



Cambridge International AS & A Level

MATHEMATICS

9709/51

Paper 5 Probability & Statistics 1

May/June 2022

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **15** printed pages.

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These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
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- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

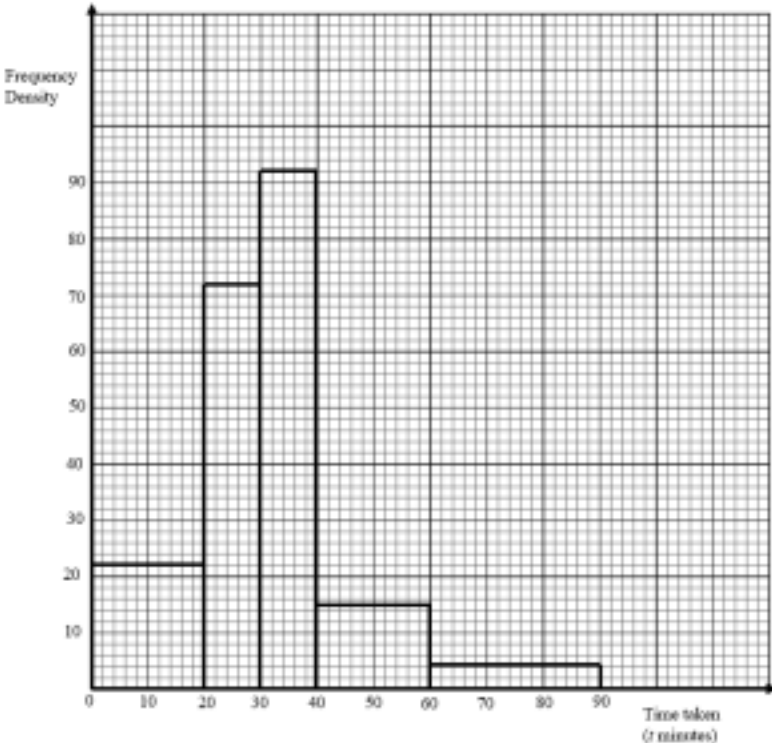
- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	5!	M1	$k!$ where $k = 5, 6$ or 7 Condone $\times 1$ OE
	120	A1	
		2	
1(b)	[Total no of ways =] $\frac{8!}{2!3!}$ [= 3360]	M1	$\frac{8!}{a!b!}$, $a = 1, 2$ $b = 1, 3$ $a \neq b$
	[With 3Es together =] $\frac{6!}{2!}$ [= 360]	M1	$\frac{6!}{c!}$, $c = 1, 2$ seen in an addition/subtraction
	[With 3Es not together] = $3360 - 360$	M1	$\frac{8!}{d!e!} - \frac{6!}{f!}$ where $d, f = 1, 2$ & $e = 1, 3$
	3000	A1	
		4	

Question	Answer	Marks	Guidance
2(a)	$^{12}C_4 \times 2$	M1	$^gC_4 \times h$ $g = 12, 13, h = 1, 2$
	990	A1	
	Alternative method for question 2(a)		
	[total – both on – neither on] $^{14}C_5 - (^{12}C_3 + ^{12}C_5) = [2002 - 220 - 792]$	M1	$^kC_5 - (^aC_3 + ^aC_5)$ $a = 12, 13$ and $k = 13, 14$
	990	A1	
		2	
2(b)	[Mrs Lan plus] 2W 2M $^7C_2 \times ^6C_2 = 315$ 3W 1M $^7C_3 \times ^6C_1 = 210$ 4W $^7C_4 = 35$	M1	$^7C_r \times ^6C_{4-r}$ for $r = 2, 3$ or 4
		B1	Outcome for one identifiable scenario correct, accept unevaluated
		M1	Add outcomes for 3 identifiable correct scenarios Note: if scenarios not labelled, they may be identified by seeing $^7C_r \times ^6C_s$ $r + s = 4$ to imply r women and s men for both B & M marks only
	[Total =] 560	A1	
		4	

Question	Answer	Marks	Guidance												
3(a)	<table><tr><td>Class width</td><td>20</td><td>10</td><td>10</td><td>20</td><td>30</td></tr><tr><td>Frequency density</td><td>22</td><td>72</td><td>92</td><td>15</td><td>4</td></tr></table>	Class width	20	10	10	20	30	Frequency density	22	72	92	15	4	M1	At least 4 frequency densities calculated (Frequency ÷ class width, e.g. $\frac{440}{20} \left(\textit{condone} \frac{440}{19.5}, \frac{440}{20.5} \right)$ Accept unsimplified, may be read from graph using <i>their</i> scale
	Class width	20	10	10	20	30									
	Frequency density	22	72	92	15	4									
		A1	All heights correct on graph NOT FT												
B1		Bar ends at [0,] 20, 30, 40, 60, 90 at axis with a horizontal linear scale with at least 3 values indicated. $0 \leq \text{horizontal scale} \leq 90$													
B1		Axes labelled frequency density (fd), time (<i>t</i>) and minutes (mins) or in a title. Linear vertical scale, with at least 3 values indicated $0 \leq \text{vertical axes} \leq 92$ (condone 90 used).													
		4													

Question	Answer	Marks	Guidance						
3(b)	<table border="1"><tr><td>Midpoints</td><td>10</td><td>25</td><td>35</td><td>50</td><td>75</td></tr></table>	Midpoints	10	25	35	50	75	B1	At least 4 correct midpoints seen
	Midpoints	10	25	35	50	75			
	[Mean = 31.44 given] [Variance = $\frac{440 \times 10^2 + 720 \times 25^2 + 920 \times 35^2 + 300 \times 50^2 + 120 \times 75^2}{2500} - 31.44^2$] = $\frac{44000 + 450000 + 1127000 + 750000 + 675000}{2500} - 31.44^2$ [= $\frac{3046000}{2500} - 31.44^2 = 229.9264$] Or Variance = $\frac{440(10 - 31.44)^2 + 720(25 - 31.44)^2 + 920(35 - 31.44)^2 + 300(50 - 31.44)^2 + 120(75 - 31.44)^2}{2500}$ = $\frac{202256 + 29860 + 11659 + 103342 + 227697}{2500} = \frac{574814}{2500} = 229.9264$	M1	Correct formula for variance or standard deviation (– mean ² included with <i>their</i> midpoints (not upper bound, lower bound, class width, frequency density, frequency or cumulative frequency) and <i>their</i> $\sum f$ if calculated. Condone 1 data error.						
	Standard deviation = 15.2	A1	WWW, allow 15.16[3...]						
		3							
3(c)	30–40	B1							
		1							
3(d)	Stays the same, data still in same intervals	B1	Frequencies unchanged						
		1							

