

Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

- 1 (a)** Express $x^2 - 8x + 11$ in the form $(x + p)^2 + q$ where p and q are constants. [2]

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- (b)** Hence find the exact solutions of the equation $x^2 - 8x + 11 = 1$. [2]

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- 2** The thirteenth term of an arithmetic progression is 12 and the sum of the first 30 terms is -15 .

Find the sum of the first 50 terms of the progression.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 3** The coefficient of x^4 in the expansion of $\left(2x^2 + \frac{k^2}{x}\right)^5$ is a . The coefficient of x^2 in the expansion of $(2kx - 1)^4$ is b .

(a) Find a and b in terms of the constant k .

[3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

(b) Given that $a + b = 216$, find the possible values of k .

[3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 (a)** Prove the identity $\frac{\sin^3 \theta}{\sin \theta - 1} - \frac{\sin^2 \theta}{1 + \sin \theta} \equiv -\tan^2 \theta (1 + \sin^2 \theta)$. [4]

[illegible]

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(b) Hence solve the equation

$$\frac{\sin^3 \theta}{\sin \theta - 1} - \frac{\sin^2 \theta}{1 + \sin \theta} = \tan^2 \theta (1 - \sin^2 \theta)$$

for $0 < \theta < 2\pi$.

[2]

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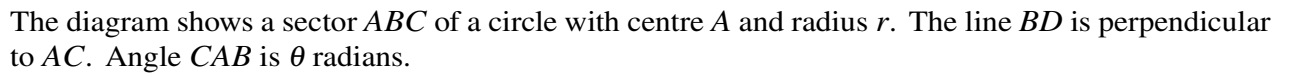
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6 The function f is defined as follows:

$$f(x) = \frac{x^2 - 4}{x^2 + 4} \quad \text{for } x > 2.$$

(a) Find an expression for $f^{-1}(x)$. [3]

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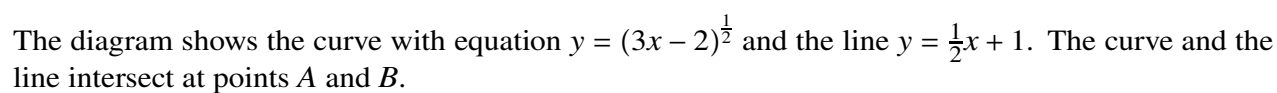
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- (b) Show that $1 - \frac{8}{x^2 + 4}$ can be expressed as $\frac{x^2 - 4}{x^2 + 4}$ and hence state the range of f . [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings present.

- (c) Explain why the composite function ff cannot be formed. [1]



- [illegible]

(b) Hence find the area of the region enclosed between the curve and the line.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 8 (a)** The curve $y = \sin x$ is transformed to the curve $y = 4 \sin(\frac{1}{2}x - 30^\circ)$.

Describe fully a sequence of transformations that have been combined, making clear the order in which the transformations are applied. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Find the exact solutions of the equation $4 \sin(\frac{1}{2}x - 30^\circ) = 2\sqrt{2}$ for $0^\circ \leq x \leq 360^\circ$. [3]

[illegible]

- 9** The equation of a circle is $x^2 + y^2 + 6x - 2y - 26 = 0$.

- (a) Find the coordinates of the centre of the circle and the radius. Hence find the coordinates of the lowest point on the circle. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- (b)** Find the set of values of the constant k for which the line with equation $y = kx - 5$ intersects the circle at two distinct points. [6]

This image shows a full page of a worksheet designed for handwriting practice. It consists of approximately 20 evenly spaced, horizontal dotted lines across the entire width of the page. The background is plain white, and there are no other markings, text, or illustrations present.

10 The equation of a curve is such that $\frac{d^2y}{dx^2} = 6x^2 - \frac{4}{x^3}$. The curve has a stationary point at $(-1, \frac{9}{2})$.

