

Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** Solve the equation $4 \sin \theta + \tan \theta = 0$ for $0^\circ < \theta < 180^\circ$. [3]

[illegible]

- 2 (a) Find the first three terms in the expansion, in ascending powers of x , of $(2 + 3x)^4$. [2]

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- (b) Find the first three terms in the expansion, in ascending powers of x , of $(1 - 2x)^5$. [2]

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- (c) Hence find the coefficient of x^2 in the expansion of $(2 + 3x)^4(1 - 2x)^5$. [2]

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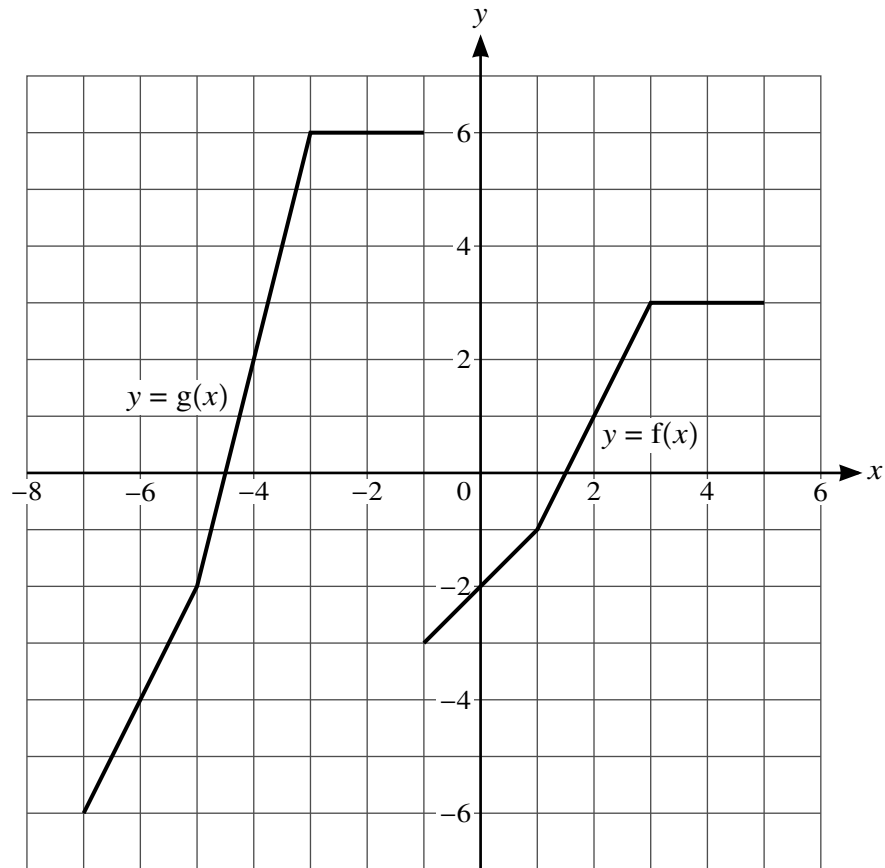
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The diagram shows graphs with equations $y = f(x)$ and $y = g(x)$.

Describe fully a sequence of two transformations which transforms the graph of $y = f(x)$ to $y = g(x)$. [4]

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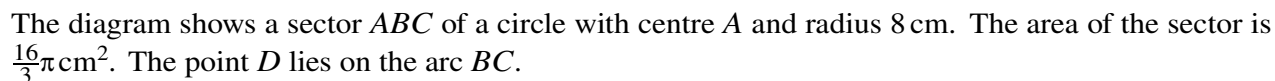
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[4]

[illegible]

- 5** The line with equation $y = kx - k$, where k is a positive constant, is a tangent to the curve with equation $y = -\frac{1}{2x}$.

Find, in either order, the value of k and the coordinates of the point where the tangent meets the curve. [5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6** The first three terms of an arithmetic progression are $\frac{p^2}{6}$, $2p - 6$ and p .

(a) Given that the common difference of the progression is not zero, find the value of p . [3]

[illegible]

(b) Using this value, find the sum to infinity of the geometric progression with first two terms $\frac{p^2}{6}$ and $2p - 6$. [2]

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7 A curve has equation $y = 2 + 3 \sin \frac{1}{2}x$ for $0 \leq x \leq 4\pi$.

(a) State greatest and least values of y .

