

Cambridge International AS & A Level

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MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2

May/June 2023

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

- 1 Use logarithms to solve the equation $12^x = 3^{2x+1}$. Give your answer correct to 3 significant figures. [4]

[illegible]

- 2** A curve has equation $y = \frac{2 + 3 \ln x}{1 + 2x}$.

Find the equation of the tangent to the curve at the point $(1, \frac{2}{3})$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. [5]

[illegible]

3 It is given that $\int_0^a (3e^{2x} - 1) dx = 12$, where a is a positive constant.

(a) Show that $a = \frac{1}{2} \ln(9 + \frac{2}{3}a)$. [4]

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(b) Use an iterative formula, based on the equation in (a), to find the value of a correct to 4 significant figures. Use an initial value of 1 and give the result of each iteration to 6 significant figures. [3]

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- 4 The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + 3x^2 + kx - 30,$$

where k is a constant. It is given that $(x - 3)$ is a factor of $p(x)$.

- (a) Find the value of k . [2]

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- (b) Hence find the quotient when $p(x)$ is divided by $(x - 3)$ and factorise $p(x)$ completely. [3]

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- (c) It is given that a is one of the roots of the equation $p(x) = 0$.

Given also that the equation $|4y - 5| = a$ is satisfied by two real values of y , find these two values of y . [3]

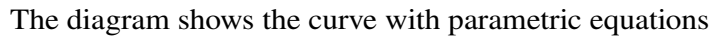
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for $-\frac{1}{4}\pi \leq t \leq \frac{1}{4}\pi$. The curve has a maximum point M .

- [illegible]

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6 Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{3}\pi} \left(4 \cos^2 2x + \frac{1}{\cos^2 x} \right) dx = \frac{3}{4}\sqrt{3} + \frac{1}{6}\pi - 1.$ [7]

[illegible]

- 7 (a) Express $7 \cos \theta + 24 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the value of α correct to 2 decimal places. [3]

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- (b) Solve the equation $7 \cos \theta + 24 \sin \theta = 18$ for $0^\circ < \theta < 360^\circ$. [4]

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$$\frac{150}{7 \cos \frac{1}{2}\beta + 24 \sin \frac{1}{2}\beta + 50}$$

is denoted by V .

Find the value of V and determine the smallest positive value of β (in degrees) for which the value of V occurs. [4]

[illegible]

[illegible]

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MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

May/June 2023

1 hour 15 minutes

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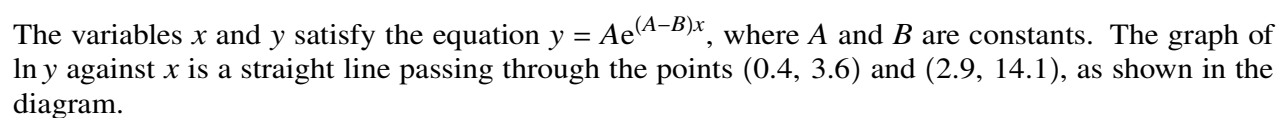
1 Solve the equation

$$\sec^2 \theta + 5 \tan^2 \theta = 9 + 17 \sec \theta$$

for $0^\circ < \theta < 360^\circ$.

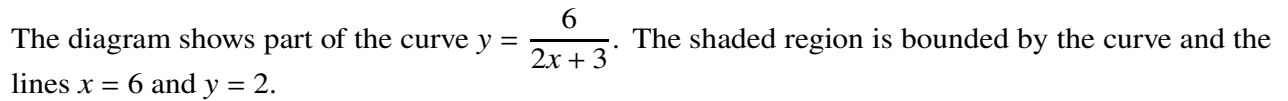
[5]

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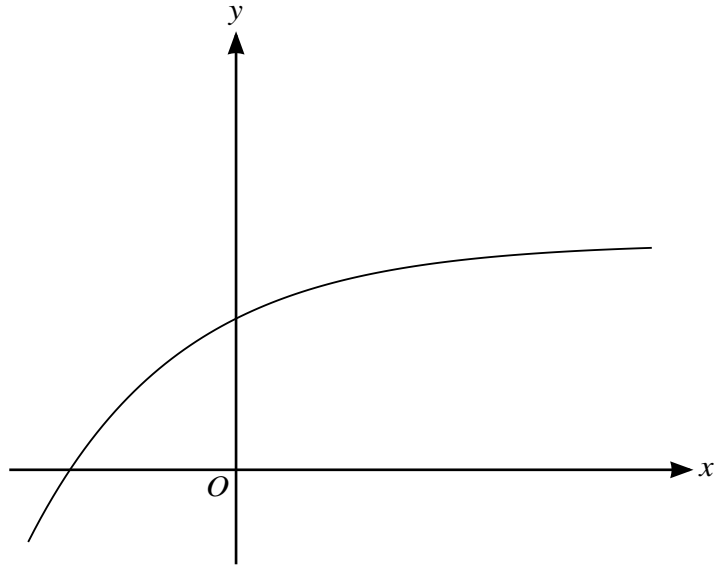
[5]

[illegible]



This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

4 (a)



The diagram shows the graph of $y = 3 - e^{-\frac{1}{2}x}$.

