



Cambridge International AS & A Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2023

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$4\sin\theta + \tan\theta = 0 \Rightarrow 4\sin\theta + \frac{\sin\theta}{\cos\theta} [= 0]$	M1	For use of $\tan\theta = \frac{\sin\theta}{\cos\theta}$. BOD if θ missing.
	$\Rightarrow \sin\theta(4\cos\theta + 1) [= 0 \Rightarrow \sin\theta = 0 \text{ or } \cos\theta = -\frac{1}{4}]$	M1	WWW Factorise, not divide by $\sin\theta$ or $\tan\theta$. May see $\tan\theta(4\cos\theta + 1) [= 0]$ or $\sin\theta(4 + \sec\theta) [= 0]$.
	$\theta = 104.5^\circ$	A1	AWRT 1.82 rads A0. Ignore answers outside $(0, 180^\circ)$. If M1 M0, SC B1 for $\theta = 104.5^\circ$ max 2/3.
		3	

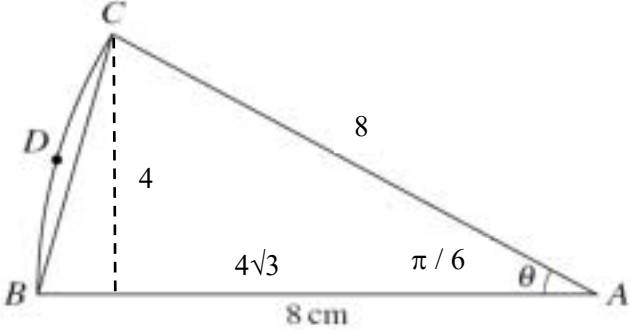
Question	Answer	Marks	Guidance
2(a)	$16 + 96x + 216x^2$	B2, 1, 0	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(b)	$1 - 10x + 40x^2$	B2, 1, 0	ISW (higher powers of x). Terms may be in any order or presented as a list.
		2	
2(c)	$(16 \times 40) - (10 \times 96) + (1 \times 216)$	M1	<i>Their</i> 3 products which would give the term in x^2 (FT <i>their</i> values). Look for $640 - 960 + 216$.
	-104	A1	Condone $-104x^2$.
		2	

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Question	Answer	Marks	Guidance
3	{Stretch} {factor 2} {in y-direction}	B2, 1, 0	B2 for fully correct, B1 with two elements correct. { } indicates different elements.
	{Translation} $\begin{pmatrix} \{-6\} \\ \{0\} \end{pmatrix}$	B2, 1, 0	B2 for fully correct, B1 with two elements correct. { } indicates different elements.
		4	Transformations may be in either order.

Question	Answer	Marks	Guidance
4	$\frac{1}{2} \times 8^2 \times \theta = \frac{16\pi}{3} \Rightarrow \theta = \frac{\pi}{6}$	B1	SOI OE e.g. $\frac{2\pi}{12}$, 0.524(3s.f.) Use of degrees acceptable throughout provided conversion used in formulae for sector area and arc length.
	Arc length = $8 \times \text{their } \frac{\pi}{6}$ [= 4.1887...]	M1	OE FT <i>their</i> θ . Look for $\frac{4\pi}{3}$.
	[BC =] $2 \times 8 \sin\left(\frac{1}{2} \times \text{their } \frac{\pi}{6}\right)$ [= 4.1411...]	M1	Attempt to find BC or BC^2 (see alt. methods below) FT <i>their</i> θ . Look for $16 \sin \frac{\pi}{12}$ or $4\sqrt{6} - 4\sqrt{2}$.
	Perimeter = 8.33	A1	AWRT Must be combined into one term.

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Question	Answer	Marks	Guidance
4	Alternative methods for Question 4: 2nd M1 mark (use normal scheme for the other marks)		
	ALT 1 $BC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \cos\left(\text{their } \frac{\pi}{6}\right) [\Rightarrow BC = 4.14\dots]$ ALT 2 $BC^2 = (8 - 4\sqrt{3})^2 + 4^2 [\Rightarrow BC = 4.14\dots]$  ALT 3 $\frac{BC}{\sin\left(\frac{\pi}{6}\right)} = \frac{8}{\sin\left(\frac{5\pi}{12}\right)} [\Rightarrow BC = 4.14\dots]$		ALT 1 Substitute into correct cosine rule. FT <i>their</i> θ Look for $128 - 64\sqrt{3}$ ALT 2 Find lengths 4 and $4\sqrt{3}$ then use Pythagoras in the left hand triangle. ALT 3 Substitute into correct sine rule.
		4	

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Question	Answer	Marks	Guidance
5	$kx - k = -\frac{1}{2x} \Rightarrow 2kx^2 - 2kx + 1 = 0$ OR quadratic in y : $x = \frac{y+k}{k} \Rightarrow y = -\frac{1}{2\left(\frac{y+k}{k}\right)} \Rightarrow 2y^2 + 2ky + k = 0$	*M1	OE e.g. $kx^2 - kx + \frac{1}{2} = 0$, $x^2 - x + \frac{1}{2k} = 0$ Equate line and curve to form 3-term quadratic (all terms on one side).
	$b^2 - 4ac = 0 \Rightarrow (-2k)^2 - 4(2k)(1) = 0$ or $4k^2 - 8k = 0 \Rightarrow 4k(k - 2) = 0$ OR using equation in y : $(2k)^2 - 4(2)(k) = 0$	DM1	Use discriminant correctly with their a, b, c not in quadratic formula. DM0 if x still present. May see $k^2 - 4(k)\left(\frac{1}{2}\right) = 0$ or $1 - 4\left(\frac{1}{2k}\right) = 0$.
	$k = 2$ only	A1	If DM0 then $k = 2$, award A0 XP then B0 B0 Allow A1 even if divides by k to solve. If $k = 0$ also present but uses $k = 2$, award A1.
	$4x^2 - 4x + 1 = 0 \Rightarrow (2x - 1)^2 = 0 \Rightarrow x = \frac{1}{2}$	B1	
	$y = 2 \times \frac{1}{2} - 2 = -1$	B1	

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Question	Answer	Marks	Guidance
5	Alternative method for Q5		
	$\frac{dy}{dx} = \frac{1}{2x^2}$ or $\frac{1}{2}x^{-2}$	*M1	Differentiate $-\frac{1}{2x}$ M0 for $2x^{-2}$. No errors.
	$[y =] \frac{1}{2x^2}x - \frac{1}{2x^2} = -\frac{1}{2x}$ or $\frac{1}{x} = \frac{1}{2x^2} [\Rightarrow 2x^2 - x = 0]$	DM1	Sub <i>their</i> $\frac{dy}{dx}$ into equation of line or set gradient = k to form equation in x .
	$x = \frac{1}{2}$ only	A1	If DM0 then $x = \frac{1}{2}$, award A0XP then B0 B0.
	$y = \left[2 \times \frac{1}{2} - 2 \right] = -1$	B1	
	$k = 2$	B1	
		5	

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Question	Answer	Marks	Guidance
6(a)	$2(2p-6) = p + \frac{p^2}{6} \Rightarrow \frac{p^2}{6} - 3p + 12 [= 0]$ OR $(2p-6) - \frac{p^2}{6} = p - (2p-6) \Rightarrow \frac{p^2}{6} - 3p + 12 [= 0]$ OR $\frac{1}{6}d^2 + d [= 0]$	*M1	Correct method leading to formation of a 3-term quadratic in p (all terms on one side) or 2-term quadratic in d . OE e.g. $p^2 - 18p + 72 [= 0]$, $\frac{1}{2}p^2 - 9p + 36 [= 0]$.
	$p^2 - 18p + 72 [= 0] \Rightarrow (p-6)(p-12) [= 0]$ or $\frac{18 \pm \sqrt{(-18)^2 - 4(1)(72)}}{2}$ OR $d\left(\frac{1}{6}d + 1\right) [= 0] \Rightarrow d = -6$	DM1	Solve a 3-term quadratic in p by factorisation, formula or completing the square or solve a 2-term quadratic in d by factorisation.
	$p = 12$ only	A1	Since $p = 6$ gives $d = 0$. If *M1 DM0 then $p = 12$ only, award SC B1 , max 2/3 marks. A0 XP if error in either factor and $p = 12$ only. $p = 12$ only by trial and improvement 3/3.
		3	