[2]

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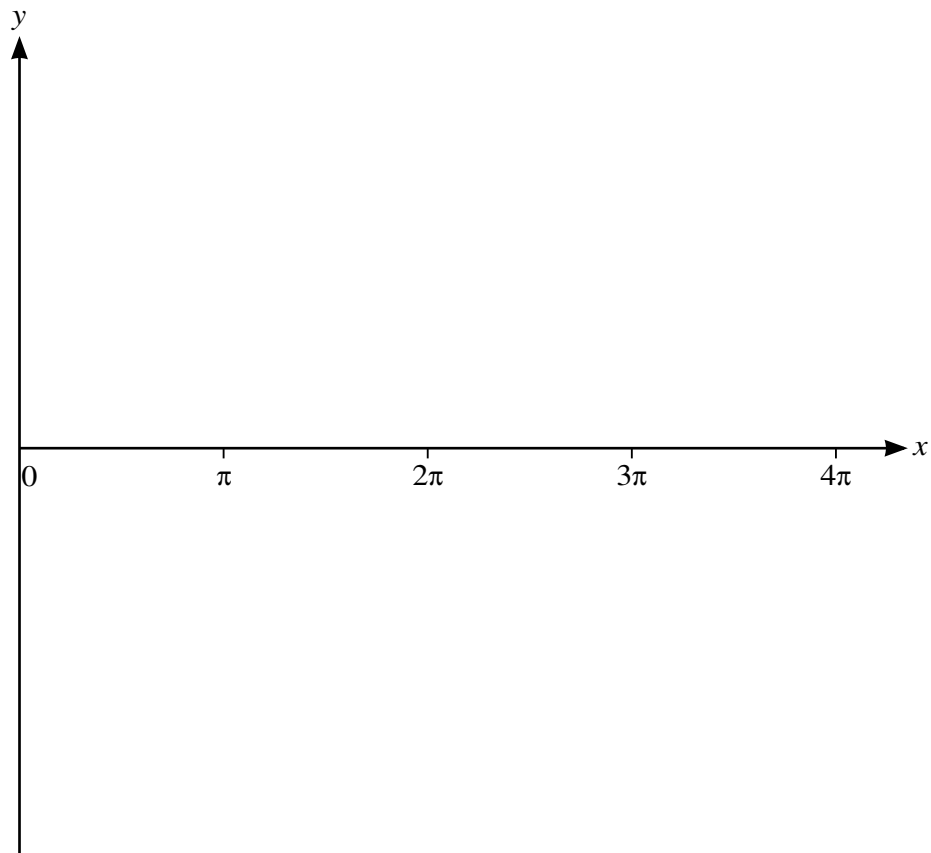
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(b) Sketch the curve.

[2]



(c) State the number of solutions of the equation

$$2 + 3 \sin \frac{1}{2}x = 5 - 2x$$

for $0 \leq x \leq 4\pi$.

[1]

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- 8** The functions f and g are defined as follows, where a and b are constants.

$$f(x) = 1 + \frac{2a}{x-a} \text{ for } x > a$$

$$g(x) = bx - 2 \text{ for } x \in \mathbb{R}$$

- (a) Given that $f(7) = \frac{5}{2}$ and $gf(5) = 4$, find the values of a and b . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

For the rest of this question, you should use the value of a which you found in (a).

- (b) Find the domain of f^{-1} . [1]

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- (c) Find an expression for $f^{-1}(x)$. [3]

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- 9 Water is poured into a tank at a constant rate of 500 cm^3 per second. The depth of water in the tank, t seconds after filling starts, is h cm. When the depth of water in the tank is h cm, the volume, $V \text{ cm}^3$, of water in the tank is given by the formula $V = \frac{4}{3}(25 + h)^3 - \frac{62500}{3}$.

(a) Find the rate at which h is increasing at the instant when $h = 10$ cm. [3]

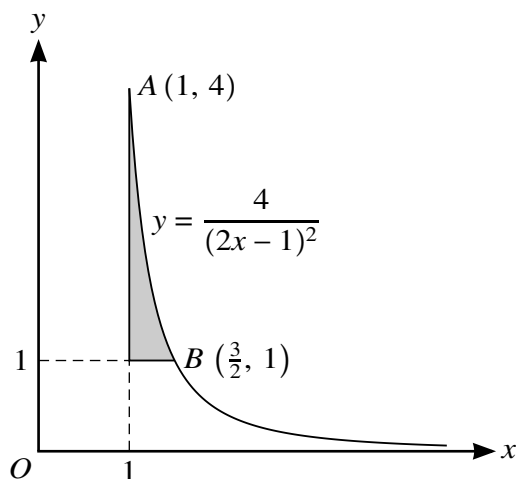
[illegible]

- (b)** At another instant, the rate at which h is increasing is 0.075 cm per second.

