



Cambridge International AS & A Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1

May/June 2021

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **17** printed pages.

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
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- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
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- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
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ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$[y =] -\frac{1}{x^3} + 8x^4 [+ c]$	B1 B1	OE. Accept unsimplified.
	$4 = -8 + \frac{1}{2} + c$	M1	Substituting $\left(\frac{1}{2}, 4\right)$ into an integrated expression
	$y = -\frac{1}{x^3} + 8x^4 + \frac{23}{2}$	A1	OE. Accept $-x^{-3}$; must be 8; $y =$ must be seen in working.
		4	

Question	Answer	Marks	Guidance
2	$10(2a + 19d) = 405$	B1	
	$20(2a + 39d) = 1410$	B1	
	Solving simultaneously two equations obtained from using the correct sum formulae [$a = 6, d = 1.5$]	M1	Reach $a =$ or $d =$
	Using the correct formula for 60th term with their a and d	M1	
	60th term = 94.5	A1	OE, e.g. $\frac{189}{2}$
		5	

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Question	Answer	Marks	Guidance
3(a)	243	B1	
	$-810x$	B1	
	$+1080x^2$	B1	
		3	
3(b)	$(4 + x)^2 = 16 + 8x + x^2$	B1	
	Coefficient of x^2 is $16 \times 1080 + 8 \times (-810) + 243$	M1	Allow if at least 2 pairs used correctly
	11043	A1	Allow $11043x^2$
		3	

Question	Answer	Marks	Guidance
4	$a = 2$	B1	
	$b = \frac{\pi}{4}$	B1	or $\frac{2\pi}{8}$
	$c = 1$	B1	
		3	

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Question	Answer	Marks	Guidance
5	$(-12)^2 = 8k \times 2k$	M1	Forming an equation in k
	$k = -3$	A1	
	Using correct formula for S_{∞} [$r = 0.5$, $a = -384$]	M1	With $-1 < r < 1$
	$S_{\infty} = -768$	A1	
	Alternative method for Question 5		
	$r^2 = \frac{2k}{8k}$	M1	
	$r = [\pm]0.5$	A1	
	Using correct formula for S_{∞} [$r = 0.5$, $a = -384$]	M1	$-1 < r < 1$
	$S_{\infty} = -768$	A1	
		4	

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Question	Answer	Marks	Guidance
6	$(2k-3)x^2 - kx - (k-2) = 3x - 4$	*M1	Equating curve and line
	$(2k-3)x^2 - (k+3)x - (k-6) [= 0]$	DM1	Forming a 3-term quadratic
	$(k+3)^2 + 4(2k-3)(k-6) [= 0]$	DM1	Use of discriminant (dependent on both previous M marks)
	$9k^2 - 54k + 81 [= 0]$ [leading to $k^2 - 6k + 9 = 0$]	M1	Simplifying and solving <i>their</i> 3-term quadratic in k
	$k = 3$	A1	
	Alternative method for Question 6		
	$(2k-3)x^2 - kx - (k-2) = 3x - 4$	*M1	Equating curve and line
	$2(2k-3)x - k = 3 \Rightarrow x = \frac{k+3}{4k-6} \text{ or } k = \frac{3+6x}{4x-1}$	DM1	Differentiating and solving for x or k
	Either $(2k-3)\left(\frac{k+3}{4k-6}\right)^2 - k\left(\frac{k+3}{4k-6}\right) - (k-2) = 3\left(\frac{k+3}{4k-6}\right) - 4$ Or $4x\left(\frac{3x^2+3x-6}{2x^2-x-1}\right) - 6x - \left(\frac{3x^2+3x-6}{2x^2-x-1}\right) = 3$	DM1	Substituting <i>their</i> x into equation or <i>their</i> $k = \frac{3x^2+3x-6}{2x^2-x-1}$ or $k = \frac{3x+6}{2x+1}$ into derivative equation (dependent on both previous M marks)
	$9k^2 - 54k + 81 [= 0]$ [leading to $k^2 - 6k + 9 = 0$]	M1	Simplifying and solving <i>their</i> 3-term quadratic in k (or solving for x)
	$k = 3$	A1	
			SC If M0, B1 for differentiating, equating to 3 and solving for x or k
		5	

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Question	Answer	Marks	Guidance
7(a)	Reach $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}$ or $\frac{1 - \sin^2\theta}{1 - \sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$ or $\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} - 2\tan^2\theta$ or $\sec^2\theta - \frac{2\sin^2\theta}{\cos^2\theta}$ or $2 - \sec^2\theta$ or $\frac{\cos 2\theta}{\cos^2\theta}$	M1	May start with $1 - \tan^2\theta$
	$1 - \tan^2\theta$	A1	AG, must show sufficient stages
		2	
7(b)	$1 - \tan^2\theta = 2\tan^4\theta \Rightarrow 2\tan^4\theta + \tan^2\theta - 1 [= 0]$	M1	Forming a 3-term quadratic in $\tan^2\theta$ or e.g. u
	$\tan^2\theta = 0.5$ or -1 leading to $\tan\theta = [\pm]\sqrt{0.5}$	M1	
	$\theta = 35.3^\circ$ and 144.7° (AWRT)	A1	Both correct. Radians 0.615, 2.53 scores A0.
		3	