(a) Determine the nature of the stationary point at $(-1, \frac{9}{2})$. [1]

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(b) Find the equation of the curve. [5]

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- (c) Show that the curve has no other stationary points.

[3]

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- (d) A point A is moving along the curve and the y -coordinate of A is increasing at a rate of 5 units per second.

Find the rate of increase of the x -coordinate of A at the point where $x = 1$.

[3]

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[illegible]

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Cambridge International AS & A Level

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages.

- 1** The coefficient of x^4 in the expansion of $(3+x)^5$ is equal to the coefficient of x^2 in the expansion of $\left(2x + \frac{a}{x}\right)^6$.

Find the value of the positive constant a .

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 2** The second and third terms of a geometric progression are 10 and 8 respectively.

Find the sum to infinity.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3** The equation of a curve is such that $\frac{dy}{dx} = 3(4x - 7)^{\frac{1}{2}} - 4x^{-\frac{1}{2}}$. It is given that the curve passes through the point $(4, \frac{5}{2})$.

Find the equation of the curve.

[4]

[illegible]

- 4** The first, second and third terms of an arithmetic progression are k , $6k$ and $k + 6$ respectively.

(a) Find the value of the constant k .

[2]

This image shows a single sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

(b) Find the sum of the first 30 terms of the progression.

[3]

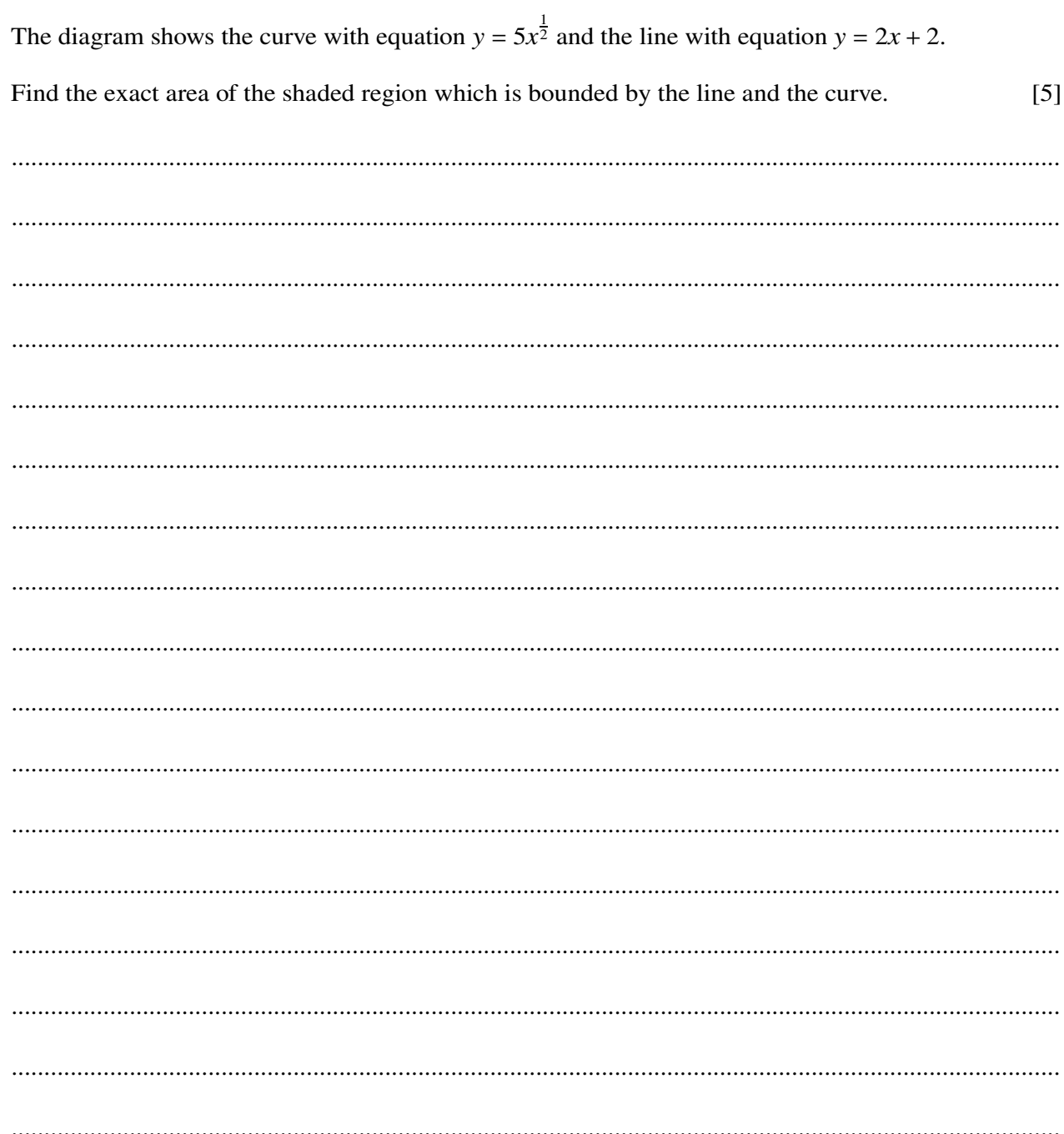
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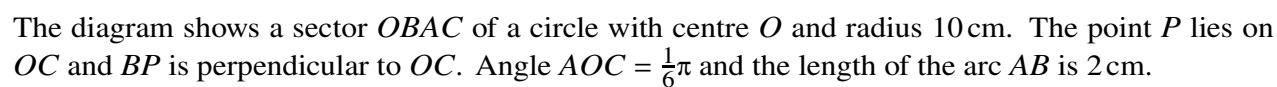
- (a) Given that the curve and the line intersect at the points with x -coordinates 0 and $\frac{3}{4}$, find the values of k and a . [4]