Find the value of V at this instant.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



The diagram shows part of the curve with equation $y = \frac{4}{(2x-1)^2}$ and parts of the lines $x = 1$ and $y = 1$. The curve passes through the points $A(1, 4)$ and $B(\frac{3}{2}, 1)$.

- (a) Find the exact volume generated when the shaded region is rotated through 360° about the x -axis. [5]

[illegible]

- (b) A triangle is formed from the tangent to the curve at B , the normal to the curve at B and the x -axis.

Find the area of this triangle.

[6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 11** The equation of a curve is such that $\frac{dy}{dx} = 6x^2 - 30x + 6a$, where a is a positive constant. The curve has a stationary point at $(a, -15)$.

(a) Find the value of a . [2]

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(b) Determine the nature of this stationary point. [2]

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(c) Find the equation of the curve.

[3]

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(d) Find the coordinates of any other stationary points on the curve.

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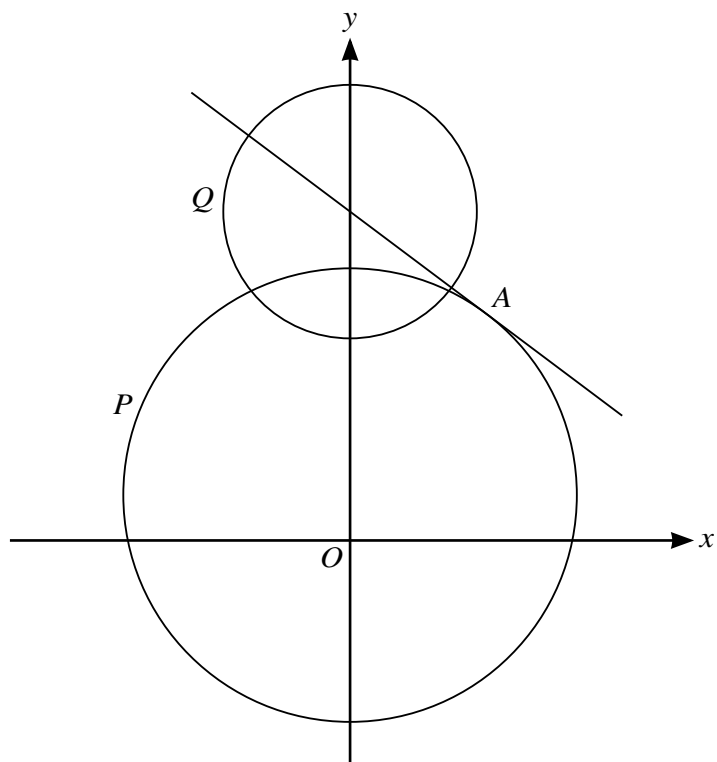
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The diagram shows a circle P with centre $(0, 2)$ and radius 10 and the tangent to the circle at the point A with coordinates $(6, 10)$. It also shows a second circle Q with centre at the point where this tangent meets the y -axis and with radius $\frac{5}{2}\sqrt{5}$.

- (a) Write down the equation of circle P . [1]

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- (b) Find the equation of the tangent to the circle P at A . [2]

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- (c) Find the equation of circle Q and hence verify that the y -coordinates of both of the points of intersection of the two circles are 11. [3]

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- (d) Find the coordinates of the points of intersection of the tangent and circle Q , giving the answers in surd form. [3]

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Cambridge International AS & A Level

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

May/June 2023

1 hour 50 minutes

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You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** The equation of a curve is such that $\frac{dy}{dx} = \frac{4}{(x-3)^3}$ for $x > 3$. The curve passes through the point $(4, 5)$.

Find the equation of the curve.

[3]

[illegible]

- 2** The coefficient of x^4 in the expansion of $(x + a)^6$ is p and the coefficient of x^2 in the expansion of $(ax + 3)^4$ is q . It is given that $p + q = 276$.

Find the possible values of the constant a .