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Question	Answer	Marks	Guidance
8(a)	Either Let midpoint of PQ be H : $\sin HCP = \frac{2}{4} \Rightarrow \text{Angle } HCP = \frac{\pi}{6}$ Or $\sin PSQ = \frac{4}{8} \Rightarrow \text{Angle } PSQ = \frac{\pi}{6}$ Or using cosine rule: $\text{angle } PCQ = \frac{\pi}{3}$ Or by inspection: triangle PCQ or PCT is equilateral so $\text{angle } PCQ = \frac{\pi}{3}$	M1	
	$\text{Angle } PCS = \pi - \frac{\pi}{6} - \frac{\pi}{6} = \frac{2}{3}\pi$	A1	AG
		2	
8(b)	$\text{Perimeter} = 2 \times 4 \times \frac{2\pi}{3} \text{ or } 8\pi - \frac{8\pi}{3}$	M1	Length of two arcs PS and QR
	$+2\pi \times 2$	M1	Adding circumference of two semicircles
	$\frac{28\pi}{3}$	A1	Must be a single term
		3	

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Question	Answer	Marks	Guidance
8(c)	Area sector $CPQ = \frac{1}{2} \times 4^2 \times \frac{\pi}{3} = \frac{8\pi}{3}$	M1	Uses correct formula for sector
	Area of segment of large circle beyond CPQ $= \frac{8\pi}{3} - \frac{1}{2} \times 4^2 \times \sin\left(\frac{\pi}{3}\right) = \frac{8\pi}{3} - 4\sqrt{3}$	M1	Attempts to find area of segment
	Area of small semicircle = $\pi \times 2$ or area of small circle = $\pi \times 2^2$	M1	
	Area of plate = Large circle – [2 ×] small semicircle – [2 ×] segment area	M1	
	$\pi \times 4^2 - \pi \times 2^2 - 2 \times \left(\frac{8\pi}{3} - 4\sqrt{3}\right) = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
	Alternative method for Question 8(c)		
	Area of sector $PCS = \frac{1}{2} \times 4^2 \times \frac{2\pi}{3} = \frac{16\pi}{3}$	M1	Uses correct formula for sector
	Area of triangle $PCQ = \frac{1}{2} \times 4^2 \times \sin\frac{\pi}{3} = 4\sqrt{3}$	M1	Uses correct formula for triangle
	Area of small semicircle = $\pi \times 2$ or area of circle = $\pi \times 2^2$	M1	
	Area of plate = [2 ×] large sector + [2 ×] triangle – [2 ×] small semicircle	M1	
	$2\left(\frac{16\pi}{3}\right) + 2(4\sqrt{3}) - \pi \times 2^2 = \frac{20\pi}{3} + 8\sqrt{3}$	A1	AG
		5	

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Question	Answer	Marks	Guidance
9(a)	Range of f is $f(x) \geq -4$	B1	Allow y , f or 'range' or $[-4, \infty)$
		1	
9(b)	$y = (x - 2)^2 - 4 \Rightarrow (x - 2)^2 = y + 4 \Rightarrow x - 2 = +\sqrt{(y + 4)} \text{ or } \pm\sqrt{(y + 4)}$	M1	May swap variables here
	$[f^{-1}(x)] = \sqrt{(x + 4)} + 2$	A1	
		2	
9(c)	$(x - 2)^2 - 4 = -\frac{5}{3}x + 2 \Rightarrow x^2 - 4x + 4 - 4 = -\frac{5}{3}x + 2 [\Rightarrow x^2 - \frac{7}{3}x - 2 = 0]$	M1	Equating and simplifying to a 3-term quadratic
	$(3x + 2)(x - 3)[= 0] \text{ or } \frac{7 \pm \sqrt{7^2 - 4(3)(-6)}}{6} \text{ OE}$	M1	Solving quadratic
	$x = 3 \text{ only}$	A1	
		3	

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Question	Answer	Marks	Guidance
9(d)	$f^{-1}(12) = 6$	M1	Substitute 12 into <i>their</i> $f^{-1}(x)$ and evaluate
	$g(f^{-1}(12)) = 6a + 2$	M1	Substitute <i>their</i> '6' into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	M1	Substitute the result into $g(x)$ and = 62
	$6a^2 + 2a - 60 [= 0]$	M1	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	A1	
	Alternative method for Question 9(d)		
	$g(f^{-1}(x)) = a(\sqrt{x+4} + 2) + 2$ or $gg(x) = a(ax + 2) + 2$	M1	Substitute <i>their</i> $f^{-1}(x)$ or $g(x)$ into $g(x)$
	$g(g(f^{-1}(x))) = a(a(\sqrt{x+4} + 2) + 2) + 2$	M1	Substitute the result into $g(x)$
	$g(g(f^{-1}(12))) = a(6a + 2) + 2 = 62$	M1	Substitute 12 and = 62
	$6a^2 + 2a - 60 [= 0]$	M1	Forming and solving a 3-term quadratic
	$a = -\frac{10}{3}$ or 3	A1	
		5	

Question	Answer	Marks	Guidance
10(a)	When $y = 0$ $x^2 - 4x - 77 = 0$ [$\Rightarrow (x+7)(x-11) = 0$ or $(x-2)^2 = 81$]	M1	Substituting $y = 0$
	So x -coordinates are -7 and 11	A1	
		2	

