

Cambridge International AS & A Level

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MATHEMATICS

9709/21

Paper 2 Pure Mathematics 2

May/June 2022

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

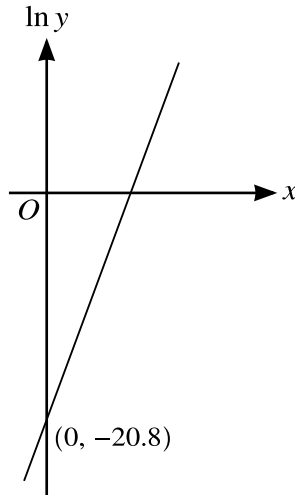
- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **12** pages.

1



The variables x and y satisfy the equation $y = 4^{2x-a}$, where a is an integer. As shown in the diagram, the graph of $\ln y$ against x is a straight line passing through the point $(0, -20.8)$, where the second coordinate is given correct to 3 significant figures.

- (a) Show that the gradient of the straight line is $\ln 16$. [2]

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- (b) Determine the value of a . [2]

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- 2 (a) Express the equation $7 \tan \theta + 4 \cot \theta - 13 \sec \theta = 0$ in terms of $\sin \theta$ only. [3]

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- (b) Hence solve the equation $7 \tan \theta + 4 \cot \theta - 13 \sec \theta = 0$ for $0^\circ < \theta < 360^\circ$. [3]

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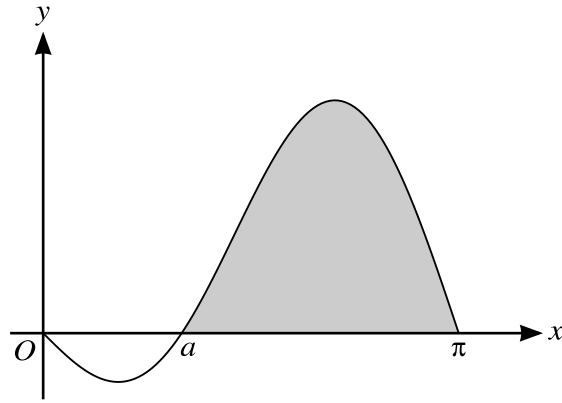
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The diagram shows the curve with equation $y = 3 \sin x - 3 \sin 2x$ for $0 \leq x \leq \pi$. The curve meets the x -axis at the origin and at the points with x -coordinates a and π .

- (a) Find the exact value of a . [3]

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- (b) Find the area of the shaded region. [4]

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- 4** A curve has equation $x^2y + 2y^3 = 48$.

Find the equation of the normal to the curve at the point $(4, 2)$, giving your answer in the form $ax + by + c = 0$ where a, b and c are integers. [7]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 (a) By sketching the graphs of

$$y = |5 - 2x| \quad \text{and} \quad y = 3 \ln x$$

on the same diagram, show that the equation $|5 - 2x| = 3 \ln x$ has exactly two roots. [3]

(b) Show that the value of the larger root satisfies the equation $x = 2.5 + 1.5 \ln x$. [1]

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- (c) Show by calculation that the value of the larger root lies between 4.5 and 5.0. [2]

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- (d) Use an iterative formula, based on the equation in part (b), to find the value of the larger root correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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6 A curve has equation $y = \frac{9e^{2x} + 16}{e^x - 1}$.

(a) Show that the x -coordinate of any stationary point on the curve satisfies the equation

$$e^x(3e^x - 8)(3e^x + 2) = 0. \quad [4]$$

[illegible]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

7 The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + 5x^2 + ax + 2a,$$

where a is an integer.

- (a) Find, in terms of x and a , the quotient when $p(x)$ is divided by $(x + 2)$, and show that the remainder is 4. [3]

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- (b) It is given that $\int_{-1}^1 \frac{p(x)}{x+2} dx = \frac{22}{3} + \ln b$, where b is an integer.

Find the values of a and b . [6]

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[illegible]

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MATHEMATICS

9709/22

Paper 2 Pure Mathematics 2

May/June 2022

1 hour 15 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages. Any blank pages are indicated.

- 1** Given that $y = \frac{\ln x}{x^2}$, find the exact value of $\frac{dy}{dx}$ when $x = e$. [3]

[illegible]

- 2 (a) Sketch, on the same diagram, the graphs of $y = |2x - 9|$ and $y = 5x - 3$. [2]

- (b) Solve the equation $|2x - 9| = 5x - 3$. [2]

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- 3** A curve has equation $e^{2x} \cos 2y + \sin y = 1$.