[4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 3 (a) Express $4x^2 - 24x + p$ in the form $a(x + b)^2 + c$, where a and b are integers and c is to be given in terms of the constant p . [2]

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- (b) Hence or otherwise find the set of values of p for which the equation $4x^2 - 24x + p = 0$ has no real roots. [1]

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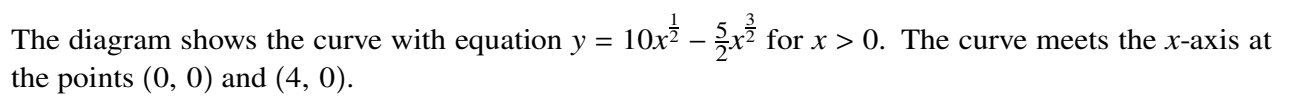
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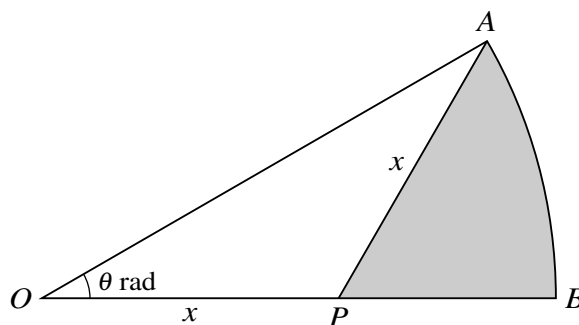
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4 Solve the equation $8x^6 + 215x^3 - 27 = 0$.

[3]

[illegible]

[illegible]



(a) Show that the arc length AB is $2x\theta \cos \theta$. [2]

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(b) Find the area of the shaded region APB in terms of x and θ . [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 7 (a) (i) By first expanding $(\cos \theta + \sin \theta)^2$, find the three solutions of the equation

$$(\cos \theta + \sin \theta)^2 = 1$$

for $0 \leq \theta \leq \pi$.

[3]

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- (ii) Hence verify that the only solutions of the equation $\cos \theta + \sin \theta = 1$ for $0 \leq \theta \leq \pi$ are 0 and $\frac{1}{2}\pi$. [2]

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- (b) Prove the identity $\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} \equiv \frac{\cos \theta + \sin \theta - 1}{1 - 2 \sin^2 \theta}$. [3]

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- (c) Using the results of (a)(ii) and (b), solve the equation

$$\frac{\sin \theta}{\cos \theta + \sin \theta} + \frac{1 - \cos \theta}{\cos \theta - \sin \theta} = 2(\cos \theta + \sin \theta - 1)$$

for $0 \leq \theta \leq \pi$.

[3]

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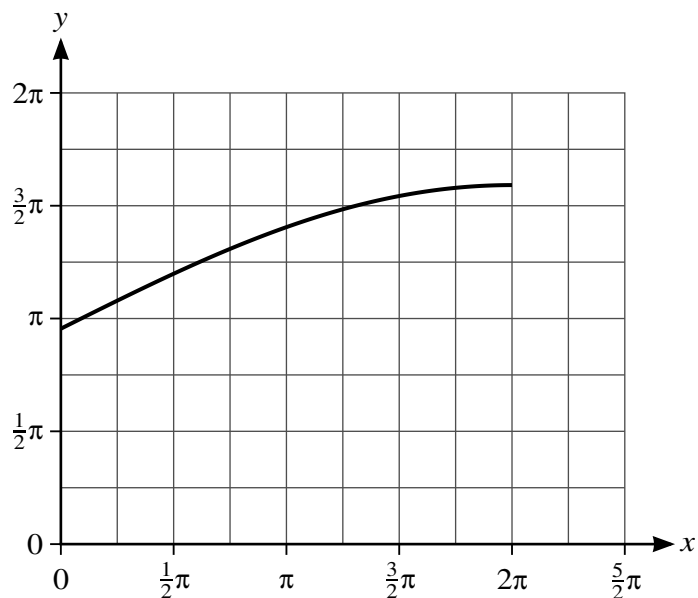
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The diagram shows the graph of $y = f(x)$ where the function f is defined by

$$f(x) = 3 + 2 \sin \frac{1}{4}x \text{ for } 0 \leq x \leq 2\pi.$$

(a) On the diagram above, sketch the graph of $y = f^{-1}(x)$. [2]

(b) Find an expression for $f^{-1}(x)$. [2]