[illegible]

- (b)** Given instead that $a = -\frac{7}{2}$, find the values of k for which the line is a tangent to the curve. [5]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dashed lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.





- [illegible]

[illegible]

- 8** The equation of a circle is $x^2 + y^2 + ax + by - 12 = 0$. The points $A(1, 1)$ and $B(2, -6)$ lie on the circle.

- (a)** Find the values of a and b and hence find the coordinates of the centre of the circle. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b)** Find the equation of the tangent to the circle at the point A , giving your answer in the form $px + qy = k$, where p , q and k are integers. [4]

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- 9** The equation of a curve is $y = 3x + 1 - 4(3x + 1)^{\frac{1}{2}}$ for $x > -\frac{1}{3}$.

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. [3]

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- (b)** Find the coordinates of the stationary point of the curve and determine its nature. [4]

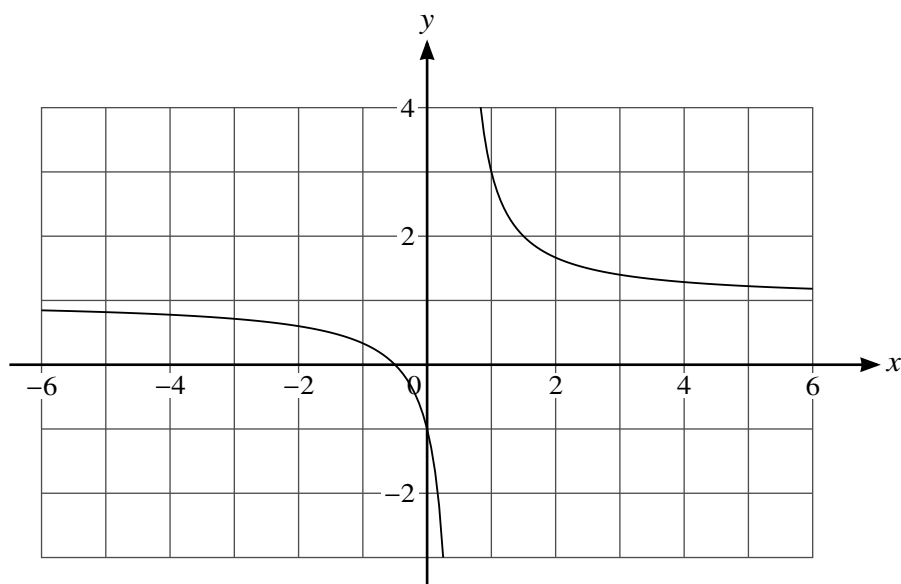
[illegible]