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Question	Answer	Marks	Guidance
6(b)	For GP $r = \left[\frac{2p-6}{\frac{p^2}{6}} \right] = \frac{18}{24} \left[= \frac{3}{4} \right]$	B1	OE SOI.
	Sum to infinity = $\frac{24}{1 - \frac{3}{4}} = 96$	B1 FT	FT <i>their value</i> of p if used correctly to find r (B0 if ‘ p ’ used) provided $ r < 1$. e.g. $p = 18 \Rightarrow [S_\infty =] \frac{54}{1 - \frac{5}{9}} = 121.5$.
		2	

Question	Answer	Marks	Guidance
7(a)	[Greatest =] 5	B1	No inequality required.
	[Least =] -1	B1	No inequality required.
			Condone $(-1, 5)$ or equivalent.
		2	

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Question	Answer	Marks	Guidance
7(b)		B1	One complete cycle starting and finishing at $y = 2$. Maximum and minimum in correct quadrants. Shape and curvature approximately correct.
		B1 FT	Maximum and minimum (indicated on y -axis with numbers or lines, or labelled on graph). FT <i>their</i> greatest and least values. Award B1 for 5 and -1 even if <i>their</i> values were incorrect in (a) .
		2	
7(c)	1	B1	WWW
		1	

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Question	Answer	Marks	Guidance
8(a)	$1 + \frac{2a}{7-a} = \frac{5}{2} \Rightarrow \frac{2a}{7-a} = \frac{3}{2} \Rightarrow 7a = 21 \Rightarrow a = \dots$ OR $1 + \frac{2a}{7-a} = \frac{5}{2} \Rightarrow (7-a) + 2a = \frac{5}{2}(7-a) [\Rightarrow 7a = 21] \Rightarrow a = \dots$	M1	OE Substitute $x = 7$ then solve for a via legitimate mathematical steps. Condone sign errors only.
	$a = 3$	A1	If M0, SC B1 for $a = 3$ with no working.
	$f(5) = 1 + \frac{2(\text{their}3)}{5 - \text{their}3} = 4 [\Rightarrow 4b - 2 = 4] \Rightarrow b = \dots$ OR $gf(5) = b \left(1 + \frac{2(\text{their}3)}{5 - \text{their}3} \right) - 2 [\Rightarrow 4b - 2 = 4] \Rightarrow b = \dots$	M1	Evaluate $f(5)$, either separately or within gf then solve for b via legitimate mathematical steps. Condone sign errors only. FT <i>their a</i> value.
	$b = \frac{3}{2}$	A1	OE e.g. $\frac{6}{4}$, 1.5 .
		4	
8(b)	$x > 1$	B1	Accept $(1, \infty)$ or $\{*: * > 1\}$ where $*$ is any variable. B0 for $f^{-1}(x) > 1$ or $f(x) > 1$ or $y > 1$.
		1	

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Question	Answer	Marks	Guidance
8(c)	EITHER $x - 1 = \frac{6}{y - 3} \Rightarrow (y - 3)(x - 1) = 6$ OR $x = 1 + \frac{6}{y - 3} \Rightarrow x(y - 3) = (y - 3) + 6$	*M1	OE $y - 1 = \frac{6}{x - 3} \Rightarrow (x - 3)(y - 1) = 6$. OE $y = 1 + \frac{6}{x - 3} \Rightarrow y(x - 3) = (x - 3) + 6$. Allow *M1 for use of <i>their</i> 3 from (a).
	$y - 3 = \frac{6}{x - 1}$ or $y(x - 1) = 3x + 3$	DM1	OE $x - 3 = \frac{6}{y - 1}$ or $x(y - 1) = 3y + 3$. Allow DM1 for use of <i>their</i> 3 from (a).
	$[f^{-1}(x)] = 3 + \frac{6}{x - 1}$	A1	OE Correct answer e.g. $\frac{3x + 3}{x - 1}$ ISW. Must be in terms of x .
			*M1 DM1 possible for 'a' used, but A0 so max 2/3.
		3	

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Question	Answer	Marks	Guidance
9(a)	$\frac{dV}{dh} = \frac{4}{3} \times 3(25+h)^2$ [= 4900 when $h = 10$]	B1	Correct expression for $\frac{dV}{dh}$.
	$\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt} \Rightarrow \text{their } "4(25+10)^2" \times \frac{dh}{dt} = 500 \Rightarrow \frac{dh}{dt} = \left[\frac{500}{4900} \right]$	M1	Use chain rule correctly to find a numerical expression for $\frac{dh}{dt}$. Accept e.g. $\frac{500}{2500+2000+400}$.
	$\frac{dh}{dt} = 0.102 [\text{cms}^{-1}]$	A1	AWRT OE e.g. $\frac{5}{49}$ ISW.
		3	
9(b)	$\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt} \Rightarrow 500 = \text{their } "4(25+h)^2" \times 0.075$	*M1	SOI Use chain rule correctly to form equation in h .
	$\left[(25+h)^2 = \frac{5000}{3} \right] \Rightarrow h = [15.8\ 248\dots]$	DM1	Solve quadratic to find h . Exact value of h is $\sqrt{\frac{5000}{3}} - 25$ or $\frac{50\sqrt{6}}{3} - 25$ $h + 25 = 40.82\dots$
	$V = 69900 \text{ cm}^3$	A1	AWRT ISW Look for 698(88.5).
		3	

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Question	Answer	Marks	Guidance
10(a)	$[\pi] \int \frac{16}{(2x-1)^4} [dx] = [\pi] \int 16(2x-1)^{-4} [dx] = [\pi] \left(-\frac{16}{3 \times 2 \times (2x-1)^3} \right)$	*M1	Integrate y^2 (power incr. by 1 or div by <i>their</i> new power). M0 if more than 1 error or $-\frac{16}{6}x(2x-1)^{-3}$.
	$[\pi] \left(-\frac{16}{3 \times 2 \times (2x-1)^3} \right)$	A1	OE e.g. $\left(-\frac{8}{3}(2x-1)^{-3} \right)$.
	$[\pi] \left(-\frac{16}{6 \times 8} + \frac{16}{6 \times 1} \right) \left[= [\pi] \frac{112}{48} = [\pi] \frac{7}{3} \right]$	DM1	Sub correct limits into <i>their</i> integral: $F\left(\frac{3}{2}\right) - F(1)$. Must see at least $\left(-\frac{1}{3} + \frac{8}{3} \right)$. Allow 1 sign error. Decimal: 2.33 π or 7.33.
	Volume of cylinder $\left[= \pi \times 1^2 \times \frac{1}{2} \right] = \frac{1}{2}\pi$ OR $[\pi] \int_1^{1.5} 1 [dx] = \frac{1}{2}\pi$	B1	$\frac{1}{2}\pi$ or $\pm\pi\left(\frac{3}{2}-1\right)$ seen.
	Volume of revolution $\left[= \frac{7}{3}\pi - \frac{1}{2}\pi \right] = \frac{11}{6}\pi$	A1	A0 for 5.76 (not exact). If DM0 for insufficient substitution, or B0, SC B1 for $\frac{11}{6}\pi$.
		5	