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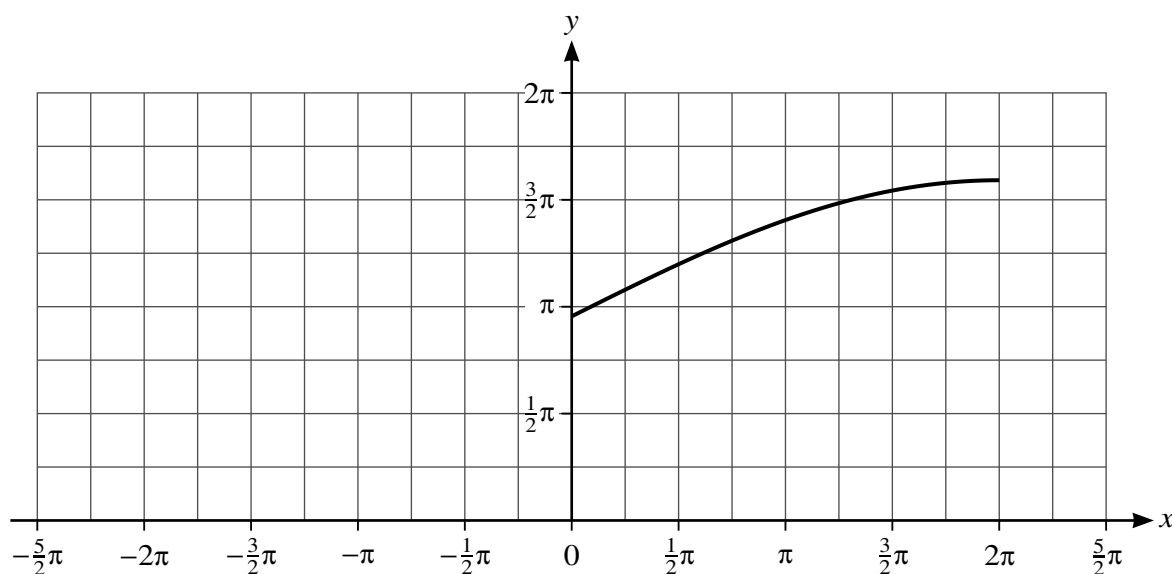
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(c)



The diagram above shows part of the graph of the function $g(x) = 3 + 2 \sin \frac{1}{4}x$ for $-2\pi \leq x \leq 2\pi$.

Complete the sketch of the graph of $g(x)$ on the diagram above and hence explain whether the function g has an inverse. [2]

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(d) Describe fully a sequence of three transformations which can be combined to transform the graph of $y = \sin x$ for $0 \leq x \leq \frac{1}{2}\pi$ to the graph of $y = f(x)$, making clear the order in which the transformations are applied. [6]

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- 9** The second term of a geometric progression is 16 and the sum to infinity is 100.

(a) Find the two possible values of the first term.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Show that the n th term of one of the two possible geometric progressions is equal to 4^{n-2} multiplied by the n th term of the other geometric progression. [4]

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- 10** The equation of a circle is $(x - a)^2 + (y - 3)^2 = 20$. The line $y = \frac{1}{2}x + 6$ is a tangent to the circle at the point P .

- (a) Show that one possible value of a is 4 and find the other possible value. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) For $a = 4$, find the equation of the normal to the circle at P . [4]

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- (c) For $a = 4$, find the equations of the two tangents to the circle which are parallel to the normal found in (b). [4]

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- 11** The equation of a curve is

$$y = k\sqrt{4x+1} - x + 5,$$

where k is a positive constant.

- (a)** Find $\frac{dy}{dx}$. [2]

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- (b)** Find the x -coordinate of the stationary point in terms of k . [2]

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- (c) Given that $k = 10.5$, find the equation of the normal to the curve at the point where the tangent to the curve makes an angle of $\tan^{-1}(2)$ with the positive x -axis. [4]

[illegible]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2023

1 hour 50 minutes

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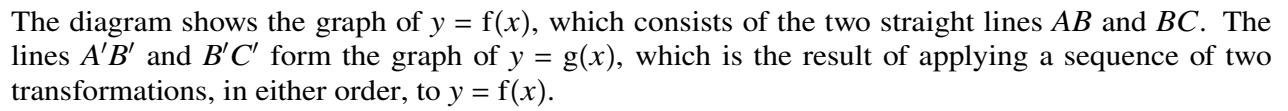
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INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



[4]

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- 2** The function f is defined for $x \in \mathbb{R}$ by $f(x) = x^2 - 6x + c$, where c is a constant. It is given that $f(x) > 2$ for all values of x .

Find the set of possible values of c .

[4]

[illegible]

- 3 (a)** Give the complete expansion of $\left(x + \frac{2}{x}\right)^5$. [2]

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- (b)** In the expansion of $(a + bx^2)\left(x + \frac{2}{x}\right)^5$, the coefficient of x is zero and the coefficient of $\frac{1}{x}$ is 80.