Question	Answer	Marks	Guidance
4(a)	$a = P(1 \text{ head}) = 0.7 \times (0.5)^3 + 0.3 \times (0.5)^3 \times 3 = \frac{1}{5}$	B1	Clear statement of unevaluated correct calculation $= \frac{1}{5}$. AG
	$b = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 \times 3 = \frac{3}{8}$	M1	Clear statement of unevaluated calculation for either b or c
	$c = 0.7 \times 0.5^3 \times 3 + 0.3 \times 0.5^3 = \frac{3}{10}$	A1	For either b or c correct
	$\left[\text{or } c = \frac{27}{40} - b \right]$	B1 FT	$\text{their } b + \text{their } c = \frac{27}{40}$
		4	
4(b)	$\left[E(X) = \frac{3 \times 0 + 16 \times 1 + 30 \times 2 + 24 \times 3 + 7 \times 4}{80} = \right] \frac{176}{80} \text{ or } 2.2$	B1 FT	Correct or accept unsimplified calculation using <i>their</i> values for b and c seen (sum of probabilities = 1)
		1	

Question	Answer	Marks	Guidance
4(c)	$[P(0, 1, 2) =]^{10}C_0 0.2^0 0.8^{10} + ^{10}C_1 0.2^1 0.8^9 + ^{10}C_2 0.2^2 0.8^8$	M1	One term $^{10}C_x p^x (1-p)^{10-x}$, for $0 < x < 10, 0 < p < 1$
	$0.107374 + 0.268435 + 0.301989$	A1	Correct expression, accept unsimplified leading to final answer
	0.678	B1	$0.677 < p \leq 0.678$
	Alternative method for question 4(c)		
	$1 - [^{10}C_{10} 0.2^{10} 0.8^0 + ^{10}C_9 0.2^9 0.8^1 + ^{10}C_8 0.2^8 0.8^2 + ^{10}C_7 0.2^7 0.8^3 + ^{10}C_6 0.2^6 0.8^4 + ^{10}C_5 0.2^5 0.8^5 + ^{10}C_4 0.2^4 0.8^6 + ^{10}C_3 0.2^3 0.8^7]$	M1	One term $^{10}C_x p^x (1-p)^{10-x}$, for $0 < x < 10, 0 < p < 1$
		A1	Correct expression, accept unsimplified
	0.678	B1	$0.677 < p \leq 0.678$
		4	
4(d)	$0.8^6 \times 0.2 + 0.8^7 \times 0.2 = 0.0524288 + 0.041943$	M1	$p^l \times (1-p) + p^m \times (1-p)$, $l = 6, 7$ $m = l + 1, 0 < p < 1$
	0.0944	A1	$0.09437 \leq p \leq 0.0944$
		2	

Question	Answer	Marks	Guidance
5(a)	$P(X < 6) = P(Z < \frac{6-5.2}{1.5}) = P(Z < 0.5333)$	M1	6, 5.2, 1.5 substituted into $\pm$ standardisation formula, condone 1.5^2 , continuity correction ± 0.5
	0.703	A1	
		2	
5(b)	$z_1 = \frac{3-\mu}{\sigma} = -1.329$	B1	$1.328 < z_1 \leq 1.329$ or $-1.329 \leq z_1 < -1.328$
	$z_2 = \frac{8-\mu}{\sigma} = 0.878$	B1	$0.877 < z_2 \leq 0.878$ or $-0.878 \leq z_2 < -0.877$
	Solve to find at least one unknown: $\frac{3-\mu}{\sigma} = -1.329$ $\frac{8-\mu}{\sigma} = 0.878$	M1	Use of the $\pm$ standardisation formula once with μ , σ , a z-value (not 0.8179, 0.7910, 0.5367, 0.5753, 0.19, 0.092 etc.) and 3 or 8, condone continuity correction but not σ^2 or $\sqrt{\sigma}$
		M1	Use either the elimination method or the substitution method to solve their two equations in μ and σ
	$\sigma = 2.27, \mu = 6.01$	A1	$2.26 \leq \sigma \leq 2.27, 6.01 \leq \mu \leq 6.02$
		5	

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Question	Answer	Marks	Guidance
5(c)	$[P(Z < -1) + P(Z > 1)] \Phi(1) - \Phi(-1) =$ $= 2 - 2 \Phi(1)$ $= 2 - 2 \times 0.8413$	M1	Identify 1 and -1 as the appropriate z -values.
		M1	Calculating the appropriate area from stated phis of z -values which must be $\pm$ the same number
	0.3174	A1	Accept AWRT 0.317
	Number of leaves: $2000 \times 0.3174 = 634.8$ so 634 or 635	B1 FT	FT <i>their</i> 4 s.f. (or better) probability, final answer must be positive integer no approximation or rounding stated
		4	

Question	Answer	Marks	Guidance
6(a)	$0.6 + 0.4 \times 0.3 = 0.72$ or $1 - 0.4 \times 0.7 = 0.72$	B1	Clear identified calculation AG
		1	
6(b)	$0.72 \times (0.4 + 0.6 \times 0.2)$	M1	$0.72 \times u, 0 < u < 1$
		M1	$v \times (0.4 + 0.6 \times 0.2)$, or $v \times (1 - 0.6 \times 0.8) 0 < v \leq 1$ no additional terms SC B1 for $0.72 \times (0.4 + 0.12)$ or $0.72 \times (1 - 0.48)$
	0.3744	A1	WWW. Condone 0.374. SC B1 for 0.3744 only
		3	
	Alternative method for question 6(b)		
	$[p(P1P2) + p(F1P1P2) + p(P1F2P2) + p(F1P1F2P2)] =$ $0.6 \times 0.4 + 0.4 \times 0.3 \times 0.4 + 0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.6 \times 0.2$	M1	Any two terms unsimplified and correct
		M1	Summing 4 appropriate scenarios by listing or on a tree diagram SC B1 for $0.24 + 0.048 + 0.072 + 0.0144$
	0.3744	A1	WWW. Condone 0.374. SC B1 for 0.3744 only
		3	

