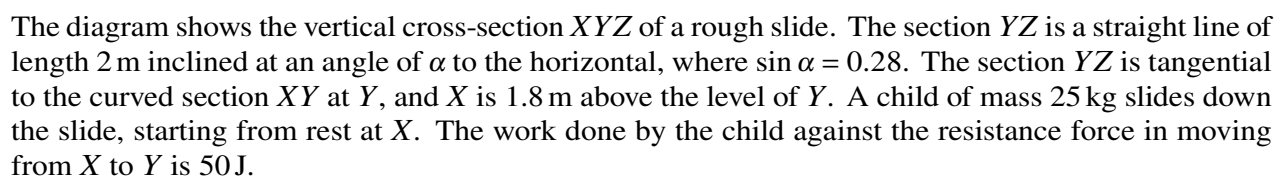
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- [illegible]

It is given that the child comes to rest at Z .

- (b)** Use an energy method to find the coefficient of friction between the child and YZ , giving your answer as a fraction in its simplest form. [6]

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[illegible]

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