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Question	Answer	Marks	Guidance
10(b)	$\left[\frac{dy}{dx} = \right] \left\{ -8(2x-1)^{-3} \right\} \{ \times 2 \}$	B2, 1, 0	OE B1 for each correct element in $\{ \}$.
	At B gradient = -2	B1	
	Eqn of tangent $y-1 = \text{their } "-2" \left(x - \frac{3}{2} \right)$ OR Eqn of normal $y-1 = \text{their } "\frac{1}{2}" \left(x - \frac{3}{2} \right)$	M1	SOI Following differentiation OE e.g. $y = -2x + 4$ or $y = \frac{1}{2}x + \frac{1}{4}$. (Must have $m_N = -\frac{1}{m_T}$ for M1).
	Tangent crosses x -axis at 2 or normal crosses x -axis at $-\frac{1}{2}$	A1	SOI For at least one intercept correct or correct integration.
	Area = $\frac{5}{4}$	A1	From intercepts: $\frac{1}{2} \times \frac{5}{2} \times 1 = \frac{5}{4}$ or $1 + \frac{1}{4} = \frac{5}{4}$, from lengths: $\frac{1}{2} \times \sqrt{5} \times \frac{\sqrt{5}}{2} = \frac{5}{4}$ or by integration.
		6	

Question	Answer	Marks	Guidance
11(a)	$6a^2 - 30a + 6a = 0 \quad [\Rightarrow 6a(a-4) = 0]$	B1	Sub $x = a$ into $\frac{dy}{dx} = 0$. May see $a^2 - 5a + a = 0$.
	$a = 4$ only	B1	
		2	

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Question	Answer	Marks	Guidance
11(b)	$\frac{d^2y}{dx^2} = 12x - 30$ or correct values of $\frac{dy}{dx}$ either side of $x = 4$	M1	Differentiate $\frac{dy}{dx}$ (mult. by power or dec. power by 1) M0 if no values of $\frac{dy}{dx}$, only signs.
	At $x = 4, \frac{d^2y}{dx^2} > 0 \therefore$ minimum or $\frac{d^2y}{dx^2} = 18 \therefore$ minimum or concludes minimum from $\frac{dy}{dx}$ values	A1	WWW A0 XP if $a = 4$ obtained incorrectly in (a) Must see 'minimum'. If M0, SC B1 for 'minimum' from $\frac{dy}{dx}$ sign diagram.
		2	
11(c)	$[y =] \frac{6}{3}x^3 - \frac{30}{2}x^2 + 6(\text{their } a)x [+c]$	B1 FT	Expect $2x^3 - 15x^2 + 24x [+c]$. B1 poss. even if uses ' a ' – no value in (a) – max 1/3.
	$-15 = 2(\text{their "4"})^3 - 15(\text{their "4"})^2 + 6(\text{their "4"})^2 + c$	M1	Sub $x = \text{their "4"}$, $y = -15$ into integral (must incl $+c$) Look for $-15 = 128 - 240 + 96 + c \Rightarrow c = 1$.
	$y = 2x^3 - 15x^2 + 24x + 1$	A1	Coefficients must be correct and simplified. Need to see ' $y =$ ' or ' $f(x) =$ ' in the working.
		3	
11(d)	$\frac{dy}{dx} = 6x^2 - 30x + 6(\text{their "4"}) [= 0]$ If correct, $[6](x-1)(x-4) [= 0]$ or $\frac{30 \pm \sqrt{(-30)^2 - 4(6)(24)}}{12}$	M1	OE Forming a 3-term quadratic using the given $\frac{dy}{dx}$ and solving by factorisation, formula or completing the square. Check for working in (b).
	Coordinates (1,12)	A1	Allow $x = 1, y = 12$ (ignore $x = 4$ if present). If M0, award SC B1 for (1,12).
		2	

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Question	Answer	Marks	Guidance
12(a)	$x^2 + (y - 2)^2 = 100$	B1	OE e.g. $(x - 0)^2 + (y - 2)^2 = 10^2$ ISW.
		1	
12(b)	Gradient of radius = $\left[\frac{10 - 2}{6 - 0} \right] \frac{4}{3}$ or gradient of tangent = $\frac{-3}{4}$	M1	OE SOI Use coordinates to find gradient of radius or differentiate to find m_T e.g. $2x + 2(y - 2) \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = \frac{-3}{4}$ at (6, 10) $y = 2 + \sqrt{100 - x^2} \Rightarrow \frac{dy}{dx} = \frac{1}{2}(100 - x^2)^{-\frac{1}{2}}(-2x) = -\frac{3}{4}$.
	Equation of tangent is $y - 10 = -\frac{3}{4}(x - 6) \quad \left[\Rightarrow y = -\frac{3}{4}x + \frac{29}{2} \right]$	A1	OE ISW Allow e.g. $\frac{58}{4}$.
		2	
12(c)	Coordinates of centre of circle Q are $\left(0, \text{their } \frac{29}{2} \right)$	M1	SOI From a linear equation in (b).
	Equation of circle Q is $x^2 + \left(y - \text{their } \frac{29}{2} \right)^2 = \left(\frac{5\sqrt{5}}{2} \right)^2 \left[= \frac{125}{4} \right]$	A1FT	OE e.g. $(x - 0)^2 + (y - 14.5)^2 = 31.25$ ISW.
	$x^2 + (11 - 2)^2 = 100 \Rightarrow x^2 = 19$ and $x^2 + \left(11 - \frac{29}{2} \right)^2 = \frac{125}{4} \Rightarrow x^2 = 19$ OR e.g. $\frac{125}{4} - \left(y - \frac{29}{2} \right)^2 + (y - 2)^2 = 100 \Rightarrow 25y = 275 \Rightarrow y = 11$	B1	OE e.g. $x = [\pm]\sqrt{19}$, $x^2 - 19 = x^2 - 19$ Correct argument to verify both y -coords are 11 ISW.
		3	

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Question	Answer	Marks	Guidance
12(d)	$x^2 + \left(-\frac{3}{4}x + \frac{29}{2} - \frac{29}{2}\right)^2 = \frac{125}{4} \left[\Rightarrow \frac{25}{16}x^2 = \frac{125}{4} \Rightarrow x^2 = 20 \right]$ or $y^2 - 29y + 199 [= 0]$	M1	Substitute equation of <i>their</i> tangent into equation of <i>their</i> circle. May see $y = \sqrt{31.25 - x^2} + 14.5$.
	$x = \pm 2\sqrt{5}$ or $y = \frac{29 \mp 3\sqrt{5}}{2}$	A1	OE e.g. $x = \pm\sqrt{20}$ For 2 x -values or 2 y -values or correct (x, y) pair.
	$y \left[= \left(-\frac{3}{4} \times \pm\sqrt{20}\right) + \frac{29}{2} \right] = \frac{29 \mp 3\sqrt{5}}{2}$	A1	OE e.g. $\frac{58}{4} + \frac{3\sqrt{20}}{4}$, $\frac{58}{4} - \frac{3\sqrt{20}}{4}$ Correct (x, y) pairs.
		3	



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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

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MARK SCHEME

Maximum Mark: 75

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This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$[y] = \frac{4}{-2}(x-3)^{-3+1}$ or $\frac{4}{-2(x-3)^2}[+c]$	B1	OE Allow $\frac{4}{-3+1}$ and $-3+1$ for the power.
	$5 = \frac{4}{-2}(4-3)^{-2} + c$ or $5 = \frac{4}{-2(4-3)^2} + c$ leading to $c =$	M1	Correct use of (4,5) to find c in an integrated expression (defined by the correct power and no extra x 's or terms).
	$y = \frac{-2}{(x-3)^2} + 7$ or $y = -2(x-3)^{-2} + 7$	A1	OE $-\frac{4}{2}$ must be simplified to -2 . Condone $c = 7$ as their final line as long as either y or $f(x) =$ is seen elsewhere. Do not ISW if the result is of the form $y = mx+c$.
		3	

