



Cambridge International AS & A Level

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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1** The equation of a curve is such that $\frac{dy}{dx} = \frac{3}{x^4} + 32x^3$. It is given that the curve passes through the point $(\frac{1}{2}, 4)$.

Find the equation of the curve.

[4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** The sum of the first 20 terms of an arithmetic progression is 405 and the sum of the first 40 terms is 1410.

Find the 60th term of the progression.

[5]

[illegible]

- 3 (a) Find the first three terms in the expansion of $(3 - 2x)^5$ in ascending powers of x . [3]

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- (b) Hence find the coefficient of x^2 in the expansion of $(4 + x)^2(3 - 2x)^5$. [3]

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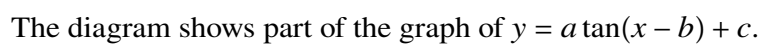
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[3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 5 The fifth, sixth and seventh terms of a geometric progression are $8k$, -12 and $2k$ respectively.

Given that k is negative, find the sum to infinity of the progression.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 6** The equation of a curve is $y = (2k - 3)x^2 - kx - (k - 2)$, where k is a constant. The line $y = 3x - 4$ is a tangent to the curve.

Find the value of k .

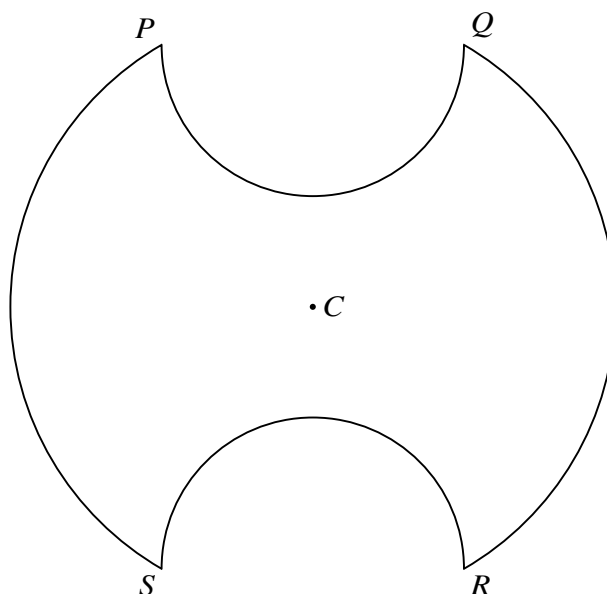
[5]

This image shows a full page of a worksheet designed for handwriting practice. It features 20 evenly spaced, horizontal dashed lines across the entire width of the page. The background is plain white, and there are no margins, text, or other markings present.

7 (a) Prove the identity $\frac{1 - 2 \sin^2 \theta}{1 - \sin^2 \theta} \equiv 1 - \tan^2 \theta$. [2]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

[illegible]



The diagram shows a symmetrical metal plate. The plate is made by removing two identical pieces from a circular disc with centre C . The boundary of the plate consists of two arcs PS and QR of the original circle and two semicircles with PQ and RS as diameters. The radius of the circle with centre C is 4 cm, and $PQ = RS = 4$ cm also.

- (a) Show that angle $PCS = \frac{2}{3}\pi$ radians. [2]

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- (b) Find the exact perimeter of the plate. [3]

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[5]

[illegible]

- 9 Functions f and g are defined as follows:

$$f(x) = (x - 2)^2 - 4 \text{ for } x \geq 2,$$

$$g(x) = ax + 2 \text{ for } x \in \mathbb{R},$$

where a is a constant.

- (a) State the range of f . [1]

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- (b) Find $f^{-1}(x)$. [2]

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- (c) Given that $a = -\frac{5}{3}$, solve the equation $f(x) = g(x)$. [3]

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[illegible]

10 The equation of a circle is $x^2 + y^2 - 4x + 6y - 77 = 0$.

- (a)** Find the x -coordinates of the points A and B where the circle intersects the x -axis. [2]

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- (b)** Find the point of intersection of the tangents to the circle at A and B . [6]

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11 The equation of a curve is $y = 2\sqrt{3x + 4} - x$.