10 Functions f and g are defined as follows:

$$f(x) = \frac{2x+1}{2x-1} \quad \text{for } x \neq \frac{1}{2},$$

$$g(x) = x^2 + 4 \quad \text{for } x \in \mathbb{R}.$$

(a)



The diagram shows part of the graph of $y = f(x)$.

State the domain of f^{-1} .

[1]

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(b) Find an expression for $f^{-1}(x)$.

[3]

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(c) Find $gf^{-1}(3)$.

[2]

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- (d)** Explain why $g^{-1}(x)$ cannot be found. [1]

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- (e) Show that $1 + \frac{2}{2x-1}$ can be expressed as $\frac{2x+1}{2x-1}$. Hence find the area of the triangle enclosed by the tangent to the curve $y = f(x)$ at the point where $x = 1$ and the x - and y -axes. [6]

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- 11** The function f is given by $f(x) = 4\cos^4 x + \cos^2 x - k$ for $0 \leq x \leq 2\pi$, where k is a constant.

(a) Given that $k = 3$, find the exact solutions of the equation $f(x) = 0$.

[5]

[illegible]

- (b)** Use the quadratic formula to show that, when $k > 5$, the equation $f(x) = 0$ has no solutions. [5]

[illegible]

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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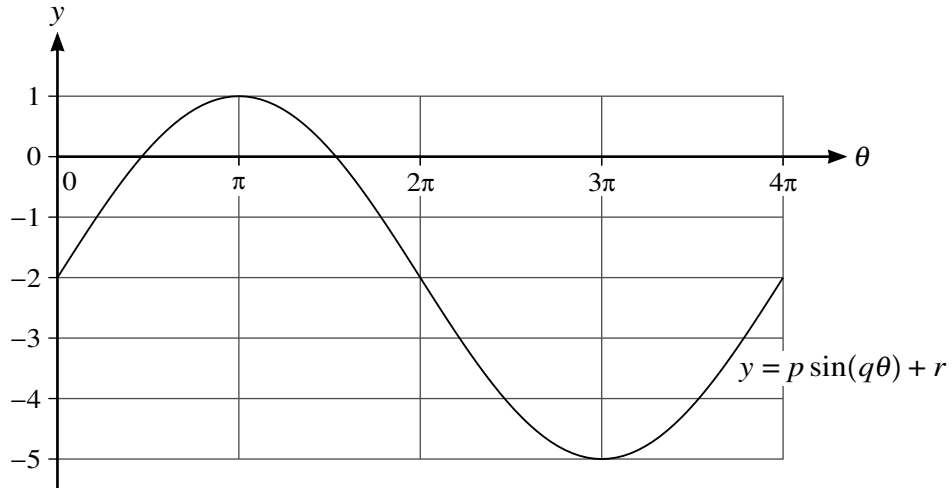
- 1** The coefficient of x^3 in the expansion of $\left(p + \frac{1}{p}x\right)^4$ is 144.

Find the possible values of the constant p .

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

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The diagram shows part of the curve with equation $y = p \sin(q\theta) + r$, where p , q and r are constants.

- (a) State the value of p . [1]

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- (b) State the value of q . [1]

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- (c) State the value of r . [1]

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- 3** An arithmetic progression has first term 4 and common difference d . The sum of the first n terms of the progression is 5863.