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Question	Answer	Marks	Guidance
10(b)	Centre of circle C is $(2, -3)$	B1	
	Gradient of AC is $-\frac{1}{3}$ or Gradient of BC is $\frac{1}{3}$	M1	For either gradient (M1 sign error, M0 if x -coordinate(s) in numerator)
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	M1	For either perpendicular gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	A1	For either equation
	Meet when $3x + 21 = -3x + 33$	M1	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	A1	
	Alternative method for Question 10(b)		
	Implicit differentiation: $2y \frac{dy}{dx}$ seen	B1	
	$2x - 4 + 2y \frac{dy}{dx} + 6 \frac{dy}{dx} = 0$	M1	Fully differentiated = 0 with at least one term involving y differentiated correctly
	Gradient of tangent at A is 3 or Gradient of tangent at B is -3	M1	For either gradient
	Equations of tangents are $y = 3x + 21$, $y = -3x + 33$	A1	For either equation
	Meet when $3x + 21 = -3x + 33$	M1	OR: (centre of circle has x coordinate 2) so x coordinate of point of intersection is 2
	Coordinates of point of intersection $(2, 27)$	A1	
		6	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = 3(3x+4)^{-0.5} - 1$	B1 B1	B1 All correct with 1 error, B2 if all correct
	Gradient of tangent = $-\frac{1}{4}$ and Gradient of normal = 4	*M1	Substituting $x = 4$ into a differentiated expression and using $m_1 m_2 = -1$
	Equation of line is $(y - 4) = 4(x - 4)$ or evaluate c	DM1	With (4, 4) and <i>their</i> gradient of normal
	So $y = 4x - 12$	A1	
		5	
11(b)	$3(3x+4)^{-0.5} - 1 = 0$	M1	Setting <i>their</i> $\frac{dy}{dx} = 0$
	Solving as far as $x =$	M1	Where $\frac{dy}{dx}$ contains $a(bx+c)^{-0.5}$ a, b, c any values
	$x = \frac{5}{3}, \quad y = 2\left(3 \times \frac{5}{3} + 4\right)^{0.5} - \frac{5}{3} = \frac{13}{3}$	A1	
		3	
11(c)	$\frac{d^2y}{dx^2} = -\frac{9}{2}(3x+4)^{-1.5}$	M1	Differentiating <i>their</i> $\frac{dy}{dx}$ OR checking $\frac{dy}{dx}$ to find +ve and -ve either side of their $x = \frac{5}{3}$
	At $x = \frac{5}{3}$ $\frac{d^2y}{dx^2}$ is negative so the point is a maximum	A1	
		2	

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Question	Answer	Marks	Guidance
11(d)	$\text{Area} = \left[\int 2(3x+4)^{0.5} - x \, dx = \right] \frac{4}{9}(3x+4)^{1.5} - \frac{1}{2}x^2$	B1 B1	B1 for each correct term (unsimplified)
	$\left(\frac{4}{9}(16)^{1.5} - \frac{1}{2}(4)^2 \right) - \frac{4}{9}(4)^{1.5} = \frac{256}{9} - 8 - \frac{32}{9}$	M1	Substituting limits 0 and 4 into an expression obtained by integrating y
	$16\frac{8}{9}$	A1	Or $\frac{152}{9}$
		4	



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1(a)	$(4x - 3)^2$ or $(4x + (-3))^2$ or $a = -3$	B1	$k(4x - 3)^2$ where $k \neq 1$ scores B0 but mark final answer, allow recovery.
	$+ 1$ or $b = 1$	B1	
		2	
1(b)	[For one root] $k = 1$ or ‘ <i>their b</i> ’	B1 FT	Either by inspection or solving or from $24^2 - 4 \times 16 \times (10 - k) = 0$ WWW
	[Root or $x =$] $\frac{3}{4}$ or 0.75	B1	SC B2 for correct final answer WWW.
		2	

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Question	Answer	Marks	Guidance
2(a)	Translation $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$	B1	Allow shift and allow by 1 in x -direction or [parallel to/on/in/along/against] the x -axis or horizontally. 'Translation by 1 to the right' only, scores B0
	Stretch	B1	Stretch. SC B2 for amplitude doubled.
	Factor 2 in y -direction	B1	With/by factor 2 in y -direction or [parallel to/on/in/along/against] the y -axis or vertically or with x axis invariant 'With/by factor 2 upwards' only, scores B0. Accept SF as an abbreviation for scale factor.
		3	Note: Transformations can be in either order
2(b)	$[-\sin 6x][+15x]$ or $[\sin(-6x)][+15x]$ OE	B1 B1	Accept an unsimplified version. ISW. B1 for each correct component – square brackets indicate each required component.
			If B0, SC B1 for either $\sin(-2x) + 5x$ or $-\sin(2x) + 5x$ or $\sin 6x - 15x$ or $\sin\left(-\frac{2}{3}x\right) + \frac{5}{3}x$
		2	

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Question	Answer	Marks	Guidance
3(a)	1.2679	B1	AWRT. ISW if correct answer seen. $3 - \sqrt{3}$ scores B0
		1	
3(b)	1.7321	B1	AWRT. ISW if correct answer seen.
		1	
3(c)	Sight of 2 or 2.0000 or two in reference to the gradient	*B1	
	This is because the gradient at E is the limit of the gradients of the chords as the x -value tends to 3 or Δx tends to 0.	DB1	Allow it gets nearer/approaches/tends/almost/approximately 2
		2	