Question	Answer	Marks	Guidance
6(c)	$P(\text{fails first or second level} \text{finishes game}) = \frac{P(\text{fails first or second level} \cap \text{finishes game})}{\text{their (b)}}$	M1	Either $0.6 \times 0.6 \times 0.2$ or $0.4 \times 0.3 \times 0.4$ seen Condone 0.072 or 0.048 if seen in (b)
	Numerator = $P(S \text{ SF}) + P(FS \text{ S}) = 0.6 \times 0.6 \times 0.2 + 0.4 \times 0.3 \times 0.4 = 0.072 + 0.048 = 0.12$	A1	Both correct accept unsimplified expression. No additional terms
	Required probability = $\frac{0.12}{\text{their (b)}}$	M1	<u>Their</u> sum of two 3-term probabilities as numerator <u>their (b)</u> or correct
	0.321 or $\frac{25}{78}$	A1	$0.3205 < p \leq 0.321$
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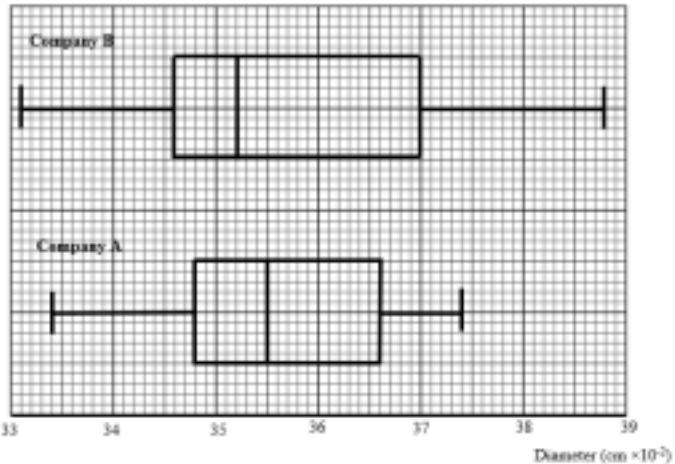
Question	Answer	Marks	Guidance
1	$\sum x - \sum 200 = \sum (x - 200)$	B1	Forming a correct 3-term (linear) equation from $\sum x$, $\sum 200$ and $\sum (x - 200)$. Accept $6846 - 200n = 446$ OE. Condone 1 sign error.
	$\sum 200 = 200n$	B1	SOI
	$[200n = 6846 - 446 = 6400] \quad n = 32$	B1	WWW
		3	

Question	Answer						Marks	Guidance
2(a)	x	2	3	4	5	6	B1	Table with correct X values and at least one probability. Condone any additional X values if probability stated as 0.
	p	$\frac{1}{36}$	$\frac{4}{36}$	$\frac{10}{36}$	$\frac{12}{36}$	$\frac{9}{36}$	B1	3 correct probabilities linked with correct outcomes. Accept 3 sf decimals.
		0.02778	0.1111	0.2778	0.3333	0.25	B1	2 further correct probabilities linked with correct outcomes. Accept 3 sf decimals.
							3	SC B1 for 5 probabilities ($0 < p < 1$) that sum to 1 with less than 3 correct probabilities.

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Question	Answer	Marks	Guidance
2(b)	If method FT from <i>their</i> incorrect (a), expressions for $E(X)$ and $\text{Var}(X)$ must be seen at the stage shown in bold (or less simplified) in the scheme with all probabilities < 1 .		
	$\left[E(X) = \frac{1 \times 2 + 4 \times 3 + 10 \times 4 + 12 \times 5 + 9 \times 6}{36} = \right] \frac{\mathbf{2 + 12 + 40 + 60 + 54}}{\mathbf{36}}$	M1	Accept unsimplified expression. May be calculated in variance. FT <i>their</i> table with 4 or more probabilities summing to $0.999 \leq \text{total} \leq 1$ ($0 < p < 1$).
	$\left[\text{Var}(X) = \frac{1 \times 2^2 + 4 \times 3^2 + 10 \times 4^2 + 12 \times 5^2 + 9 \times 6^2}{36} - \left(\text{their } E(X) \right)^2 = \right]$ $\frac{\mathbf{1 \times 4 + 4 \times 9 + 10 \times 16 + 12 \times 25 + 9 \times 36}}{\mathbf{36}} - \left(\text{their } \frac{\mathbf{14}}{\mathbf{3}} \right)^2$ $\left[\frac{4 + 36 + 160 + 300 + 324}{36} - \left(\text{their } \frac{14}{3} \right)^2 \right]$	M1	Appropriate variance formula using <i>their</i> $(E(X))^2$ value. FT <i>their</i> table with 3 or more probabilities ($0 < p < 1$) which need not sum to 1 and the calculation in bold (or less simplified) seen.
	$E(X) = \frac{168}{36}, \frac{14}{3}, 4.67$ $\text{Var}(X) = \frac{10}{9}, 1\frac{1}{9}, 1.11, \frac{1440}{1296}$	A1	Answers for $E(X)$ and $\text{Var}(X)$ must be identified. $E(X)$ may be identified by correct use in Variance. Condone E, V, μ , σ^2 etc. If M0 earned SC B1 for identified correct final answers.
		3	

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Question	Answer	Marks	Guidance
3(a)	Median = 0.355	B1	Identified condone Q2.
	[IQR =] 0.366 – 0.348	M1	$0.365 \leq UQ \leq 0.369 - 0.343 \leq LQ \leq 0.349$. Subtraction may be implied by answer.
	0.018	A1	If 0/3 scored SC B1 for figs Median = 355 IQR = 18.
		3	
3(b)	Box-and-whisker plot on provided grid 	B1	All 5 key values for <i>B</i> plotted accurately in standard format using <i>their</i> scale. Labelled <i>B</i> . Check accuracy in the middle of vertical line.
		B1 FT	All 5 key values for <i>A</i> , FT from part 3(a), plotted in standard format accurately using <i>their</i> scale. Labelled <i>A</i> . Check accuracy in the middle of vertical line.
		B1	Whiskers not through box for both, not drawn at corners of boxes, single linear scale with at least 3 values stated, covering at least 0.34 to 0.38 and labelled diameter (<i>d</i> etc) and cm. Accept as a title.
		3	If both plots attempted and plot(s) not labelled, SC B1 for at least 1 fully correct set of values plotted.
3(c)	A comparison in context	B1	Single comment comparing spread or central tendency in context. Must reference either diameter or pipes. Not a simple numerical comparison of statistical values such as median, range, IQR or min/max.
		1	