- (a)** Find the equation of the normal to the curve at the point (4, 4), giving your answer in the form $y = mx + c$. [5]

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- (b)** Find the coordinates of the stationary point. [3]

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- (c) Determine the nature of the stationary point. [2]

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- (d) Find the exact area of the region bounded by the curve, the x -axis and the lines $x = 0$ and $x = 4$. [4]

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- 1 (a) Express $16x^2 - 24x + 10$ in the form $(4x + a)^2 + b$. [2]

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- (b) It is given that the equation $16x^2 - 24x + 10 = k$, where k is a constant, has exactly one root.

Find the value of this root. [2]

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- 2 (a) The graph of $y = f(x)$ is transformed to the graph of $y = 2f(x - 1)$.

Describe fully the two single transformations which have been combined to give the resulting transformation. [3]

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- (b) The curve $y = \sin 2x - 5x$ is reflected in the y -axis and then stretched by scale factor $\frac{1}{3}$ in the x -direction.

Write down the equation of the transformed curve. [2]

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- 3 The equation of a curve is $y = (x - 3)\sqrt{x + 1} + 3$. The following points lie on the curve. Non-exact values are rounded to 4 decimal places.

$A(2, k)$ $B(2.9, 2.8025)$ $C(2.99, 2.9800)$ $D(2.999, 2.9980)$ $E(3, 3)$

- (a) Find k , giving your answer correct to 4 decimal places. [1]

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- (b) Find the gradient of AE , giving your answer correct to 4 decimal places. [1]

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The gradients of BE , CE and DE , rounded to 4 decimal places, are 1.9748, 1.9975 and 1.9997 respectively.

- (c) State, giving a reason for your answer, what the values of the four gradients suggest about the gradient of the curve at the point E . [2]

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- 4** The coefficient of x in the expansion of $\left(4x + \frac{10}{x}\right)^3$ is p . The coefficient of $\frac{1}{x}$ in the expansion of $\left(2x + \frac{k}{x^2}\right)^5$ is q .

Given that $p = 6q$, find the possible values of k .

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

5 The function f is defined by $f(x) = 2x^2 + 3$ for $x \geq 0$.

(a) Find and simplify an expression for $ff(x)$. [2]

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(b) Solve the equation $ff(x) = 34x^2 + 19$. [4]

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[4]

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- 7 The point A has coordinates $(1, 5)$ and the line l has gradient $-\frac{2}{3}$ and passes through A . A circle has centre $(5, 11)$ and radius $\sqrt{52}$.

(a) Show that l is the tangent to the circle at A . [2]

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(b) Find the equation of the other circle of radius $\sqrt{52}$ for which l is also the tangent at A . [3]

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- 8 The first, second and third terms of an arithmetic progression are a , $\frac{3}{2}a$ and b respectively, where a and b are positive constants. The first, second and third terms of a geometric progression are a , 18 and $b + 3$ respectively.

(a) Find the values of a and b . [5]

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(b) Find the sum of the first 20 terms of the arithmetic progression. [3]

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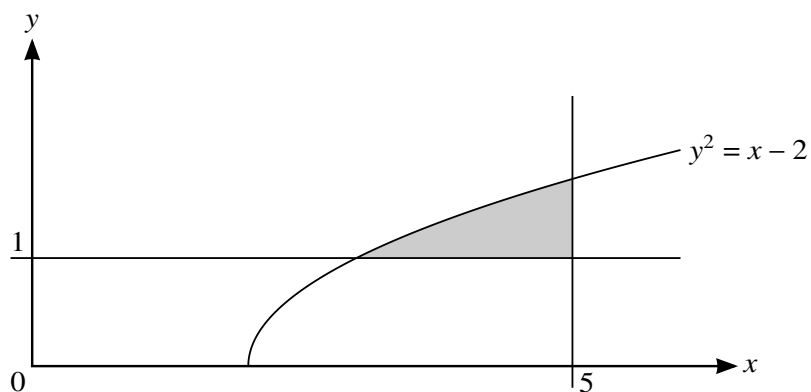
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The diagram shows part of the curve with equation $y^2 = x - 2$ and the lines $x = 5$ and $y = 1$. The shaded region enclosed by the curve and the lines is rotated through 360° about the x -axis.