Question	Answer	Marks	Guidance
2	[Coefficient of $x^4 = p =$] $15a^2$	B1	May be seen in an expansion or with x^4 .
	[Coefficient of $x^2 = q =$] $54a^2$	B1	May be seen in an expansion or with x^2 .
	Equating <i>their p + their q</i> to 276 leading to an equation in a^2 only	M1	No x terms and no extra terms. If p and q are not identified then it needs to be clear from the expansion that the appropriate coefficients are being used. $69a^2 = 276$ implies the first 3 marks.
	$a = \pm 2$	A1	CAO
		4	

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Question	Answer	Marks	Guidance
3(a)	$4(x-3)^2$ seen or $a = 4$ and $b = -3$	B1	OE Award marks for the correct expression or their values a , b and c . Condone $4(x-3) + p - 36 = 0$ and $4\left(\frac{p}{4} - 9\right)$.
	$-36 + p$ or $p - 36$ seen or $c = p - 36$	B1	
		2	
3(b)	$p - 36 > 0$ leading to $p > 36$ or $24^2 - 4 \times 4p < 0 \Rightarrow p > 36$ or $36 < p$	B1	Allow $(36, \infty)$ or $36 < p < \infty$. Consider final answer only.
		1	

Question	Answer	Marks	Guidance
4	$[8x^6 + 215x^3 - 27 = 0]$ leading to $(8x^3 - 1)(x^3 + 27) = 0$ OR $\frac{-215 \pm \sqrt{215^2 - 4 \cdot 8 \cdot -27}}{2 \cdot 8}$ or $\frac{-215 \pm \sqrt{47089}}{2 \cdot 8}$	M1	OE If a substitution is used then the correct coefficients must be retained. Condone substitution of $x = x^3$.
	$\frac{1}{8}, -27$	A1	Both correct values seen. SC: if M0 scored SC B1 is available for sight of $\frac{1}{8}$ and -27 OE
	$\frac{1}{2}$ or $0.5, -3$	A1	SC: if M0SCB1 scored then SCB1 is available for the correct answers and no others. Do not ISW if answers given as a range.
		3	

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Question	Answer	Marks	Guidance
5	$\left[\int \left(10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} \right) dx \right] = \left\{ \frac{10}{\frac{3}{2}}x^{\frac{3}{2}} \right\} \left\{ -\frac{5}{2 \times \frac{5}{2}}x^{\frac{5}{2}} \right\} \left[= \frac{20}{3}x^{\frac{3}{2}} - x^{\frac{5}{2}} \right]$	B1 B1	B1 for contents of each { } then ISW.
	$= \left(\text{their } \frac{20}{3} \times 8 - 32 \right) [-0]$	M1	Using limit(s) correctly in an integrated expression (defined by one correct power). Minimum acceptable working is their $\left(\frac{160}{3} - 32 \right)$.
	[Area of shaded region =] $\frac{64}{3}, 21\frac{1}{3}$ or 21.3[333...]	A1	Condone the presence of π for the first 3 marks. Condone using the limits the wrong way around for the M mark and if -21.3 is corrected to 21.3 allow the A mark. SC: if M0 scored SCB1 is available for correct final answer If $\int \left(10x^{\frac{1}{2}} - \frac{5}{2}x^{\frac{3}{2}} \right) dx = 21.3$ and no integration seen B1 only.
		4	

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Question	Answer	Marks	Guidance
6(a)	$\frac{1}{2}OA = x \cos \theta$ or $\frac{OA}{\sin(\pi - 2\theta)} = \frac{x}{\sin \theta}$ or $OA^2 = x^2 + x^2 - 2x^2 \cos(\pi - 2\theta)$ or $x^2 = r^2 + x^2 - 2rx \cos \theta$ or other valid method.	*B1	Correct expression containing $\frac{1}{2}OA$, OA or OA^2 (allow p , a or r for OA) containing only terms with x and θ but not just $OA = 2x \cos \theta$. Do not condone $\sin \pi - 2\theta$ until missing brackets recovered or $\cos(180 - 2\theta)$ until it becomes $-\cos 2\theta$ etc.
	$OA = 2x \cos \theta$ leading to Arc length = $2x\theta \cos \theta$	DB1	AG Complete correct method showing all necessary working. Condone $2x \cos \theta \times \theta$.
		2	If B0 but www then SCB1 for $OA = 2x \cos \theta$ leading to Arc length = $2x\theta \cos \theta$.
6(b)	Sector area = $\frac{1}{2}(2x \cos \theta)^2 \times \theta$	M1	OE Using sector formula with a correct OA. Condone $\cos^2 \theta$ for $\cos^2 \theta$ and missing brackets.
	Triangle area = $\frac{1}{2} \times 2x \cos \theta x \sin \theta$ OR $\frac{1}{2}x^2 \sin(\pi - 2\theta)$	M1	Using a correct triangle formula for the correct triangle. Condone missing brackets and 180 for π .
	[Area APB =] Their sector area – their triangle area	M1	Both expressions must be areas involving terms with x^2 and θ only. Condone missing brackets and 180 for π for the triangle. Condone calling the sector a segment.
	[Area APB =] $\frac{1}{2}(2x \cos \theta)^2 \times \theta - \frac{1}{2}x^2 \sin(\pi - 2\theta)$ [= $x^2(2\theta \cos^2 \theta - \frac{1}{2} \sin 2\theta)$ or $x^2 \cos \theta(2\theta \cos \theta - \sin \theta)$]	A1	OE A correct expression. Mark the first unsimplified result of subtraction and ISW any incorrect ‘simplifications’.
		4	

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Question	Answer	Marks	Guidance
7(a)(i)	$\cos^2\theta + 2\sin\theta\cos\theta + \sin^2\theta = 1$ leading to $2\sin\theta\cos\theta = 0$ or $\sin 2\theta = 0$	*B1	Or arriving at $\cos\theta = 0$ or $\sin\theta = 0$ or $\tan\theta = 0$ after first expanding and www.
	$[\theta =] 0, \frac{\pi}{2}, \pi$	DB 2,1,0	B2 for three correct answers only. B1 for two correct answers and one incorrect or 3 correct answers plus other values in the range. SC DB1 for correct 3 answers in degrees and no others. Ignore extras outside of the range and allow decimal equivalents.
		3	Verifying 3 answers rather than expanding and solving 0/3.
7(a)(ii)	$\cos 0 + \sin 0 = [1 + 0] = 1$ and $\cos \frac{\pi}{2} + \sin \frac{\pi}{2} [= 0 + 1] = 1$	B1	Checking both correct values. Do not allow solving an equation. Condone use of 90 degrees.
	$\cos \pi + \sin \pi [= -1 + 0] = -1$ or $\neq 1$	B1	www
		2	