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Question	Answer	Marks	Guidance
4(a)	$[P(1.98 < X < 2.03) =]P\left(\frac{1.98 - 2.02}{0.03} < z < \frac{2.03 - 2.02}{0.03}\right)$ $[= P(-1.333 < z < 0.333)]$	M1	Use of $\pm$ standardisation formula once with 2.02, 0.03 and either 1.98 or 2.03 substituted appropriately. Condone 0.03^2 and continuity correction ± 0.005 , not $\sqrt{0.03}$.
	$[= \Phi(0.333) - (1 - \Phi(1.333))]$ $= 0.6304 + 0.9087 - 1$	M1	Calculating the appropriate probability area from <i>their</i> z-values. (or $0.6304 - 0.09121$ or $(0.9087 - 0.5) + (0.6304 - 0.5)$ etc)
	0.539	A1	$0.539 \leq z < 0.5395$ Only dependent upon 2nd M mark. If M0 scored SC B1 for $0.539 \leq z < 0.5395$.
		3	
4(b)	$[P(X > 2.6) = \frac{134}{5000} = 0.0268]$ $[P(X < 2.6) = 1 - 0.0268 =] 0.9732$	B1	0.9732 or $\frac{4866}{5000}$ or $\frac{2433}{2500}$ seen.
	$\frac{2.6 - 2.55}{\sigma} = 1.93$	M1	Use of $\pm$ standardisation formula with 2.6 and 2.55 substituted, no $\sigma^2, \sqrt{\sigma}$ or continuity correction.
		M1	<i>Their</i> standardisation formula with values substituted equated to z-value which rounds to ± 1.93 .
	$\sigma = 0.0259$	A1	AWRT 0.0259 or $\frac{5}{193}$. If M0 earned, SC B1 for correct final answer.
		4	

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Question	Answer	Marks	Guidance
5(a)	$[P(10, 11, 12) =]$ ${}^{12}C_{10} 0.72^{10} 0.28^2 + {}^{12}C_{11} 0.72^{11} 0.28^1 + {}^{12}C_{12} 0.72^{12} 0.28^0$	M1	One term ${}^{12}C_x p^x (1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
	$= 0.193725 + 0.0905726 + 0.0194084$	A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leq 0.304$.
	Alternative method for question 5(a)		
	$[1 - P(0, 1, 2, 3, 4, 5, 6, 7, 8, 9) =]$ $1 - ({}^{12}C_0 0.72^0 0.28^{12} + {}^{12}C_1 0.72^1 0.28^{11} + {}^{12}C_2 0.72^2 0.28^{10} +$ ${}^{12}C_3 0.72^3 0.28^9 + {}^{12}C_4 0.72^4 0.28^8 + {}^{12}C_5 0.72^5 0.28^7 +$ ${}^{12}C_6 0.72^6 0.28^6 + {}^{12}C_7 0.72^7 0.28^5 + {}^{12}C_8 0.72^8 0.28^4 +$ ${}^{12}C_9 0.72^9 0.28^3)$	M1	One term ${}^{12}C_x p^x (1-p)^{12-x}$, for $0 < x < 12$, $0 < p < 1$.
		A1	Correct expression, accept unsimplified, no terms omitted, leading to final answer.
	0.304	B1	Final answer $0.3036 < p \leq 0.304$.
		3	
5(b)	Mean = $[0.52 \times 90 =]46.8$, var = $[0.52 \times 0.48 \times 90] = 22.464$	B1	46.8 and 22.464 or 22.46 seen, allow unsimplified, $(4.739 < \sigma \leq 4.740)$ imply correct variance).
	$[P(X < 40) =] P\left(z < \frac{39.5 - 46.8}{\sqrt{22.464}}\right)$	M1	Substituting <i>their</i> mean and <i>their</i> variance into $\pm$ standardisation formula (any number for 39.5), not σ^2 , $\sqrt{\sigma}$.
		M1	Using continuity correction 39.5 or 40.5 in <i>their</i> standardisation formula.
	$= [P(Z < -1.540)] = 1 - 0.9382$	M1	Appropriate area Φ , from final process, must be probability.
	0.0618	A1	$0.06175 \leq p \leq 0.0618$
		5	

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Question	Answer	Marks	Guidance
6(a)	$\left[\frac{9!}{2!2!} \right] 90\,720$	B1	
		1	
6(b)	Method 1 Arrangements Cs at ends – Arrangements Cs at ends and Os together		
	[Os not together =] $\frac{7!}{2!} - 6!$ [= 2520 – 720]	M1	$\frac{w!}{2!} - y$, $w = 6, 7$ y an integer. Condone $2 \times \left(\frac{w!}{2!} \right) - y$.
		M1	$a - 6!$ or $a - 720$, a an integer resulting in a positive answer.
	1800	A1	
	Method 2 identified scenarios R ^ ^ ^ R		
	[Os not together =] $5! \times \frac{6 \times 5}{2!} =$	M1	$5! \times b$, b integer > 1 .
		M1	$c \times \left(\frac{6 \times 5}{2!} \text{ or } {}^6C_2 \text{ or } \frac{{}^6P_2}{2!} \text{ or } 15 \right)$, c integer > 1 .
	1800	A1	
		3	