Question	Answer	Marks	Guidance
4	[Coefficient of x or p =] 480	B1	SOI. Allow $480x$ even in an expansion.
	$\left[\text{Term in } \frac{1}{x} \text{ or } q = \right] [10 \times] (2x)^3 \left(\frac{k}{x^2} \right)^2$	M1	Appropriate term identified and selected.
	$[10 \times 2^3 k^2 =] 80k^2$	A1	Allow $\frac{80k^2}{x}$
	$p = 6q$ used ($480 = 6 \times 80k^2$ or $80 = 80k^2$)	M1	Correct link used for <i>their</i> coefficient of x and $\frac{1}{x}$ (p and q) with no x 's.
	$[k^2 = 1 \Rightarrow] k = \pm 1$	A1	A0 if a range of values given. Do not allow $\pm\sqrt{1}$.
		5	

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Question	Answer	Marks	Guidance
5(a)	$ff(x) = 2(2x^2 + 3)^2 + 3$	M1	Condone = 0.
	$8x^4 + 24x^2 + 21$	A1	ISW if correct answer seen. Condone = 0.
		2	
5(b)	$8x^4 + 24x^2 + 21 = 34x^2 + 19 \Rightarrow 8x^4 + 24x^2 - 34x^2 + 21 - 19 [= 0]$	M1	Equating $34x^3 + 19$ to <i>their</i> 3-term $ff(x)$ and collect all terms on one side condone $\pm$ sign errors.
	$8x^4 - 10x^2 + 2 [= 0]$	A1	
	$[2](x^2 - 1)(4x^2 - 1)$	M1	Attempt to solve 3-term quartic or 3-term quadratic by factorisation, formula or completing the square or factor theorem.
	$\left[x^2 = 1 \text{ or } \frac{1}{4} \text{ leading to } \right] x = 1 \text{ or } x = \frac{1}{2}$	A1	If factorising, factors must expand to give $8x^4$ or $4x^4$ 4 or <i>their</i> ax^4 otherwise M0A0 due to calculator use. Condone ± 1 , $\pm \frac{1}{2}$ but not $\sqrt{\frac{1}{4}}$ or $\sqrt{1}$.
		4	

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Question	Answer	Marks	Guidance
6	Gradient $AB = \frac{1}{2}$	B1	SOI
	Lines meet when $-2x + 4 = \frac{1}{2}(x - 8) + 3$ Solving as far as $x =$	*M1	Equating given perpendicular bisector with the line through (8, 3) using <i>their</i> gradient of AB (but not -2) and solving. Expect $x = 2, y = 0$.
	Using mid-point to get as far as $p =$ or $q =$	DM1	Expect $\frac{8+p}{2} = 2$ or $\frac{3+q}{2} = 0$
	$p = -4, q = -3$	A1	Allow coordinates of B are $(-4, -3)$.
	Alternative method for Question 6		
	Gradient $AB = \frac{1}{2}$	B1	SOI
	$\frac{q-3}{p-8} = \frac{1}{2}$ [leading to $2q = p - 2$], $\frac{q+3}{2} = -2\left(\frac{8+p}{2}\right) + 4$ [leading to $q = -11 - 2p$]	*M1	Equating gradient of AB with <i>their</i> gradient of AB (but not -2) and using mid-point in equation of perpendicular bisector.
	Solving simultaneously <i>their</i> 2 linear equations	DM1	Equating and solving 2 correct equations as far as $p =$ or $q =$.
	$p = -4, q = -3$	A1	Allow coordinates of B are $(-4, -3)$.

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Question	Answer	Marks	Guidance
6	Alternative method for Question 6		
	Gradient $AB = \frac{1}{2}$	B1	
	$\frac{q-3}{p-8} = \frac{1}{2}$ [leading to $p = 2q + 2$], $y - \frac{q+3}{2} = -2(x - (q+5))$ [leading to $y = -2x + \frac{5q+23}{2}$]	*M1	Equating gradient of AB with <i>their</i> gradient of AB (but not -2) and using mid-point in equation of perpendicular bisector.
	<i>their</i> $\frac{5q+23}{2} = 4 \Rightarrow q =$	DM1	Equating and solving as far as q or $p =$
	$p = -4, q = -3$	A1	Allow coordinates of B are $(-4, -3)$.
		4	

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Question	Answer	Marks	Guidance
7(a)	$(5 - 1)^2 + (11 - 5)^2 = 52$ or $\frac{11-5}{5-1}$	M1	For substituting (1,5) into circle equation or showing gradient = $\frac{3}{2}$.
	For both circle equation and gradient, and proving line is perpendicular and stating that A lies on the circle	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$(x - 5)^2 + (y - 11)^2 = 52$ and $y - 5 = -\frac{2}{3}(x - 1)$	M1	Both equations seen and attempt to solve. May see $y = -\frac{2}{3}x + \frac{17}{3}$
	Solving simultaneously to obtain $(y - 5)^2 = 0$ or $(x - 1)^2 = 0 \Rightarrow 1$ root or tangent or discriminant = 0 $\Rightarrow 1$ root or tangent	A1	Clear reasoning.
	Alternative method for Question 7(a)		
	$\frac{dy}{dx} = \frac{10 - 2x}{2y - 22} = \frac{10 - 2}{10 - 22}$	M1	Attempting implicit differentiation of circle equation and substitute $x = 1$ and $y = 5$.
	Showing gradient of circle at A is $-\frac{2}{3}$	A1	Clear reasoning.
7(b)	Centre is $(-3, -1)$	B1 B1	B1 for each correct co-ordinate.
	Equation is $(x + 3)^2 + (y + 1)^2 = 52$	B1 FT	FT <i>their</i> centre, but not if either (1, 5) or (5, 11). Do not accept $\sqrt{52^2}$.
		3	