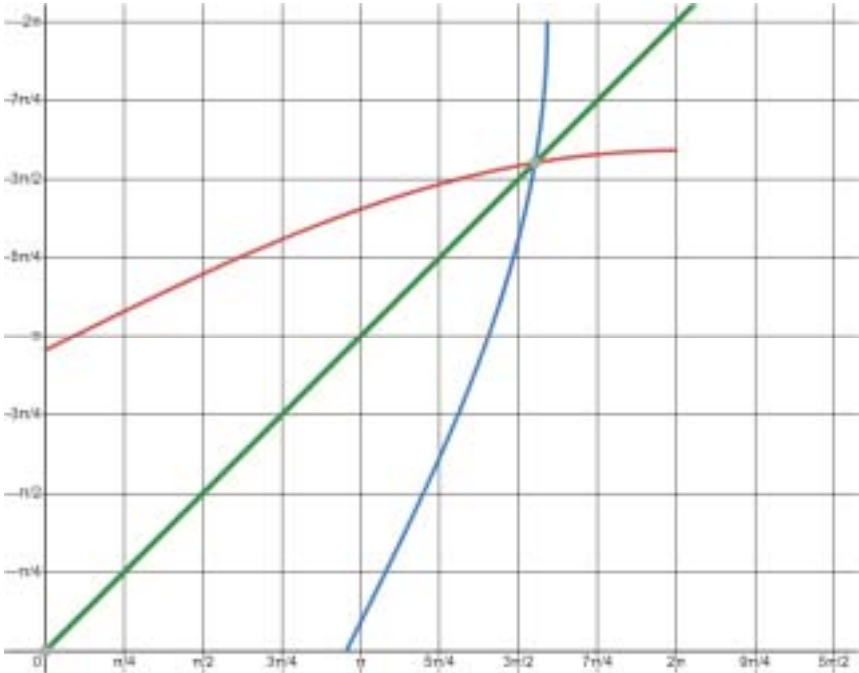
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Question	Answer	Marks	Guidance
7(b)	$\frac{(\cos\theta - \sin\theta)\sin\theta + (\cos\theta + \sin\theta)(1 - \cos\theta)}{(\cos\theta + \sin\theta)(\cos\theta - \sin\theta)}$	M1	Correct common denominator and correct products in the numerator and no missing terms. Correct factors in the denominator can be implied by $\cos^2\theta - \sin^2\theta$. Condone brackets missing if recovered.
	$= \frac{\cos\theta\sin\theta - \sin^2\theta + \cos\theta - \cos^2\theta + \sin\theta - \sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta}$	A1	
	$= \frac{\sin\theta + \cos\theta - \cos^2\theta - \sin^2\theta}{\cos^2\theta - \sin^2\theta} = \frac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta}$	A1	AG Clear evidence of using $\sin^2\theta + \cos^2\theta = 1$ in either the numerator or denominator. Condone c, s and/or omission of θ . Working from both sides of the identity and correctly arriving at the same expression can score M1A1. A final statement is then required for the A1.
		3	

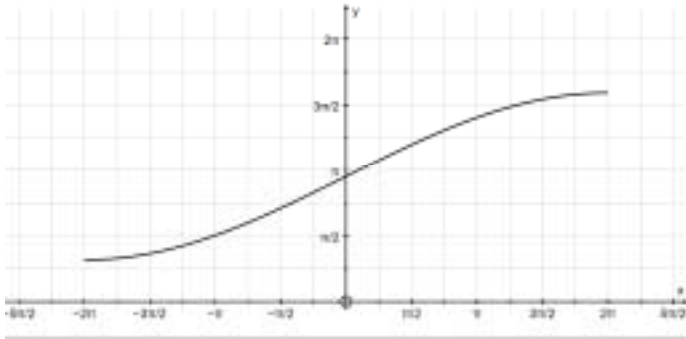
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Question	Answer	Marks	Guidance
7(c)	$\frac{\cos\theta + \sin\theta - 1}{1 - 2\sin^2\theta} = 2(\cos\theta + \sin\theta - 1)$ leading to $1 = 2(1 - 2\sin^2\theta)$	*M1	Replacing LHS with the expression from (b) and attempting to simplify i.e. condone omission of $(\cos\theta + \sin\theta - 1) = 0$ at this stage. M0 for $0 = 2(1 - 2\sin^2\theta)$
	$k\sin^2\theta = 1 \text{ or } 3 \text{ leading to } \sin\theta = [\pm]\sqrt{\frac{1 \text{ or } 3}{k}}$ $\left[4\sin^2\theta = 1 \text{ leading to } \sin\theta = \pm\frac{1}{2} \right]$	DM1	Dividing by k and taking the square root of a positive value < 1. This mark can be implied by the solutions $\frac{1}{6}\pi, \frac{5}{6}\pi$.
	Solutions $0, \frac{1}{6}\pi, \frac{1}{2}\pi, \frac{5}{6}\pi$	A1	Allow 0, 0.524, 1.57, 2.62 AWR. If M0 SCB1 for $(\cos\theta + \sin\theta - 1) = 0 \Rightarrow 0, \frac{1}{2}\pi$. If M0 SCB1 for all four correct answers and no others. Ignore answers outside of the range. Answers in degrees A0.
		3	

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Question	Answer	Marks	Guidance
8(a)		*B1	The line $y = x$ correctly drawn. Can be implied by reasonably correct graph of $f^{-1}(x)$.
		DB1	Fully correct (needs to reach $y = 2\pi$ and x -axis and cross the line $y = x$ in the correct squares).
		2	

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Question	Answer	Marks	Guidance
8(b)	$y = 3 + 2\sin\frac{1}{4}x$ leading to $\sin\frac{1}{4}x = \frac{y \pm 3}{2}$	M1	Attempting to arrive at an expression for $\sin\frac{1}{4}x$; condone $\pm$ sign errors. Variables may be interchanged initially. M1 not implied by $x = \frac{y \pm 3}{2\sin\frac{1}{4}}$.
	$x = 4\sin^{-1}\left(\frac{y-3}{2}\right)$ leading to $[f^{-1}(x) \text{ or } y =] 4\sin^{-1}\left(\frac{x-3}{2}\right)$	A1	ISW Must clearly be $\sin^{-1}\left(\frac{x-3}{2}\right)$ NOT $\frac{\sin^{-1}(x-3)}{2}$. Allow $\left(\frac{3-x}{-2}\right)$ but not $\div\frac{1}{4}$.
		2	
8(c)		B1	Continuing given graph from y intercept to -2π . The correct shape needed between 0 and -2π , including starting to level off (gradient in the final two squares needs to be reducing) as -2π is approached. The y co-ordinate at -2π must be in the correct square.
	Yes it does have an inverse, because the graph is always increasing OR because it is one-one OR because it passes the horizontal line test OR it is not a many to one [function].	B1 FT	If there is no graph to the left of the y axis, no mark is available. FT an incorrect graph and if the answer is now 'No' provide an appropriate reason.
		2	

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Question	Answer	Marks	Guidance
8(d)	{ } indicates different elements throughout.		
	{Stretch} {factor 4} {in x -direction}	B2, 1, 0	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of x -axis, horizontally or y -axis invariant. Wavelength or period increased by a factor of 4 for B2 or by 4 for B1.
	{Stretch} {factor 2} {in y -direction}	B2, 1, 0	B2 for fully correct, B1 with two elements correct. Condone use of 'sf' instead of factor and 'co-ordinates' stretched instead of graph stretched. Allow any mention of y -axis, vertically or x -axis invariant. Allow y 'co-ordinates' doubled or amplitude doubled for B2.
	{Translation} $\begin{pmatrix} \{0\} \\ \{3\} \end{pmatrix}$	B2, 1, 0	B2 for fully correct, B1 with two elements correct. Allow shift. Any mention of y axis, y -direction or vertically implies $\{0\}$, so shift by 3 vertically is B2, but shift by a factor of 3 vertically or a translation of 3 'up' is B1.
		6	After scoring B2, B2 the final transformation can only be awarded B2 if the order is fully correct i.e. the translation must not be applied before the y stretch. If all correct except the order award B2B2B1.