On the diagram, sketch the graph of $y = |5x - 4|$, and show that the equation $3 - e^{-\frac{1}{2}x} = |5x - 4|$ has exactly two real roots. [2]

It is given that the two roots of $3 - e^{-\frac{1}{2}x} = |5x - 4|$ are denoted by α and β , where $\alpha < \beta$.

(b) Show by calculation that α lies between 0.36 and 0.37. [2]

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(c) Use the iterative formula $x_{n+1} = \frac{1}{5}(7 - e^{-\frac{1}{2}x_n})$ to find β correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]

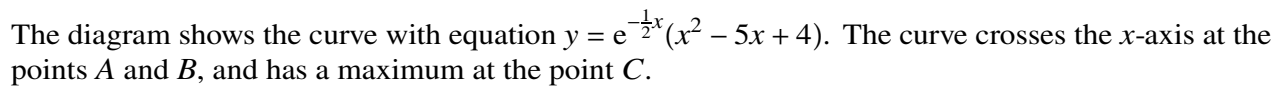
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(b) Find the exact coordinates of C .

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6 (a)** Show that $4 \sin\left(\theta + \frac{1}{3}\pi\right) \cos\left(\theta - \frac{1}{3}\pi\right) \equiv \sqrt{3} + 2 \sin 2\theta$. [4]

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- (b)** Find the exact value of $4 \sin \frac{17}{24}\pi \cos \frac{1}{24}\pi$. [2]

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[Turn over

where a is a constant. It is given that, at the point P on the curve, the gradient is 1.

(a) Show that the value of t at P satisfies the equation

$$2t^3 + (a+8)t^2 + (4a+8)t + 4a-1 = 0. \quad [4]$$

[illegible]

- (b) It is given that $(t + 1)$ is a factor of

$$2t^3 + (a + 8)t^2 + (4a + 8)t + 4a - 1.$$

Find the value of a .

[2]

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- (c) Hence show that P is the only point on the curve at which the gradient is 1.

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[illegible]

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9709/23

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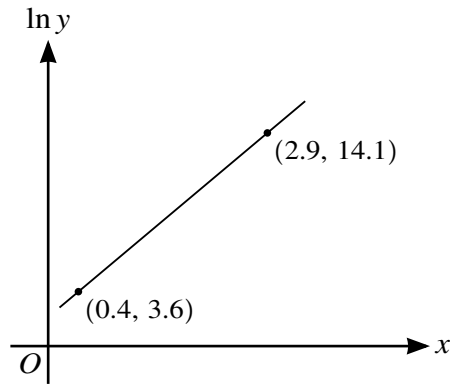
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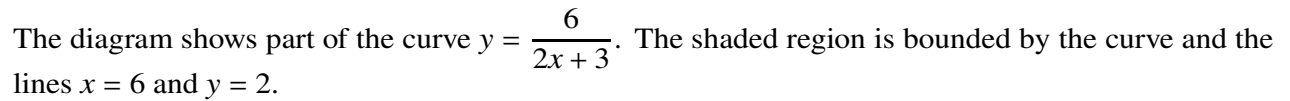


The variables x and y satisfy the equation $y = Ae^{(A-B)x}$, where A and B are constants. The graph of $\ln y$ against x is a straight line passing through the points $(0.4, 3.6)$ and $(2.9, 14.1)$, as shown in the diagram.

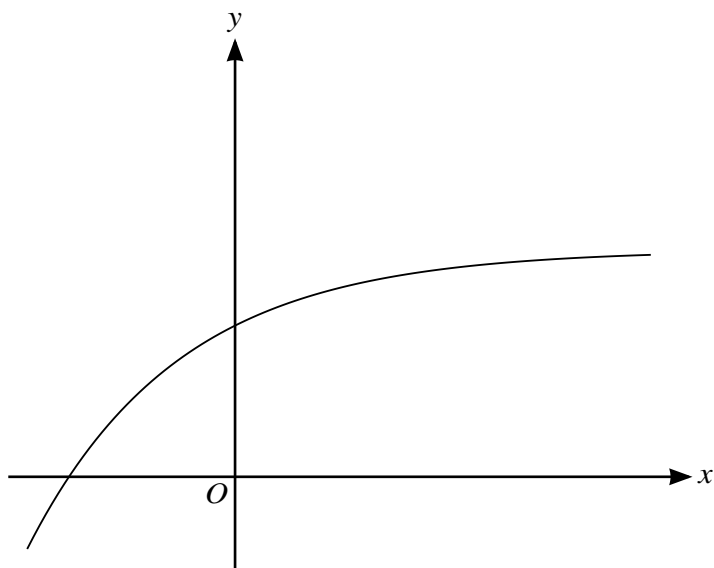
Find the values of A and B correct to 3 significant figures.

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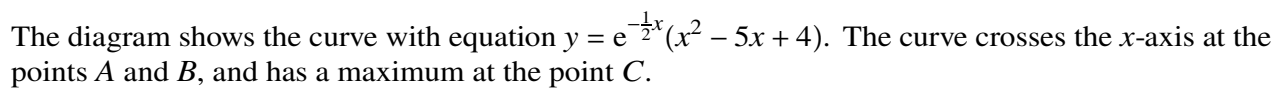
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