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Question	Answer	Marks	Guidance
6(c)	CCO _ ${}^5C_1 = 5$ CC _ _ ${}^5C_2 = 10$ OOC _ ${}^5C_1 = 5$ OO _ _ ${}^5C_2 = 10$ C _ _ _ ${}^5C_3 = 10$ O _ _ _ ${}^5C_3 = 10$	B1	Correct outcome/value for 1 identified scenario. Accept unsimplified. WWW
		M1	Add 5 or 6 values of appropriate scenarios only, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified. Condone use of permutations.
	[Total =] 50	A1	
		3	
6(d)	Both Os in group with a C ${}^5C_2 = 10$ Both Os in group without a C ${}^5C_2 \times {}^3C_2 = 30$ One O in a C group, one not ${}^5C_1 \times {}^4C_2 = 30$ One O with each C $({}^5C_1 \times {}^4C_1) \div 2! = 10$	B1	A correct scenario calculated accurately. Accept unsimplified.
		M1	Add 3 or 4 correct scenario values, no incorrect scenarios, accept repeated scenarios. Accept unsimplified.
	[Total =] 80	A1	
	Alternative method for question 6(d)		
	CCO O ^ ^ ^ = ${}^5C_2 = 10$ CC ^ O ^ ^ O ^ = ${}^5C_1 \times {}^4C_2 = 30$ CC ^ OO ^ ^ ^ = ${}^5C_1 \times {}^4C_1 = 20$	B1	A correct scenario calculated accurately. Accept unsimplified.
	Total ways of making three groups $\frac{{}^9C_6 \times {}^6C_3}{2 \times 2 \times 3} = 140$ 140 – (their 10+ their 30+ their 20)	M1	Total subtract 2 or 3 correct scenario values, no incorrect scenarios. Accept unsimplified.
	80	A1	
		3	

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Question	Answer	Marks	Guidance
7(a)	YYY: $\frac{5}{12} \times \frac{4}{11} \times \frac{3}{10} = \frac{60}{1320}, \frac{1}{22}$	M1	Either $12 \times 11 \times 10$ in denominator or $a \times (a-1) \times (a-2)$, $a = 5, 4, 3$ in numerator seen in at least one expression.
	OOO: $\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} = \frac{24}{1320}, \frac{1}{55}$	A1	One expression $\frac{a}{12} \times \frac{a-1}{11} \times \frac{a-2}{10}$, $a = 5, 4, 3$ (consistent in expression). Correct order of values in the numerator is essential.
	RRR: $\frac{3}{12} \times \frac{2}{11} \times \frac{1}{10} = \frac{6}{1320}, \frac{1}{220}$	M1	$\frac{5}{12} \times \frac{4}{d} \times \frac{3}{e} + \frac{4}{12} \times \frac{3}{d} \times \frac{2}{e} + \frac{3}{12} \times \frac{2}{d} \times \frac{1}{e}$, either $d = 11, e = 10$ or $d = 12, e = 12$. Condone $\frac{1}{22} + \frac{1}{55} + \frac{1}{220}$ OE
	[Total =] $\frac{90}{1320}, \frac{3}{44}, 0.0682$	A1	0.06818. Dependent only upon the second M mark.

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Question	Answer	Marks	Guidance
7(a)	Alternative method for question 7(a)		
	YYY: $\frac{{}^5C_3}{{}^{12}C_3} = \frac{10}{220}, \frac{1}{22}$	M1	Either ${}^{12}C_3$ in denominator or ${}^aC_3$ in numerator seen in at least one expression.
	OOO: $\frac{{}^4C_3}{{}^{12}C_3} = \frac{4}{220}, \frac{1}{55}$	A1	One expression $\frac{{}^aC_3}{{}^{12}C_3}$ $a = 5, 4, 3$
	RRR: $\frac{{}^3C_3}{{}^{12}C_3} = \frac{1}{220}$	M1	$\frac{{}^5C_3}{{}^{12}C_3} + \frac{{}^4C_3}{{}^{12}C_3} + \frac{{}^3C_3}{{}^{12}C_3}$ Condone $\frac{1}{22} + \frac{1}{55} + \frac{1}{220}$ OE
	[Total =] $\frac{90}{1320}, \frac{3}{44}, 0.0682$	A1	0.06818. Dependent only upon the second M mark.
		4	
7(b)	$P(\text{YYY} \mid \text{all same colour}) = \frac{60}{1320} \div \frac{90}{1320}$	M1	$\frac{\text{their } P(\text{YYY}) \text{ or } \frac{60}{1320} \text{ or } \frac{1}{22}}{\text{their } 7(a) \text{ or } \frac{90}{1320} \text{ or } \frac{3}{44}}$
	$\frac{2}{3}, 0.667$	A1	OE
		2	

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Question	Answer	Marks	Guidance
7(c)	In each method, the M mark requires the scenarios to be identifiable. This may be implied by a list of scenarios and then the calculations which will be assumed to be in the same order. A correct value/expression will be condoned as identifying the connected scenario.		
	Method 1		
	$[1 - \text{no orange} =] 1 - \frac{8}{12} \times \frac{7}{11} \times \frac{6}{10} \text{ or } 1 - \frac{{}^8C_3}{{}^{12}C_3} = 1 - \frac{14}{55}$	B1	$\frac{8}{12} \times \frac{7}{11} \times \frac{6}{10} \text{ or } \frac{{}^8C_3}{{}^{12}C_3} \text{ seen, condone } \frac{336}{1320} \text{ or } \frac{56}{220} \text{ only, not OE.}$
		M1	$1 - \frac{f}{12} \times \frac{g}{d} \times \frac{h}{e}$ Either $d = 11, e = 10$ or $d = 12, e = 12$ $\text{or } 1 - \frac{{}^8C_3}{{}^{12}C_3}.$ Condone $1 - \frac{14}{55}$ OE (not $\frac{41}{55}$).
	$\frac{41}{55}$	A1	$0.745 \leq p \leq 0.74545$ If M0 scored SC B1 $0.745 \leq p \leq 0.74545$.