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Question	Answer	Marks	Guidance
9(a)	$\left[ar = 16, \frac{a}{1-r} = 100 \right]$ leading to $a = \frac{16}{r}$ and $a = 100(1-r)$	B1	Rearranging two algebraic statements to give $a =$. These can be implied by a correct equation in one variable.
	$100(1-r)r = 16$ leading to $100r^2 - 100r + 16 = 0$	*M1	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors only.
	$(5r-4)(5r-1) = 0$ OR $\frac{25 \pm \sqrt{25^2 - 4.25.4}}{2.25}$ leading to $r = \left[\frac{4}{5} \text{ or } \frac{1}{5} \right]$	DM1	Condone $(5r-4)(5r-1)$ following $100r^2 - 100r + 16$.
	$a = 20, a = 80$	A1	SC: if DM0 scored SCB1 is available for sight of 20 and 80.
	Alternative Method for Question 9(a)		
	$\left[ar = 16, \frac{a}{1-r} = 100 \right]$ leading to $r = \frac{16}{a}$ and $r = \frac{100-a}{100}$	B1	Rearranging two algebraic statements to give $r =$. These can be implied by a correct equation in one variable.
	$1600 = 100a - a^2$ leading to $a^2 - 100a + 1600 = 0$	*M1	Using their two expressions and rearranging to get a 3-term quadratic expression with all of the terms on one side. Condone sign errors and 160 instead of 1600 only.
	$(a-20)(a-80) = 0$ OR $\frac{100 \pm \sqrt{100^2 - 4.1600}}{2}$	DM1	
	$a = 20, a = 80$	A1	SC: if DM0 scored SCB1 is available for sight of 20 and 80.
		4	

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Question	Answer	Marks	Guidance
9(b)	$r = \frac{4}{5}, \frac{1}{5}$	B1	OE SOI
	$[u_n =] \text{their } 20 \times \text{their} \left(\frac{4}{5}\right)^{n-1} \quad [v_n =] \text{their } 80 \times \text{their} \left(\frac{1}{5}\right)^{n-1}$	B1FT	2 expressions for the nth term FT <i>their</i> values from part (a) if $ r $ less than 1.
	Method 1 for final 2 marks		
	$20 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1}$	M1	Correctly separating the numerator and denominator of <i>their</i> $\left(\frac{4}{5}\right)^{n-1}$ or one correct step towards the solution eg $u_n = 80 \times \frac{4^{n-2}}{5^{n-1}}$.
	$u_n = \frac{1}{4} \times 80 \times \left(\frac{1}{5}\right)^{n-1} \times 4^{n-1} = 4^{n-2} \times 80 \times \left(\frac{1}{5}\right)^{n-1} = 4^{n-2} \times v_n$	A1	AG Given result clearly shown
	Method 2 for final 2 marks		
	$\frac{20 \times 0.8^{n-1}}{80 \times 0.2^{n-1}} = \frac{1}{4} \times 4^{n-1}$	M1	Dividing two nth terms of the correct format and simplifying their terms in r .
	$= 4^{-1} \times 4^{n-1} = 4^{n-2}$	A1	AG
		4	

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Question	Answer	Marks	Guidance
10(a)	$(x-a)^2 + \left(\frac{1}{2}x + 6 - 3\right)^2 = 20$ or using $x = 2y - 12$	*M1	Obtaining an unsimplified equation in x or y only.
	$\frac{5}{4}x^2 + (3-2a)x + a^2 - 11 [= 0]$	A1	OE e.g. $5x^2 + 4(3-2a)x + 4a^2 - 44$ Rearranging to get a correct 3-term quadratic on one side. Condone terms not grouped together. $5y^2 - y(54 + 4a) + 133 + a^2 + 24$.
	$(3-2a)^2 - 4 \times \frac{5}{4}(a^2 - 11) [= 0]$	DM1	OE Using $b^2 - 4ac$ on <i>their</i> 3 term quadratic $[= 0]$.
	Method 1 for final 2 marks		
	Using $a = 4$: $(3-8)^2 - 5(5) = 0$	A1	Clearly substituting $a = 4$.
	$a = -16$	B1	Condone no method shown for this value.
	Method 2 for final 2 marks		
	$-a^2 - 12a + 64 = 0 \Rightarrow (a-4)(a+16) = 0 \Rightarrow a = 4$	A1	AG Full method clearly shown.
	$a = -16$	B1	Condone no method shown for this value.
		5	If M0, SCB1 available for substituting $a = 4$, finding P(2, 7) and verifying that $CP^2 = 20$.

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Question	Answer	Marks	Guidance
10(b)	Centre (4, 3) identified or used or the point P is (2, 7)	B1	
	$\therefore$ gradient of normal = -2	B1	SOI
	Forming normal equation using their gradient (not 0.5) and their centre or P	M1	Condone use of $(\pm 4, \pm 3)$.
	$\frac{y-3}{(x-4)} = -2$ or $y-7 = -2(x-2)$	A1	OE Condone $f(x) =$.
		4	

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Question	Answer	Marks	Guidance
10(c)	Method 1 for Question 10(c)		
	Diameter: $y - 3 = \frac{1}{2}(x - 4)$ [leading to $y = \frac{1}{2}x + 1$] Or $2(x - 4) + 2(y - 3)\frac{dy}{dx} = 0$ [leading to $y = \frac{1}{2}x + 1$]	*M1	Using gradient $\frac{1}{2}$ with their centre. By implicit differentiation.
	$(x - 4)^2 + \left(\frac{1}{2}x + 1 - 3\right)^2 = 20$ [$\frac{5}{4}x^2 - 10x = 0$]	DM1	Obtaining an unsimplified equation in x or y only. [$y^2 - 6y + 5 = 0$].
	$x = 0$ or 8 , $y = 1$ or 5 [(0, 1) and (8, 5)]	A1	Correct co-ordinates for both points. Condone no method shown for solution.
	Equations are $y - 1 = -2x$ and $y - 5 = -2(x - 8)$	A1	$2x + y = 1$ and $2x + y = 21$.
	Method 2 for Question 10(c)		
	Coordinates of points at which tangents meet curve are $(4+4, 3+2) = (8, 5)$ and $(4-4, 3-2) = (0, 1)$	*M1 A1	Vector approach using their centre and gradient = 0.5 . Condone answers only with no working.
	Equations are $y - 5 = -2(x - 8)$ and $y - 1 = -2x$	DM1 A1	Forming equations of tangents using <i>their</i> (0, 1) and (8, 5).
	Method 3 for Question 10(c)		
	$(x - 4)^2 + (-2x + c - 3)^2 = 20$ [$5x^2 + (4 - 4c)x + (c - 3)^2 - 4 = 0$]	*M1	Obtaining an unsimplified equation in x only using equation of circle with $y = -2x + c$.
	$(4 - 4c)^2 - 20((c - 3)^2 - 4) [= 0]$ [leading to $-4c^2 - 32c + 120c + 16 - 100 = 0$]	DM1	Using $b^2 - 4ac [= 0]$.

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Question	Answer	Marks	Guidance
10(c)	$4c^2 - 88c + 84 [= 0]$ [leading to $c^2 - 22c + 21 = 0$]	A1	
	$c = 21$ and $c = 1$ or $y = -2x + 21$ and $y = -2x + 1$	A1	Condone no method shown for solution.
		4	

Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \left\{ k \frac{1}{2} (4x+1)^{-\frac{1}{2}} \right\} \{ \times 4 \} \{ -1 \}$	B 2,1,0	OE e.g. $2k(4x+1)^{-\frac{1}{2}} - 1$ B2 Three correct unsimplified { } and no others. B1 Two correct { } or three correct { } and an additional term e.g. +5. B0 More than one error.
		2	
11(b)	$2k(4x+1)^{-\frac{1}{2}} - 1 = 0$ leading to $(4x+1)^{\frac{1}{2}} = 2k$ or $\frac{2k}{(4x+1)^{\frac{1}{2}}} = 1$	M1	OE Equating their $\frac{dy}{dx}$ of the form $ak(4x+1)^{-\frac{1}{2}} - 1$ where $a = 2$ or 0.5 , to 0 and dealing with the negative power correctly including k not multiplied by $(4x+1)^{\frac{1}{2}}$.
	$x = \frac{4k^2 - 1}{4}$	A1	CAO OE simplified expression ISW.
		2	

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Question	Answer	Marks	Guidance
11(c)	$2 \times 10.5(4x+1)^{-\frac{1}{2}} - 1 = 2$	M1	Putting $k = 10.5$ into their $\frac{dy}{dx}$ and equating to 2.
	$7 = (4x+1)^{\frac{1}{2}}$ leading to $4x+1 = 49$ leading to $x = 12$	A1	If M1 earned SCB1 available for $x = \frac{33}{64}$ from $a = \frac{1}{2}$.
	$y = [10.5\sqrt{4x+1} - x + 5] 66.5$ [leading to (12, 66.5)]	A1	
	$y - 66.5 = -\frac{1}{2}(x - 12)$	A1	OE
		4	



Cambridge International AS & A Level

MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2023

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	{Translation} $\begin{pmatrix} \{0\} \\ \{-2\} \end{pmatrix}$	B2, 1, 0	B2 for fully correct, B1 with two elements correct. { } indicates different elements.
	{Stretch} {[scale] factor 2} {parallel to x -axis}	B2, 1, 0	B2 for fully correct, B1 with two elements correct.
		4	Transformations can be in either order.