Find the volume obtained.

[6]

[illegible]

10 (a) Prove the identity $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} \equiv \frac{4 \tan x}{\cos x}$. [4]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (b) Hence solve the equation $\frac{1 + \sin x}{1 - \sin x} - \frac{1 - \sin x}{1 + \sin x} = 8 \tan x$ for $0 \leq x \leq \frac{1}{2}\pi$. [3]

[illegible]

- 11** The gradient of a curve is given by $\frac{dy}{dx} = 6(3x - 5)^3 - kx^2$, where k is a constant. The curve has a stationary point at $(2, -3.5)$.

(a) Find the value of k . [2]

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(b) Find the equation of the curve. [4]

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- (c) Find $\frac{d^2y}{dx^2}$. [2]

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- (d) Determine the nature of the stationary point at $(2, -3.5)$. [2]

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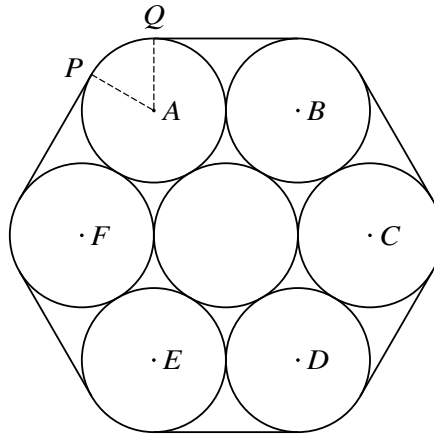
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The diagram shows a cross-section of seven cylindrical pipes, each of radius 20 cm, held together by a thin rope which is wrapped tightly around the pipes. The centres of the six outer pipes are A , B , C , D , E and F . Points P and Q are situated where straight sections of the rope meet the pipe with centre A .

- (a) Show that angle $PAQ = \frac{1}{3}\pi$ radians. [2]

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- (b) Find the length of the rope. [4]

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- (c) Find the area of the hexagon $ABCDEF$, giving your answer in terms of $\sqrt{3}$. [2]

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- (d) Find the area of the complete region enclosed by the rope. [3]

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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INFORMATION

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- 1** A curve with equation $y = f(x)$ is such that $f'(x) = 6x^2 - \frac{8}{x^2}$. It is given that the curve passes through the point $(2, 7)$.

Find $f(x)$. [3]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 2 The function f is defined by $f(x) = \frac{1}{3}(2x - 1)^{\frac{3}{2}} - 2x$ for $\frac{1}{2} < x < a$. It is given that f is a decreasing function.

Find the maximum possible value of the constant a . [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 3** A line with equation $y = mx - 6$ is a tangent to the curve with equation $y = x^2 - 4x + 3$.

Find the possible values of the constant m , and the corresponding coordinates of the points at which the line touches the curve. [6]

[illegible]

- 4 (a) Show that the equation

$$\frac{\tan x + \sin x}{\tan x - \sin x} = k,$$

where k is a constant, may be expressed as

$$\frac{1 + \cos x}{1 - \cos x} = k. \quad [2]$$

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- (b) Hence express $\cos x$ in terms of k . [2]

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- (c) Hence solve the equation $\frac{\tan x + \sin x}{\tan x - \sin x} = 4$ for $-\pi < x < \pi$. [2]

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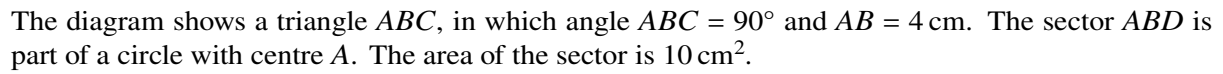
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- [illegible]