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Question	Answer	Marks	Guidance
7(c)	Method 2		
	$P(1\text{ O}) = \left(\frac{4}{12} \times \frac{3}{11} \times \frac{2}{10} + \frac{4}{12} \times \frac{5}{11} \times \frac{4}{10} + 2 \times \frac{4}{12} \times \frac{5}{11} \times \frac{3}{10} \right) \times 3 = \frac{672}{1320}$ $P(2\text{ O}) = \frac{4}{12} \times \frac{3}{11} \times \frac{8}{10} \times 3 = \frac{288}{1320}$ $P(3\text{ O}) = \frac{24}{1320}$	B1	P(1 O) or P(2 O) correct, accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12} \times \frac{g}{d} \times \frac{h}{e}$ seen, either $d = 11, e = 10$ or $d = 12, e = 12$.
	[Total =] $\frac{984}{1320} = \frac{41}{55}, 0.745$	A1	$0.745 \leq p \leq 0.74545$ If M0 scored SC B1 $0.745 \leq p \leq 0.74545$.
	Method 3		
	$\begin{aligned} \text{O Y R} &= {}^4C_1 \times {}^5C_1 \times {}^3C_1 &&= 60 \\ \text{O R R} &= {}^4C_1 \times {}^3C_2 &&= 12 \\ \text{O Y Y} &= {}^4C_1 \times {}^5C_2 &&= 40 \\ \text{O O Y} &= {}^4C_2 \times {}^5C_1 &&= 30 \\ \text{O O R} &= {}^4C_2 \times {}^3C_1 &&= 18 \\ \text{O O O} &= {}^4C_3 &&= 4 \\ \text{Total} &&&= 164 \\ \text{Prob} &= \frac{164}{{}^{12}C_3} \end{aligned}$	B1	Number of ways either 1 or 2 orange sweets obtained correctly (112 or 48). Accept unsimplified Note ${}^4C_1 \times {}^8C_2 = 112$ or ${}^4C_2 \times {}^8C_1 = 48$ are correct alternatives.
		M1	3 correct scenarios (1, 2 or 3 orange sweets) added on numerator, denominator ${}^{12}C_3$
	$\frac{984}{1320} = \frac{41}{55}, 0.745$	A1	$0.745 \leq p \leq 0.74545$ If M0 scored SC B1 $0.745 \leq p \leq 0.74545$.

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Question	Answer	Marks	Guidance
7(c)	Method 4		
	$P(R R O) = \frac{3}{12} \times \frac{2}{11} \times \frac{4}{10} = \frac{1}{55}$ $P(R O \quad) = \frac{3}{12} \times \frac{4}{11} = \frac{1}{11}$ $P(R Y O) = \frac{3}{12} \times \frac{5}{11} \times \frac{4}{10} = \frac{1}{22}$ $P(O \quad \quad) = \frac{4}{12} = \frac{1}{3}$ $P(Y R O) = \frac{5}{12} \times \frac{3}{11} \times \frac{4}{10} = \frac{1}{22}$ $P(Y O \quad) = \frac{5}{12} \times \frac{4}{11} = \frac{5}{33}$ $P(Y Y O) = \frac{5}{12} \times \frac{4}{11} \times \frac{4}{10} = \frac{2}{33}$	B1 M1	$P(R \wedge \wedge) = \frac{17}{110}$ or $P(Y \wedge \wedge) = \frac{17}{66}$. Accept unsimplified. 3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12} \times \frac{g}{d} \times \frac{h}{e}$ seen, either $d = 11, e = 10$ or $d = 12, e = 12$.
	$\frac{984}{1320} = \frac{41}{55}, 0.745$	A1	$0.745 \leq p \leq 0.74545$ If M0 scored SC B1 $0.745 \leq p \leq 0.74545$.

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Question	Answer	Marks	Guidance
7(c)	Method 5		
	$P(O) = \frac{4}{12} = \frac{1}{3}$ $P(^O) = \frac{8}{12} \times \frac{4}{11} = \frac{8}{33}$ $P(^ ^ O) = \frac{8}{12} \times \frac{7}{11} \times \frac{4}{10} = \frac{28}{165}$	B1	$P(^O) = \frac{8}{33}$ or $P(^ ^ O) = \frac{28}{165}$. Accept unsimplified.
		M1	3 correct scenarios added, with at least one 3-term product of form $\frac{f}{12} \times \frac{g}{d} \times \frac{h}{e}$ seen, either $d = 11, e = 10$ or $d = 12, e = 12$ with correct numerator.
	$\frac{984}{1320} = \frac{41}{55}, 0.745$	A1	$0.745 \leq p \leq 0.74545$ If M0 scored SC B1 $0.745 \leq p \leq 0.74545$.
		3	



Cambridge International AS & A Level

MATHEMATICS

9709/53

Paper 5 Probability & Statistics 1

May/June 2022

MARK SCHEME

Maximum Mark: 50

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2022 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

This document consists of **14** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (ISW).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

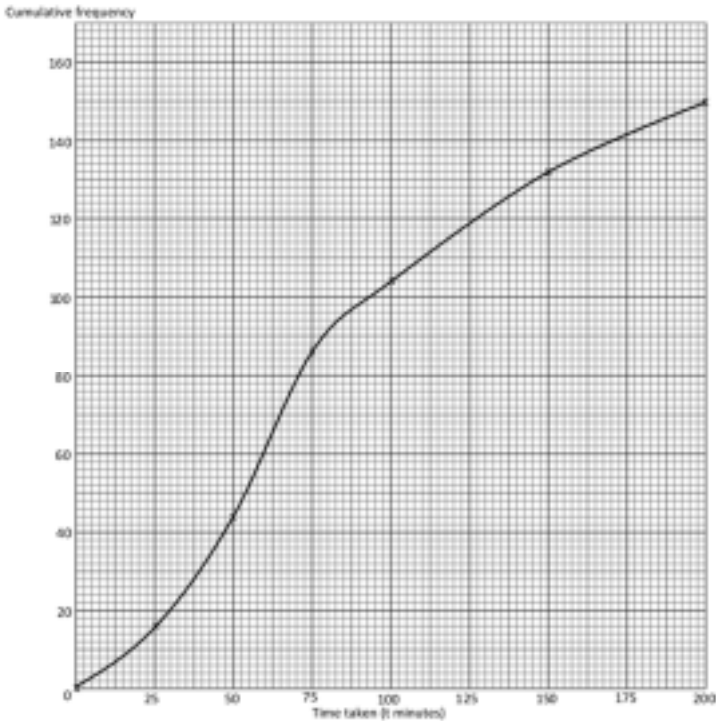
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)	Cumulative frequency (cf) graph	M1	At least 3 points plotted accurately at class upper end points (25,16) (50,44) (75,86) (100,104) (150, 132) (200, 150). Linear cf scale $0 \leq cf \leq 150$ and linear time scale $0 \leq \text{time}(\text{mins}) \leq 200$ with at least 3 values identified on each axis.
		A1	All points plotted correctly, curve drawn (within tolerance) and joined to (0,0). Axes labelled cumulative frequency (cf), time (t) and minutes (min), or a suitable title.
		2	
1(b)	Line from cumulative frequency = 30 to meet graph at t is between 37.5 and 42	B1 FT	Not from wrong working. Must be an increasing cumulative frequency graph.
		1	