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Question	Answer	Marks	Guidance
2	$x^2 - 6x + c > 2$ leading to $(x - 3)^2 - 9 + c > 2$	M1 A1	M1 for completion of the square with an equation or in equality with the '2'.
	$c > 11 - (x - 3)^2$ and $(x - 3)^2 \geq 0$	M1	SOI
	$c > 11$	A1	
	Alternative Method 1		
	$\frac{dy}{dx} = 2x - 6 = 0$	M1	M1 for differentiating and setting $\frac{dy}{dx} = 0$.
	$x = 3$	A1	
	When $x = 3$, $y = 9 - 18 + c$	M1	
	$[-9 + c > 2] \quad c > 11$	A1	
	Alternative Method 2		
	$x^2 - 6x + c > 2$ leading to $x^2 - 6x + c - 2 > 0$ then use of ' $b^2 - 4ac$ '	M1	
	$36 - 4(1)(c - 2) < 0$	M1 A1	OE Must be correct inequality for M1.
	$c > 11$	A1	
		4	

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Question	Answer	Marks	Guidance
3(a)	$x^5 + 10x^3 + 40x + \frac{80}{x} + \frac{80}{x^3} + \frac{32}{x^5}$ or $x^5 + 10x^3 + 40x + 80x^{-1} + 80x^{-3} + 32x^{-5}$	B2, 1, 0	B2, all terms correct, B1 5 terms correct. Terms must be simplified. Lists of terms allowed.
		2	
3(b)	<i>their</i> $40 \times a + (\text{their coefficient of } x^{-1}) \times b = 0$	M1	Coefficients of a and b must be non-zero, allow x 's so long as they are dealt with correctly.
	$(\text{their coefficient of } x^{-1}) \times a + (\text{their coefficient of } x^{-3}) \times b = 80$	M1	Coefficients of a and b must be non-zero, allow x 's as long as they are dealt with correctly.
	$a = 2 \quad b = -1$	A1 A1	Dependent on both M marks, may be seen without working.
		4	

Question	Answer	Marks	Guidance
4(a)	$3\sin^2 x - 3\sin^2 x \cos^2 x - 4\cos^2 x [= 0]$	M1	Replace $\tan^2 x$ with $\frac{\sin^2 x}{\cos^2 x}$ and multiply by $\cos^2 x$.
	$3(1 - \cos^2 x) - 3(1 - \cos^2 x)\cos^2 x - 4\cos^2 x [= 0]$	M1	Replace $\sin^2 x$ by $1 - \cos^2 x$ twice.
	$3\cos^4 x - 10\cos^2 x + 3 = 0$ or $-3\cos^4 x + 10\cos^2 x - 3 = 0$	A1	Or multiple of these equations.
		3	

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Question	Answer	Marks	Guidance
4(b)	$(3\cos^2 x - 1)(\cos^2 x - 3) [= 0]$	M1	OE, using <i>their</i> equation in the given form. Allow unusual notation if meaning is clear.
	$\cos x = [\pm] \frac{1}{\sqrt{3}}$	A1	SOI Answer only SC B1 .
	54.7°,	A1	
	125.3°	A1 FT	Only other answer and must be from correct factorisation for A1. FT for 180° – <i>their</i> first answer . Answers only SC B1 , SC B1 FT .
		4	

Question	Answer	Marks	Guidance
5(a)	$(x-1)^2 + (x-9+4)^2 = 40$	M1	Substitute line into circle.
	$x^2 - 6x - 7 [= 0]$ leading to $(x+1)(x-7) [= 0]$	M1	Simplify to 3-term quadratic and factorise OE.
	$(-1, -10), (7, -2)$ or $x = -1$ and $7, y = -10$ and -2	A1 A1	Answers only SC B1 , SC B1 but must see a correct quadratic equation.
		4	

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Question	Answer	Marks	Guidance
5(b)	[C is mid-point =] $\left(\frac{\text{their } x_1 + \text{their } x_2}{2}, \frac{\text{their } y_1 + \text{their } y_2}{2}\right)$	M1	Expect (3, -6).
	Radius = $\sqrt{(\text{their } x - \text{their } 3)^2 + (\text{their } y - \text{their } (-6))^2}$ OR $\text{their } \sqrt{\left((7 - (-1))^2 + (-2 - (-10))^2\right)} / 2$	M1	Expect $\sqrt{32}$.
	$(x - 3)^2 + (y + 6)^2 = 32$	A1	OE
		3	

Question	Answer	Marks	Guidance
6(a)	$\frac{\frac{1}{2}r^2\theta}{r\theta} = \frac{76.8}{9.6}$ or $\frac{1}{2}\left(\frac{9.6^2}{\theta^2}\right)\theta = 76.8$	M1	Eliminate θ or r using correct formulae SOI.
	$r = 16$	A1	
	$\theta = 0.6$	A1	Accept 34.4°
	$\Delta OAB = \frac{1}{2} \times \text{their } 16^2 \times \sin \text{their } 0.6$	M1	Allow Segment = $76.8 - \frac{1}{2} \times \text{their } 16^2 \times \sin \text{their } 0.6$. Expect 72.27.
	[Area = $76.8 - 72.27$ =] 4.53	A1	AWRT
		5	
6(b)	$AB = 2 \times 16 \times \sin 0.3$ OR $AB^2 = 16^2 + 16^2 - 2 \times 16^2 \cos 0.6$	M1	Any valid method with <i>their</i> r, θ . Expect $AB = 9.46$.
	Perimeter = $9.6 + 9.46 = 19.1$	A1	AWRT
		2	

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Question	Answer	Marks	Guidance
7(a)	$[y] < 2$ OR $[f(x)] < 2$	B1	OE e.g. $f < 2, (-\infty, 2), -\infty < f[x] < 2$. Do not accept $x < 2$ or $f(x) \leq 2$.
		1	
7(b)	$y = 2 - \frac{5}{x+2}$ leading to $y(x+2) = 2(x+2) - 5$ leading to $xy + 2y = 2x - 1$	M1	or $\frac{5}{x+2} = 2 - y$ (allow sign errors).
	$2y + 1 = 2x - xy$ leading to $2y + 1 = x(2 - y)$	DM1	or $\frac{5}{2 - y} = x + 2$ (allow sign errors).
	$x = \frac{2y+1}{2-y} \rightarrow f^{-1}(x) = \frac{2x+1}{2-x}$	A1	OE or $y = \frac{5}{2-x} - 2$.
	Domain is $x < 2$	B1 FT	FT on the numerical part of <i>their</i> range from part (a), including $x \neq 2$ not penalized. No FT for $x \in \mathcal{R}, x = k, x \neq k$.
		4	
7(c)	$fg(x) = 2 - \frac{5}{x+3+2}$	B1	
	$= \frac{2(x+5)-5}{x+5}$ or $\frac{2(x+5)}{x+5} - \frac{5}{x+5}$	M1	Use of <i>their</i> common denominator.
	$= \frac{2x+5}{x+5}$	A1	
		3	