- 6 Functions f and g are both defined for $x \in \mathbb{R}$ and are given by

$$f(x) = x^2 - 2x + 5,$$

$$g(x) = x^2 + 4x + 13.$$

- (a) By first expressing each of $f(x)$ and $g(x)$ in completed square form, express $g(x)$ in the form $f(x + p) + q$, where p and q are constants. [4]

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- (b) Describe fully the transformation which transforms the graph of $y = f(x)$ to the graph of $y = g(x)$. [2]

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- 7 (a) Write down the first four terms of the expansion, in ascending powers of x , of $(a - x)^6$. [2]

[illegible]

- (b) Given that the coefficient of x^2 in the expansion of $\left(1 + \frac{2}{ax}\right)(a - x)^6$ is -20 , find in exact form the possible values of the constant a . [5]

[illegible]

(b) Find an expression for $(fg)^{-1}(x)$.

[3]

[illegible]

- 9 (a)** A geometric progression is such that the second term is equal to 24% of the sum to infinity.

Find the possible values of the common ratio.

[3]

This image shows a full page of a document template designed for handwriting practice or general writing. It consists of approximately 20 evenly spaced, horizontal dotted lines extending across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

- (b) An arithmetic progression P has first term a and common difference d . An arithmetic progression Q has first term $2(a + 1)$ and common difference $(d + 1)$. It is given that

$$\frac{\text{5th term of } P}{\text{12th term of } Q} = \frac{1}{3} \quad \text{and} \quad \frac{\text{Sum of first 5 terms of } P}{\text{Sum of first 5 terms of } Q} = \frac{2}{3}.$$

Find the value of a and the value of d .

[6]

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- 10** Points $A(-2, 3)$, $B(3, 0)$ and $C(6, 5)$ lie on the circumference of a circle with centre D .

(a) Show that angle $ABC = 90^\circ$. [2]

[illegible]

(b) Hence state the coordinates of D . [1]

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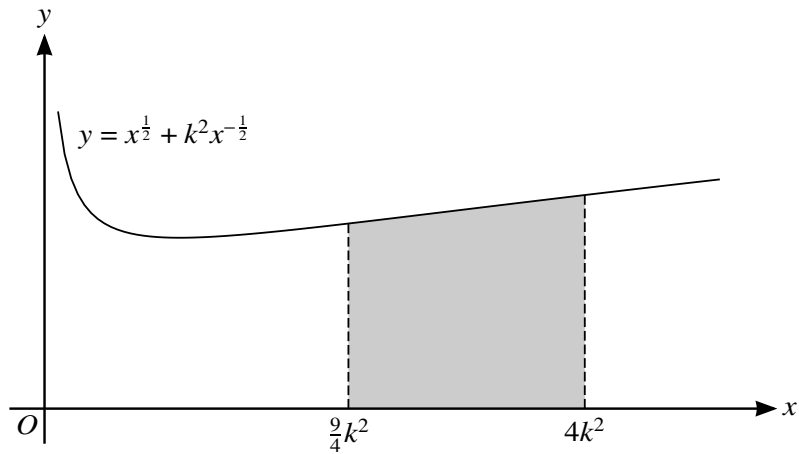
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(c) Find an equation of the circle. [2]

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[5]

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The diagram shows part of the curve with equation $y = x^{\frac{1}{2}} + k^2x^{-\frac{1}{2}}$, where k is a positive constant.

- (a) Find the coordinates of the minimum point of the curve, giving your answer in terms of k . [4]

[illegible]

The tangent at the point on the curve where $x = 4k^2$ intersects the y -axis at P .

- (b) Find the y -coordinate of P in terms of k . [4]

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The shaded region is bounded by the curve, the x -axis and the lines $x = \frac{9}{4}k^2$ and $x = 4k^2$.

- (c) Find the area of the shaded region in terms of k . [3]

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[illegible]

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