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Question	Answer	Marks	Guidance
8(a)	$\left(a + b = 2 \times \frac{3}{2}a\right) \Rightarrow b = 2a$	B1	SOI
	$18^2 = a(b + 3)$ OE or 2 correct statements about r from the GP, e.g. $r = \frac{18}{a}$ and $b + 3 = 18r$ or $r^2 = \frac{b + 3}{a}$	B1	SOI
	$324 = a(2a + 3) \Rightarrow 2a^2 + 3a - 324 [= 0]$ or $b^2 + 3b - 648 [= 0]$ or $6r^2 - r - 12 [= 0]$ or $4d^2 + 3d - 162 [= 0]$	M1	Using the correct connection between AP and GP to form a 3-term quadratic with all terms on one side.
	$(a - 12)(2a + 27) [= 0]$ or $(b - 24)(b + 27) [= 0]$ or $(2r - 3)(3r + 4) [= 0]$ or $(d - 6)(4d + 27) [= 0]$	M1	Solving <i>their</i> 3-term quadratic by factorisation, formula or completing the square to obtain answers for a , b , r or d .
	$a = 12, b = 24$	A1	WWW. Condone extra ‘solution’ $a = -13.5, b = -27$ only.
		5	

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Question	Answer	Marks	Guidance
8(b)	Common difference $d = 6$	B1 FT	SOI. FT <i>their</i> $\frac{a}{2}$
	$S_{20} = \frac{20}{2}(2 \times 12 + 19 \times 6)$	M1	Using correct sum formula with <i>their a</i> , <i>their</i> calculated d and 20.
	1380	A1	
		3	

Question	Answer	Marks	Guidance
9	Curve intersects $y = 1$ at (3, 1)	B1	Throughout Question 9: $1 < \textit{their} 3 < 5$ Sight of $x = 3$
	Volume $= [\pi] \int (x - 2) [dx]$	M1	M1 for showing the intention to integrate $(x - 2)$. Condone missing π or using 2π .
	$[\pi] \left[\frac{1}{2}x^2 - 2x \right]$ or $[\pi] \left[\frac{1}{2}(x - 2)^2 \right]$	A1	Correct integral. Condone missing π or using 2π .
	$= [\pi] \left[\left(\frac{5^2}{2} - 2 \times 5 \right) - \left(\frac{\textit{their} 3^2}{2} - 2 \times \textit{their} 3 \right) \right]$ $= [\pi] \left[\frac{5}{2} + \frac{3}{2} \right]$ as a minimum requirement for <i>their</i> values	M1	Correct use of ' <i>their</i> 3' and 5 in an integrated expression. Condone missing π or using 2π . Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	Volume of cylinder $= \pi \times 1^2 \times (5 - \textit{their} 3) [= 2\pi]$	B1 FT	Or by integrating 1 to obtain x (condone y if 5 and <i>their</i> 3 used).
	[Volume of solid $= 4\pi - 2\pi = 2\pi$ or 6.28]	A1	AWRT

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Question	Answer	Marks	Guidance
9	Alternative method for Question 9		
	Curve intersects $y = 1$ at (3, 1)	B1	Sight of $x = 3$
	Volume of solid = $\pi \int (x-2) - 1 [dx]$	M1 B1	M1 for showing the intention to integrate $(x-2)$ B1 for correct integration of -1 . Condone missing π or 2π for M1 but not for B1.
	$[\pi] \left[\frac{1}{2}x^2 - 3x \right]$ or $[\pi] \left[\frac{1}{2}(x-3)^2 \right]$	A1	Correct integral, allow as two integrals. Condone missing π or using 2π .
	$= [\pi] \left[\left(\frac{5^2}{2} - 3 \times 5 \right) - \left(\frac{\text{their } 3^2}{2} - 3 \times \text{their } 3 \right) \right]$	M1	Correct use of ‘ <i>their</i> 3’ and 5 in an integrated expression. Condone missing π or using 2π . Condone +c. Can be obtained by integrating and substituting between 5 and 2 and then 3 and 2 then subtracting.
	[Volume of solid = $4\pi - 2\pi = 2\pi$ or 6.28]	A1	AWRT
		6	