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Question	Answer	Marks	Guidance
8(a)	$r = \frac{a}{a+2}$	B1	OE SOI
	$\frac{a}{1 - \frac{a}{a+2}} = 264$	M1	Use of S_{∞} formula.
	$\frac{a(a+2)}{a+2-a} = 264$ leading to $\frac{a(a+2)}{2} = 264$ leading to $a^2 + 2a - 528 [= 0]$	M1*	Process to a 3 term quadratic or a 3 term cubic. May contain terms on LHS and RHS.
	$(a-22)(a+24) [= 0]$	DM1	Attempt to solve.
	$a = 22$ (only)	A1	22 without working SC DB1 (dep on 2 nd M1).
		5	
8(b)	$d = \frac{6^2}{6+2} - 6 = -\frac{3}{2}$	B1	
	$\frac{n}{2} \left\{ 12 + (n-1) \left(\frac{-3}{2} \right) \right\} [=] -480$	M1*	Forming an inequation with <i>their</i> numerical d . May use an equality.
	$[3](n^2 - 9n - 640) [=] 0$	A1	OE May contain terms on LHS and RHS.
	$[n =] \frac{9 \pm \sqrt{81 + 2560}}{2}$	DM1	OE. Expect 30.19 . Working for solution must be shown.
	31 only	A1	Must come from a correct first inequality (or an equality). 31 no working SC DB1 (dep on correct quadratic and correct inequality/equality).
		5	

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Question	Answer	Marks	Guidance
9(a)	$[y =] \{x\} \{+(x-1)^{-2}\} [+c]$	B1 B1	May be unsimplified.
	Sub $x = 0, y = 3$ leading to $3 = 0 + 1 + c$	M1	Substitution into an integral, expect $c = 2$.
	$y = x + (x-1)^{-2} + 2$ or $f(x) = x + (x-1)^{-2} + 2$	A1	$\frac{-2}{(-2)(x-1)^2}$ or $\frac{-2(x-1)^{-2}}{-2}$ must be simplified.
		4	
9(b)	[Gradient of tangent =] $f'(0) = 3$	B1	
	Equation of tangent is $y - 3 = \text{their gradient at } x = 0(x - 0)$	M1*	Expect $y = 3x + 3$, normal gets M0.
	Intersection given by $3x + 3 = x + (x-1)^{-2} + 2$	DM1	FT <i>their</i> equation from part (a).
	$2x + 1 = \frac{1}{(x-1)^2} \rightarrow (2x+1)(x-1)^2 - 1 = 0$ or solve equation before given form reached and show solution ($x = 3/2$) satisfies given result	A1	WWW AG
		4	
9(c)	Substitute $x = \frac{3}{2}$ leading to $(2x+1)(x-1)^2 - 1$ leading to $4 \times \frac{1}{4} - 1 = 0$. Hence $x = \frac{3}{2}$ If shown in (b) must be referenced here (in part (c))	B1	Evaluation of each bracket must be shown. Allow $\left(\frac{1}{2}\right)^2$ for second bracket. Solution of $(2x+1)(x-1)^2 - 1 = 0$ is acceptable.
	When $x = \frac{3}{2}$ $y = 7\frac{1}{2}$	B1	
		2	

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Question	Answer	Marks	Guidance
10(a)	$\left[\frac{dy}{dx} = \right] \{9\} + \left\{-\frac{3}{2}(2x+1)^{1/2} \times 2\right\}$	B1, B1	Including '+c' makes the second term B0.
	$9 - 3(2x+1)^{1/2} = 0$ leading to $2x+1=9$	M1	Set differential to zero and solve by squaring SOI. Beware $9^2 - 3^2(2x+1) = 0$ M0A0. $2x+1 = \sqrt{3}$ or $2x+1 = \pm 9$ get M0.
	Max point = (4, 9)	A1	WWW $y = 9$ must come from original equation.
		4	
10(b)	When $x = 1\frac{1}{2}$, shows substitution or $\frac{dy}{dx} = 3$	M1	Substituting $x = 1\frac{1}{2}$ into their $\frac{dy}{dx}$.
	Gradient of AB is $\frac{5\frac{1}{2} - 3\frac{1}{2}}{1\frac{1}{2} - 7\frac{1}{2}} \left[= \frac{-1}{3} \right]$	M1	Substituting into a correct expression for m_{AB} .
	$-\frac{1}{3} \times 3 = -1$. [Hence AB is the normal]	A1	
	Alternative method for Question 10(b)		
	When $x = 1\frac{1}{2}$ $\frac{dy}{dx} = 3$, [perpendicular gradient is -1/3]	M1	
	Perpendicular through A has equation $y = \frac{-x}{3} + 6$ which contains B(7.5, 3.5) leading to AB is a normal to the curve at A	M1 A1	
		3	

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Question	Answer	Marks	Guidance
10(c)	$\left\{ \frac{9x^2}{2} \right\} + \left\{ \frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2} \times 2} \right\}$	B1 B1	Integrating y with respect to x .
	$\left\{ \frac{9}{2} 7.5^2 - \frac{1}{5} (2 \times 7.5 + 1)^{2.5} \right\} - \left\{ \frac{9}{2} 1.5^2 - \frac{1}{5} (2 \times 1.5 + 1)^{2.5} \right\}$ or $\left(\frac{9}{2} \times \frac{225}{4} - \frac{1024}{5} \right) - \left(\frac{81}{8} - \frac{32}{5} \right)$ or $\frac{1933}{40} - \frac{149}{40}$ or $48.325 - 3.725$	M1	OE Apply limits $1\frac{1}{2}$ to $7\frac{1}{2}$ to an integral. Working must be seen. Expect 44.6 .
	$\frac{1}{2} \left(5\frac{1}{2} + 3\frac{1}{2} \right) \times 6$ or $\int_{\frac{3}{2}}^{\frac{15}{2}} \left(\frac{-1}{3}x + 6 \right) dx =$ $\left(\frac{-1}{6} \times \left(\frac{15}{2} \right)^2 + 6 \times \frac{15}{2} \right) - \left(\frac{-1}{6} \times \left(\frac{3}{2} \right)^2 + 6 \times \frac{3}{2} \right)$ or $\frac{285}{8} - \frac{69}{8}$ [= 27]	B1	SOI Area of trapezium. May be seen combined with the area under the curve integral.
	[Shaded area = $44.6 - 27$ =] 17.6	A1	SC B1 if no substitution of the limits seen.
		5	

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Question	Answer	Marks	Guidance
10(c)	Alternative method for Question 10(c)		
	$A = \int_{\frac{3}{2}}^{\frac{15}{2}} \left((9x - (2x+1)^{\frac{3}{2}}) - \left(\frac{-1}{3}x + 6 \right) \right) dx = \int_{\frac{3}{2}}^{\frac{15}{2}} \left(\left(\frac{28}{3}x - (2x+1)^{\frac{3}{2}} - 6 \right) \right) dx$	M1	Finding the equation of AB and subtracting from the equation of the curve.
	$= \left\{ \frac{28}{3 \times 2} x^2 - 6x \right\} + \left\{ \frac{-(2x+1)^{\frac{5}{2}}}{\frac{5}{2} \times 2} \right\}$	A1 A1	
	$\frac{127}{10} - \frac{-49}{10}$	M1	Apply limits $1\frac{1}{2}$ to $7\frac{1}{2}$ to an integral. Working must be seen.
	17.6	A1	SC B1 if no substitution of limits seen.
		5	