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Question	Answer	Marks	Guidance
2(a)	$\left[\frac{123.4}{20} = \right] 6.17$	B1	Accept 6 m 17 cm, $\frac{1234}{200}$.
		1	
2(b)	$\frac{10\text{th} + 11\text{th}}{2} = \frac{5.4 + 5.5}{2} = 5.45 \text{ (m)}$	B1	Accept 5 m 45 cm.
		1	
2(c)	The mean is unduly influenced by an extreme value, 19.4.	B1	Comment must be within context.
		1	

Question	Answer	Marks	Guidance										
3(a)	$k = \frac{1}{18} \text{ (} 4k + k + 4k + 9k = 18k = 1 \text{)}$	B1	SOI										
	<table><tr><td>x</td><td>-2</td><td>1</td><td>2</td><td>3</td></tr><tr><td>$P(X=x)$</td><td>$\frac{4}{18}$</td><td>$\frac{1}{18}$</td><td>$\frac{4}{18}$</td><td>$\frac{9}{18}$</td></tr></table>	x	-2	1	2	3	$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$	M1	Table with correct x values and at least one probability accurate using <i>their</i> k . Values need not be in order, lines may not be drawn, may be vertical, x and $P(X=x)$ may be omitted. Condone any additional X values if probability stated as 0.
	x	-2	1	2	3								
	$P(X=x)$	$\frac{4}{18}$	$\frac{1}{18}$	$\frac{4}{18}$	$\frac{9}{18}$								
		A1	Remaining probabilities correct.										
		3											

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Question	Answer	Marks	Guidance
3(b)	$\left[E(X) = \frac{4 \times -2 + 1 \times 1 + 4 \times 2 + 9 \times 3}{18} = \right]$ $\frac{-8 + 1 + 8 + 27}{18}$	M1	$-8k + k + 8k + 27k$ May be implied by use in Variance. Accept unsimplified expression. FT <i>their</i> table if probabilities sum to 1 or 0.999. SC B1 28k.
	$\left[\text{Var}(X) = \frac{4 \times (-2)^2 + 1 \times 1^2 + 4 \times 2^2 + 9 \times 3^2}{18} - (their E(X))^2 = \right]$ $= \frac{16 + 1 + 16 + 81}{18} - \left(their \frac{28}{18} \right)^2$	M1	$16k + k + 16k + 81k - (their \text{ mean})^2$ FT <i>their</i> table even if probabilities not summing to 1. Note: If table is correct, $\frac{114}{18} - (their E(X))^2$ M1. SC B1 114k – (their mean) ² .
	$E(X) = \frac{14}{9}, 1\frac{5}{9}, 1.56, \text{Var}(X) = \frac{317}{81}, 3\frac{74}{81}, 3.91$	A1	Answers for E(X) and Var(X) must be identified. $3.91 \leq \text{Var}(X) \leq 3.914$
		3	

Question	Answer	Marks	Guidance
4(a)	$\left[\left(\frac{5}{6} \right)^7 \times \frac{1}{6} = \right] 0.0465, \frac{78125}{1679616}$	B1	$0.0465 \leq p < 0.04652$
		1	

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Question	Answer	Marks	Guidance
4(b)	$P(X < 6) = 1 - \left(\frac{5}{6}\right)^5$ or $\frac{1}{6} + \left(\frac{5}{6}\right)\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right) + \left(\frac{5}{6}\right)^4\left(\frac{1}{6}\right)$	M1	$1 - p^n$, $0 < p < 1$, $n = 4, 5, 6$ or sum of 4, 5 or 6 terms $p \times (1 - p)^n$ for $n = 0, 1, 2, 3, 4(5)$.
	$0.598, \frac{4651}{7776}$	A1	
		2	
4(c)	[Probability of total less than 4 is] $\frac{3}{36}$ or $\frac{1}{12}$	B1	SOI
	$[1 - P(0, 1, 2)]$ $= 1 - \left({}^{10}C_0 \left(\frac{1}{12}\right)^0 \left(\frac{11}{12}\right)^{10} + {}^{10}C_1 \left(\frac{1}{12}\right)^1 \left(\frac{11}{12}\right)^9 + {}^{10}C_2 \left(\frac{1}{12}\right)^2 \left(\frac{11}{12}\right)^8 \right)$	M1	One term ${}^{10}C_x p^x (1 - p)^{10-x}$, for $0 < x < 10$, $0 < p < 1$.
	$1 - (0.418904 + 0.380822 + 0.155791)$	A1 FT	Correct expression. Accept unsimplified.
	0.0445	A1	$0.04448 \leq p \leq 0.0445$
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Question	Answer	Marks	Guidance
5(a)	$[P(142 < X < 205)] = P\left(\frac{142-170}{25} < z < \frac{205-170}{25}\right)$	M1	Use of $\pm$ standardisation formula once substituting 170, 25 and either 142 or 205 appropriately.. Condone 25^2 and continuity correction ± 0.5 .
	$P(-1.12 < z < 1.4)$	A1	Both correct. Accept unsimplified.
	$\Phi(1.4) - (1 - \Phi(1.12)) = 0.9192 + 0.8686 - 1$	M1	Calculating the appropriate area from stated phis of z-values.
	0.788	A1	AWRT, not from wrong working
		4	
5(b)	$P(X > 205) = 1 - 0.9192 = 0.0808$	B1 FT	Correct or FT from part 5(a).
	$(0.0808 \times 0.30 + \text{their } 0.788 \times 0.24) \times 20000$	M1	Correct or <i>their</i> $0.0808 \times 0.30 \times k + \text{their } 0.788 \times 0.24 \times k$, k positive integer.
	[\$]4266.24	A1	$4265 < \text{income} \leq 4270$, not from wrong working
		3	
5(c)	$[P(Z > \frac{w-182}{20}) = 0.72]$	B1	$0.5828 \leq z \leq 0.583$ or $-0.583 \leq z \leq -0.5828$ seen.
	$\frac{w-182}{20} = -0.583$	M1	182 and 20 substituted in $\pm$ standardisation formula, no continuity correction, not σ^2 , $\sqrt{\sigma}$, equated to a z-value.
	$w = 170$	A1	$170 \leq w < 170.35$
		3	