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Question	Answer	Marks	Guidance
10(a)	$\frac{1+\sin x}{1-\sin x} - \frac{1-\sin x}{1+\sin x} \equiv \frac{(1+\sin x)^2 - (1-\sin x)^2}{(1-\sin x)(1+\sin x)}$	*M1	For using a common denominator of $(1-\sin x)(1+\sin x)$ and reasonable attempt at the numerator(s).
	$\equiv \frac{1+2\sin x + \sin^2 x - (1-2\sin x + \sin^2 x)}{(1-\sin x)(1+\sin x)}$	DM1	For multiplying out the numerators correctly. Condone sign errors for this mark.
	$\equiv \frac{4\sin x}{1-\sin^2 x} \equiv \frac{4\sin x}{\cos^2 x}$	DM1	For simplifying denominator to $\cos^2 x$.
	$\equiv \frac{4\sin x}{\cos x \cos x} \equiv \frac{4\tan x}{\cos x}$	A1	AG. Do not award A1 if undefined notation such as s, c, t or missing x's used throughout or brackets are missing.
	Alternative method for Question 10(a)		
	$\frac{4\tan x}{\cos x} \equiv \frac{4\sin x}{\cos^2 x} \equiv \frac{4\sin x}{1-\sin^2 x}$	*M1	Using $\tan x = \frac{\sin x}{\cos x}$ and $\cos^2 x = 1 - \sin^2 x$
	$\equiv \frac{-2}{1+\sin x} + \frac{2}{1-\sin x}$	DM1	Separating into partial fractions.
	$\equiv 1 + \frac{-2}{1+\sin x} + \frac{2}{1-\sin x} - 1$	DM1	Use of 1-1 or similar
	$\equiv -\frac{1-\sin x}{1+\sin x} + \frac{1+\sin x}{1-\sin x}$	A1	
		4	

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Question	Answer	Marks	Guidance
10(b)	$\cos x = \frac{1}{2}$	*B1	OE. WWW.
	$x = \frac{\pi}{3}$	DB1	Or AWRT 1.05
	$x = 0$ from $\tan x = 0$ or $\sin x = 0$	B1	WWW. Condone extra solutions outside the domain 0 to $\frac{\pi}{2}$ but B0 if any inside.
		3	

Question	Answer	Marks	Guidance
11(a)	At stationary point $\frac{dy}{dx} = 0$ so $6(3 \times 2 - 5)^3 - k \times 2^2 = 0$	M1	Setting given $\frac{dy}{dx} = 0$ and substituting $x = 2$ into it.
	$[k =] \frac{3}{2}$	A1	OE
		2	
11(b)	$[y =] \frac{6}{4 \times 3} (3x - 5)^4 - \frac{1}{3} k x^3 [+c].$	*M1 A1FT	Integrating (increase of power by 1 in at least one term) given $\frac{dy}{dx}$. Expect $\frac{1}{2} (3x - 5)^4 - \frac{1}{2} x^3$. FT <i>their</i> non zero k .
	$-\frac{7}{2} = \frac{1}{2} (3 \times 2 - 5)^4 - \frac{1}{3} \times \frac{3}{2} \times 2^3 + c$ [leading to $-3.5 + c = -3.5$]	DM1	Using (2,-3.5) in an integrated expression. + c needed. Substitution needs to be seen, simply stating $c = 0$ is DM0.
	$y = \frac{1}{2} (3x - 5)^4 - \frac{1}{2} x^3$	A1	$y =$ or $f(x) =$ must be seen somewhere in solution.

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Question	Answer	Marks	Guidance
11(b)	Alternative method for Question 11(b)		
	$[y =] \frac{81}{2}x^4 - \frac{541}{2}x^3 + 675x^2 - 750x(+c) \text{ or } -270x^3 - k\frac{x^3}{3}$	*M1 A1 FT	From $\frac{dy}{dx} = 162x^3 - 810x^2 - kx^2 - 1350x - 750$. FT <i>their k</i>
	$-\frac{7}{2} = \frac{81}{2} \times 2^4 - \frac{541}{2} \times 2^3 + 675 \times 2^2 - 750 \times 2 + c$	DM1	Using (2, -3.5) in an integrated expression. + c needed
	$y = \frac{81}{2}x^4 - \frac{541}{2}x^3 + 675x^2 - 750x + \frac{625}{2}$	A1	y = or f(x)= must be seen somewhere in solution.
		4	
11(c)	$[3 \times] [18(3x - 5)^2] [-2kx]$	B2,1,0 FT	FT <i>their k</i> . Square brackets indicate each required component. B2 for fully correct, B1 for one error or one missing component, B0 for 2 or more errors.
	Alternative method for Question 11(c)		
	$486x^2 - 1623x + 1350 \text{ or } -1620x - 2kx$	B2,1,0 FT	FT <i>their k</i> . B2 for fully correct, B1 for one error, B0 for 2 or more errors.
		2	
11(d)	$[\text{At } x = 2] \left[\frac{d^2y}{dx^2} = \right] 54(3 \times 2 - 5)^2 - 4k \text{ or } 48$	M1	OE. Substituting $x = 2$ into <i>their</i> second differential or other valid method.
	$[> 0]$ Minimum	A1	WWW
		2	

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Question	Answer	Marks	Guidance
12(a)	[By symmetry] $[6 \times \hat{P}AQ = 2\pi]$, $[\hat{P}AQ =] 2\pi \div 6$,	M1	
	Explaining that there are six sectors around the diagram that make up a complete circle.	A1	AG
	Alternative method for Question 12(a)		
	Using area or circumference of circle centre $A \div 6$	M1	$\frac{400\pi}{6}$ or $\frac{40\pi}{6}$
	Justification for dividing by 6 followed by comparison with the sector area or arc length.	A1	AG
	Alternative method for Question 12(a)		
	Explain why $\triangle PAQ$ is an equilateral triangle	M1	Assumption of this scores M0
	Using $\triangle PAQ$ is an equilateral triangle $\therefore \hat{P}AQ = \frac{\pi}{3}$	A1	AG
	Alternative method for Question 12(a)		
	Using the internal angle of a regular hexagon $= \frac{2\pi}{3}$ Or $\hat{FAO} + \hat{OAB} = \frac{2\pi}{3}$, equilateral triangles	M1	
	$\hat{P}AQ = 2\pi - \left(\frac{\pi}{2} + \frac{2\pi}{3} + \frac{\pi}{2} \right) = \frac{\pi}{3}$	A1	AG