Find the values of the constants a and b . [4]

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- 4 (a) Show that the equation

$$3 \tan^2 x - 3 \sin^2 x - 4 = 0$$

may be expressed in the form $a \cos^4 x + b \cos^2 x + c = 0$, where a , b and c are constants to be found. [3]

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- (b) Hence solve the equation $3 \tan^2 x - 3 \sin^2 x - 4 = 0$ for $0^\circ \leq x \leq 180^\circ$. [4]

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- 5 A circle has equation $(x - 1)^2 + (y + 4)^2 = 40$. A line with equation $y = x - 9$ intersects the circle at points A and B .

(a) Find the coordinates of the two points of intersection. [4]

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(b) Find an equation of the circle with diameter AB . [3]

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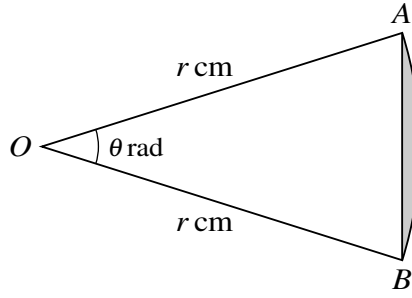
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The diagram shows a sector OAB of a circle with centre O and radius r cm. Angle $AOB = \theta$ radians. It is given that the length of the arc AB is 9.6 cm and that the area of the sector OAB is 76.8 cm^2 .

- (a) Find the area of the shaded region. [5]

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- (b) Find the perimeter of the shaded region. [2]

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7 The function f is defined by $f(x) = 2 - \frac{5}{x+2}$ for $x > -2$.

(a) State the range of f .

[1]

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(b) Obtain an expression for $f^{-1}(x)$ and state the domain of f^{-1} .

[4]

[illegible]

The function g is defined by $g(x) = x + 3$ for $x > 0$.

- (c) Obtain an expression for $fg(x)$ giving your answer in the form $\frac{ax+b}{cx+d}$, where a , b , c and d are integers. [3]

[illegible]

- 8** A progression has first term a and second term $\frac{a^2}{a+2}$, where a is a positive constant.
- (a)** For the case where the progression is geometric and the sum to infinity is 264, find the value of a . [5]

[illegible]

- (b)** For the case where the progression is arithmetic and $a = 6$, determine the least value of n required for the sum of the first n terms to be less than -480 . [5]

[illegible]

- 9** A curve which passes through $(0, 3)$ has equation $y = f(x)$. It is given that $f'(x) = 1 - \frac{2}{(x-1)^3}$.

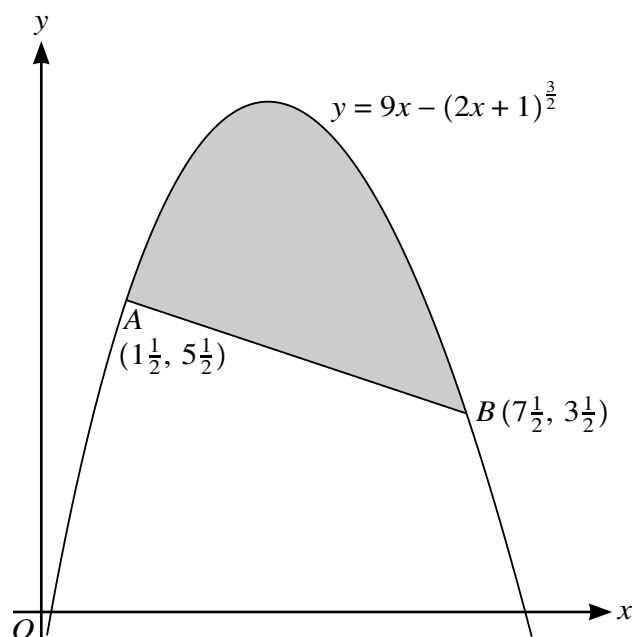
(a) Find the equation of the curve.

[4]

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- [illegible]

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The diagram shows the points $A(1\frac{1}{2}, 5\frac{1}{2})$ and $B(7\frac{1}{2}, 3\frac{1}{2})$ lying on the curve with equation $y = 9x - (2x + 1)^{\frac{3}{2}}$.

- (a) Find the coordinates of the maximum point of the curve. [4]

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- (b) Verify that the line AB is the normal to the curve at A . [3]

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- (c) Find the area of the shaded region. [5]

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