Find the exact gradient of the curve at the point $(0, \frac{1}{6}\pi)$.

[5]

[illegible]

- 4 (a) Use the trapezium rule with three intervals to show that the value of $\int_1^4 \ln x \, dx$ is approximately $\ln 12$. [4]

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- (b) Use a graph of $y = \ln x$ to show that $\ln 12$ is an under-estimate of the true value of $\int_1^4 \ln x \, dx$. [2]

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5 The polynomial $p(x)$ is defined by

$$p(x) = 2x^3 + ax^2 - 3x - 4,$$

where a is a constant. It is given that $(x - 4)$ is a factor of $p(x)$.

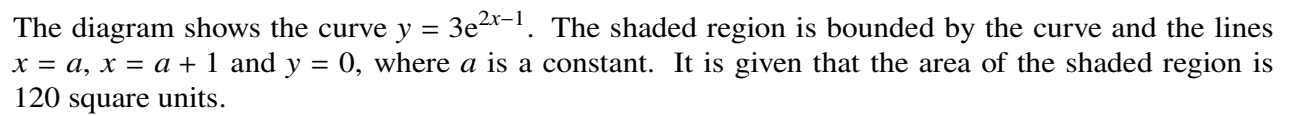
(a) Find the value of a and hence factorise $p(x)$.

[4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b)** Show that the equation $p(e^{3y}) = 0$ has only one real root and find its exact value. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



- [illegible]

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- (b) Use an iterative formula, based on the equation in part (a), to find the value of a correct to 3 significant figures. Give the result of each iteration to 5 significant figures. [3]

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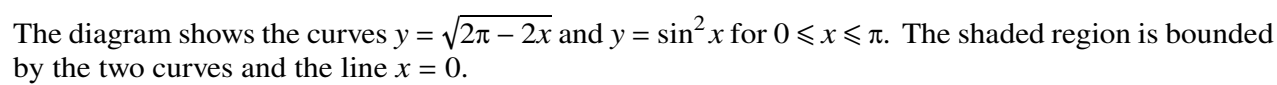
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[8]

[illegible]

- 8** (a) Express $3 \sin 2\theta \sec \theta + 10 \cos(\theta - 30^\circ)$ in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$.
Give the value of α correct to 2 decimal places. [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Hence solve the equation $3 \sin 4\beta \sec 2\beta + 10 \cos(2\beta - 30^\circ) = 2$ for $0^\circ < \beta < 90^\circ$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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MATHEMATICS

9709/23

Paper 2 Pure Mathematics 2

May/June 2022

1 hour 15 minutes

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- 1** Given that $y = \frac{\ln x}{x^2}$, find the exact value of $\frac{dy}{dx}$ when $x = e$. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 2 (a) Sketch, on the same diagram, the graphs of $y = |2x - 9|$ and $y = 5x - 3$. [2]

- (b) Solve the equation $|2x - 9| = 5x - 3$. [2]

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[5]

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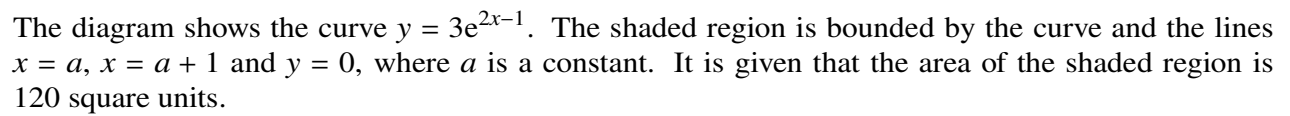
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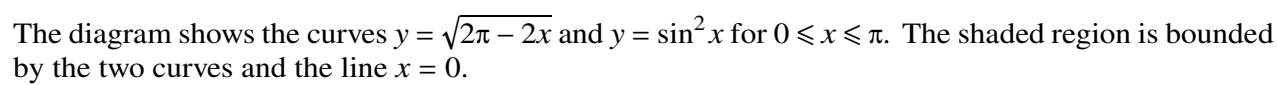
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[8]

This image shows a full page of a worksheet designed for handwriting practice. It features 18 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, providing a clear guide for letter height and placement. There are no margins, text, or other markings present.

- 8** (a) Express $3 \sin 2\theta \sec \theta + 10 \cos(\theta - 30^\circ)$ in the form $R \sin(\theta + \alpha)$ where $R > 0$ and $0^\circ < \alpha < 90^\circ$.
Give the value of α correct to 2 decimal places. [6]

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