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Question	Answer	Marks	Guidance
12(a)	Alternative method for Question 12(a)		
	$\sin\theta = \frac{20}{40}$, with θ clearly identified	M1	
	$\theta = \frac{\pi}{6}, 2\theta = \frac{\pi}{3} = \hat{FAO}$ and by similar triangles = $\hat{PAQ}$	A1	AG
		2	
12(b)	Each straight section of rope has length 40 cm	B1	SOI
	Each curved section round each pipe has length $r\theta = 20 \times \frac{\pi}{3}$	*M1	Use of $r\theta$ with $r = 20$ and θ in radians
	Total length = $6 \times ((\text{their } 40) + k\pi)$	DM1	$6 \times (\text{their straight section} + \text{their curved section})$. <i>Their</i> curved section must be from acceptable use of $r\theta$ – this could now be numeric.
	$240 + 40\pi$ or 366 (AWRT) (cm)	A1	Or directly: $(6 \times \text{diameter}) + \text{circumference}$
		4	

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Question	Answer	Marks	Guidance
12(c)	[Triangle area =] $\frac{1}{2} \times 40 \times 40 \times \sin\left(\frac{\pi}{3}\right)$ or $\frac{1}{2} \times 40 \times 20\sqrt{3}$ or $400\sqrt{3}$ or 693(AWRT)	B1	
	[Total area of hexagon = $6 \times 400\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	[Trapezium area =] $\frac{1}{2} \times (40 + 80) \times 40 \sin\left(\frac{\pi}{3}\right)$ or $1200\sqrt{3}$ or 2080 (AWRT)	B1	
	[Total area of hexagon = $2 \times 1200\sqrt{3}$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
	Alternative method for Question 12(c)		
	Area of triangle $ABC = 400\sqrt{3}$ or 693 (AWRT) or $4 \times$ Area of half of triangle $ABC = 4 \times 200\sqrt{3}$ or 1390 (AWRT) or Area of rectangle $ABDE = 1600\sqrt{3}$ or 2770 (AWRT)	B1	
	[Total area of hexagon = $2 \times 400\sqrt{3} + 1600\sqrt{3}$ =] $2400\sqrt{3}$ Or [= $4 \times 200\sqrt{3} + 1600$ =] $2400\sqrt{3}$	B1	Condone $4800\frac{\sqrt{3}}{2}$
			If B0B0, SC B1 can be scored for sight of 4160 (AWRT) as final answer.
		2	

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Question	Answer	Marks	Guidance
12(d)	Each rectangle area = 40×20 (= 800)	B1	SOI, e.g. by sight of 4800
	Each sector area = $\frac{1}{2}r^2\theta = \frac{1}{2} \times 20^2 \times \frac{\pi}{3} \left[= \frac{200\pi}{3} \right]$	B1	SOI.
	Total area = $2400\sqrt{3} + 4800 + 400\pi$ or 10 200 (cm ²) (AWRT)	B1	Or directly: part (c) + 6800 + area circle radius 20.
		3	



Cambridge International AS & A Level

MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2021

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

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This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	$[f(x) =] 2x^3 + \frac{8}{x} [+c]$	B1	Allow any correct form
	$7 = 16 + 4 + c$	M1	Substitute $f(2) = 7$ into an integral. c must be present. Expect $c = -13$
	$f(x) = 2x^3 + \frac{8}{x} - 13$	A1	Allow $y =$, $f(x)$ or y can appear earlier in answer
		3	

Question	Answer	Marks	Guidance
2	$[f^{-1}(x) =] \left((2x-1)^{1/2} \right) \times \left(\frac{1}{3} \times 2 \times \frac{3}{2} \right) (-2)$	B2, 1, 0	Expect $(2x-1)^{1/2} - 2$
	$(2x-1)^{1/2} - 2 \leq 0 \rightarrow 2x-1 \leq 4 \text{ or } 2x-1 < 4$	M1	SOI. Rearranging and then squaring, must have power of $\frac{1}{2}$ not present Allow ‘=0’ at this stage but do not allow ‘ ≥ 0 ’ or ‘ > 0 ’ If ‘-2’ missed then must see $\leq$ or $<$ for the M1
	Value [of a] is $2\frac{1}{2}$ or $a = 2\frac{1}{2}$	A1	WWW, OE e.g. $\frac{5}{2}$, 2.5 Do not allow from ‘=0’ unless some reference to negative gradient.
		4	