(a) Show that $(n - 1)d = \frac{11\,726}{n} - 8$. [1]

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- (b) Given that the n th term is 139, find the values of n and d , giving the value of d as a fraction. [4]

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- 4 (a) The curve with equation $y = x^2 + 2x - 5$ is translated by $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$.

Find the equation of the translated curve, giving your answer in the form $y = ax^2 + bx + c$. [3]

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- (b) The curve with equation $y = x^2 + 2x - 5$ is transformed to a curve with equation $y = 4x^2 + 4x - 5$.

Describe fully the single transformation that has been applied. [2]

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- 5 (a) Solve the equation $6\sqrt{y} + \frac{2}{\sqrt{y}} - 7 = 0$. [4]

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- (b) Hence solve the equation $6\sqrt{\tan x} + \frac{2}{\sqrt{\tan x}} - 7 = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]

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6 The function f is defined by $f(x) = 2x^2 - 16x + 23$ for $x < 3$.

(a) Express $f(x)$ in the form $2(x + a)^2 + b$. [2]

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(b) Find the range of f . [1]

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(c) Find an expression for $f^{-1}(x)$.

[3]

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The function g is defined by $g(x) = 2x + 4$ for $x < -1$.

(d) Find and simplify an expression for $fg(x)$.

[2]

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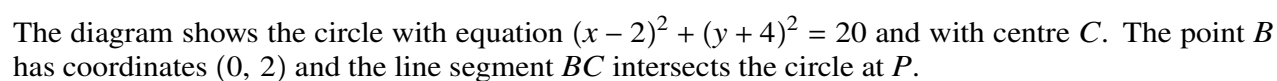
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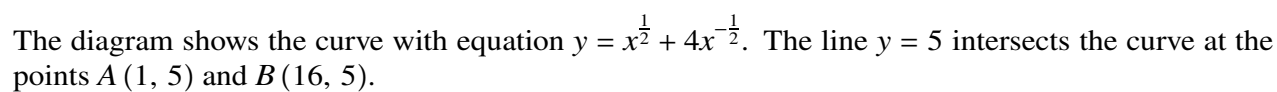
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- This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings present.

[5]

[illegible]

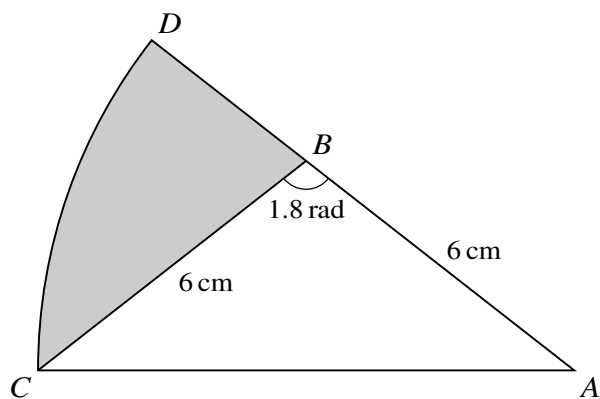


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- This image shows a full page of white paper with horizontal dashed lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Calculate the area of the shaded region.

[4]

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(a) Find the perimeter of the shaded region. [5]

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(b) Find the area of the shaded region.

[3]

[illegible]

- 10** The function f is defined by $f(x) = (4x + 2)^{-2}$ for $x > -\frac{1}{2}$.

(a) Find $\int_1^{\infty} f(x) \, dx$.

[4]

[illegible]

A point is moving along the curve $y = f(x)$ in such a way that, as it passes through the point A , its y -coordinate is **decreasing** at the rate of k units per second and its x -coordinate is **increasing** at the rate of k units per second.

(b) Find the coordinates of A .

[6]

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- 11** The point P lies on the line with equation $y = mx + c$, where m and c are positive constants. A curve has equation $y = -\frac{m}{x}$. There is a single point P on the curve such that the straight line is a tangent to the curve at P .

(a) Find the coordinates of P , giving the y -coordinate in terms of m . [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[4]

[illegible]

[illegible]

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