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Question	Answer	Marks	Guidance
6(a)	1st 2nd 3rd <pre> graph LR J1(()) -- 0.2 --> S1[S] J1 -- 0.8 --> F1[F] S1 -- 0.3 --> S2[S] S1 -- 0.7 --> F2[F] F1 -- 0.1 --> S3[S] F1 -- 0.9 --> F3[F] S2 -- 0.3 --> S4[S] S2 -- 0.7 --> F4[F] F2 -- 0.1 --> S5[S] F2 -- 0.9 --> F5[F] S3 -- 0.3 --> S6[S] S3 -- 0.7 --> F6[F] F3 -- 0.1 --> S7[S] F3 -- 0.9 --> F7[F] </pre>	B1	First and second jumps correct with probabilities and outcomes identified.
		B1	Third jump correct with probabilities and outcomes identified.
		2	
6(b)	SFF $0.2 \times 0.7 \times 0.9 = 0.126$ FSF $0.8 \times 0.1 \times 0.7 = 0.056$ FFS $0.8 \times 0.9 \times 0.1 = 0.072$	M1	Two or three correct 3 factor probabilities added, correct or FT from part 6(a). Accept unsimplified.
	[Total = probability of 1 success =] $0.254 \left(\frac{127}{500} \right)$	A1	Accept unsimplified.
	[Probability of at least 1 success = $1 - 0.8 \times 0.9 \times 0.9 =$] $0.352 \left(\frac{44}{125} \right)$	B1 FT	Accept unsimplified.
	$P(\text{exactly 1 success} \mid \text{at least 1 success}) = \frac{\text{their } 0.254}{\text{their } 0.352}$	M1	Accept unsimplified.
	$0.722, \frac{127}{176}$	A1	$0.7215 < p \leq 0.722$
		5	

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Question	Answer	Marks	Guidance
6(c)	$0.8 \times 0.9 \times 0.9 \times 0.1 \times 0.3 \times 0.3 = 0.005832$ [FFFSSS] $0.2 \times 0.3 \times 0.3 \times 0.7 \times 0.9 \times 0.9 = 0.010206$ [SSSFFF]	M1	$a \times b \times c \times d \times e \times f$ FT from <i>their</i> tree diagram. Either a, b and c all = 0.8 or 0.9 (at least one of each) and d, e and f all = 0.1 or 0.3 (at least one of each). Or $a, b, c = 0.2$ or 0.3 (at least one of each) and $d, e, f = 0.7$ or 0.9 (at least one of each).
		A1	Either correct. Accept unsimplified.
	[Total =] 0.0160[38]	A1	
		3	

Question	Answer	Marks	Guidance
7(a)	${}^{12}C_5 \times {}^7C_4 [\times {}^3C_3]$	M1	${}^{12}C_r \times q, r = 3, 4, 5$ q a positive integer > 1 , no + or - .__
		M1	${}^{12}C_s \times {}^{12-s}C_t [\times {}^{12-s-t}C_u]$ $s = 3, 4, 5; t = 3, 4, 5 \neq s; u = 3, 4, 5 \neq s, t$
	Alternative method for question 7(a)		
	$\frac{12!}{5! \times 3! \times 4!}$	M1	$12! \div$ by a product of three factorials.
		M1	$\frac{n!}{5! \times 3! \times 4!}$
	[792 $\times$ 35 =] 27 720	A1	CAO
		3	

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Question	Answer	Marks	Guidance
7(b)	$4! \text{ (Lizo)} \times 6! \text{ (Kenny)} \times 2! \text{ (Martin)} \times 2! \text{ (Nantes)}$	M1	Product involving at least 3 of 4!, 6!, 2!, 2!
	$\times 3! \text{ (orders of K, M and N)}$	M1	$w \times 3!$, w integer > 1 .
	414 720	A1	WWW CAO
		3	
7(c)	${}^7C_4 \text{ (adults)} \times {}^4C_1 \times {}^3C_1$	M1	${}^7C_4 \times b$, b integer > 1 no + or –.
	420	A1	
		2	
7(d)	K not L ${}^5C_3 \times {}^8C_3 = 560$ L not K ${}^5C_3 \times {}^8C_3 = 560$ L and K ${}^5C_2 \times {}^8C_3 = 560$	M1	${}^8C_3 \text{ (or } {}^8P_3) \times c$ for one of the products or ${}^5C_3 \text{ (or } {}^5P_3) \times c$, positive integer > 1 for first 2 products only.
		M1	Add 2 or 3 correct scenarios only values, no additional incorrect scenarios, no repeated scenarios. Accept unsimplified.
	[Total or Difference=] 1680	A1	
	Alternative method for question 7(d)		
	Total no of ways – neither L nor K Total = ${}^7C_4 \times {}^8C_3 = 1960$ Neither K nor L = ${}^5C_4 \times {}^8C_3 = 280$	M1	${}^8C_3 \times c$, c a positive integer > 1 .
		M1	Subtracting the number of ways with neither from their total number of ways.
	[Total or Difference=] 1680	A1	

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Question	Answer	Marks	Guidance
7(d)	Alternative method for question 7(d)		
	Subtracting K and L from sum of K and L K ${}^6C_3 \times {}^8C_3 = 1120$ L ${}^6C_3 \times {}^8C_3 = 1120$ L and K ${}^5C_2 \times {}^8C_3 = 560$ $1120 + 1120 - 560 = 1680$	M1	${}^8C_3 \times c$, c a positive integer >1 .
		M1	Subtracting number of ways with both from sum of number of ways with K and number of ways with L.
	[Total or Difference=] 1680	A1	
		3	