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Question	Answer	Marks	Guidance
3	$x^2 - 4x + 3 = mx - 6$ leading to $x^2 - x(4 + m) + 9$	*M1	Equating and gathering terms. May be implied on the next line.
	$b^2 - 4ac$ leading to $(4 + m)^2 - 4 \times 9$	DM1	SOI. Use of the discriminant with <i>their</i> a , b and c
	$4 + m = \pm 6$ or $(m - 2)(m + 10) = 0$ leading to $m = 2$ or -10	A1	Must come from $b^2 - 4ac = 0$ SOI
	Substitute both <i>their</i> m values into <i>their</i> equation in line 1	DM1	
	$m = 2$ leading to $x = 3$; $m = -10$ leading to $x = -3$	A1	
	$(3, 0), (-3, 24)$	A1	Accept 'when $x = 3, y = 0$; when $x = -3, y = 24$ ' If final A0A0 scored, SC B1 for one point correct WWW
	Alternative method for Question 3		
	$\frac{dy}{dx} = 2x - 4 \rightarrow 2x - 4 = m$	*M1	
	$x^2 - 4x + 3 = (2x - 4)x - 6$	DM1	
	$x^2 - 4x + 3 = 2x^2 - 4x - 6 \rightarrow 9 = x^2 \rightarrow x = \pm 3$	A1	
	$y = 0, 24$ or $(3, 0), (-3, 24)$	A1	
	Substitute both <i>their</i> x values into <i>their</i> equation in line 1	DM1	Or substitute both <i>their</i> (x, y) into $y = mx - 6$
	When $x = 3, m = 2$; when $x = -3, m = -10$	A1	If A0, DM1, A0 scored, SC B1 for one point correct WWW
		6	

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Question	Answer	Marks	Guidance
4(a)	$\frac{\tan x + \sin x}{\tan x - \sin x} [=k] \text{ leading to } \frac{\sin x + \sin x \cos x}{\sin x - \sin x \cos x} [=k]$ $\text{or } \frac{\frac{1}{\cos x} + 1}{\frac{1}{\cos x} - 1} [=k] \text{ or } \frac{\tan x + \tan x \cos x}{\tan x - \tan x \cos x} [=k]$	M1	Multiply numerator and denominator by $\cos x$, or divide numerator and denominator by $\tan x$ or $\sin x$
	$\frac{\sin x(1 + \cos x)}{\sin x(1 - \cos x)} \text{ or } \frac{\frac{1}{\cos x} + 1}{\frac{1}{\cos x} - 1} \cdot \frac{\cos x}{\cos x} \text{ or } \frac{\tan x(1 + \cos x)}{\tan x(1 - \cos x)} \text{ leading to } \frac{1 + \cos x}{1 - \cos x} [=k]$	A1	AG, WWW
		2	
4(b)	$k - k \cos x = 1 + \cos x \text{ leading to } k - 1 = k \cos x + \cos x$	M1	Gather like terms on LHS and RHS
	$k - 1 = (k + 1) \cos x \text{ leading to } \cos x = \frac{k - 1}{k + 1}$	A1	WWW, OE
		2	
4(c)	Obtaining $\cos x$ from <i>their</i> (b) or (a)	M1	Expect $\cos x = \frac{3}{5}$
	± 0.927 (only solutions in the given range)	A1	AWRT. Accept $\pm 0.295\pi$
		2	

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Question	Answer	Marks	Guidance
5(a)	$\frac{1}{2} \times 4^2 \times \text{angle BAD} = 10$	M1	Use of sector area formula
	Angle BAD = 1.25	A1	OE. Accept 0.398π , 71.6° for SC B1 only
		2	
5(b)	Arc $BD = 4 \times \text{their } 1.25$	M1	Use of arc length formula. Expect 5.
	$BC = 4 \tan(\text{their } 1.25)$	M1	Expect 12.0(4). May use $ACB = 0.321$ or 18.4°
	$CD = \frac{4}{\cos(\text{their } 1.25)} - 4$ or $\sqrt{4^2 + (\text{their } BC)^2} - 4$	M1	Expect $12.69 - 4 = 8.69$. May use ACB .
	Perimeter = $5 + 12.0(4) + 8.69 = 25.7$ (cm)	A1	AWRT
		4	

Question	Answer	Marks	Guidance
6(a)	$f(x) = (x-1)^2 + 4$	B1	
	$g(x) = (x+2)^2 + 9$	B1	
	$g(x) = f(x+3) + 5$	B1 B1	B1 for each correct element. Accept $p=3, q=5$
		4	

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Question	Answer	Marks	Guidance
6(b)	Translation or Shift	B1	
	$\begin{pmatrix} -3 \\ 5 \end{pmatrix}$ or acceptable explanation	B1 FT	If given as 2 single translations both must be described correctly e.g. $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$ & $\begin{pmatrix} 0 \\ 5 \end{pmatrix}$ FT from <i>their</i> $f(x+p)+q$ or <i>their</i> $f(x) \rightarrow g(x)$ Do not accept $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ or $\begin{pmatrix} -2 \\ 9 \end{pmatrix}$
		2	

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Question	Answer	Marks	Guidance
7(a)	$(a-x)^6 = a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 + \dots$	B2, 1, 0	Allow extra terms. Terms may be listed. Allow a^6x^0 .
		2	
7(b)	$\left(1 + \frac{2}{ax}\right)(\dots 15a^4x^2 - 20a^3x^3 + \dots)$ leading to $[x^2](15a^4 - 40a^2)$	M1	Attempting to find 2 terms in x^2
	$15a^4 - 40a^2 = -20$ leading to $15a^4 - 40a^2 + 20 [= 0]$	A1	Terms on one side of the equation
	$(5a^2 - 10)(3a^2 - 2) [= 0]$	M1	OE. M1 for attempted factorisation or solving for a^2 or u ($=a^2$) using e.g. formula or completing the square
	$a = \pm\sqrt{2}, \pm\sqrt{\frac{2}{3}}$	B1 B1	OE exact form only If B0B0 scored then SC B1 for $\sqrt{2}, \sqrt{\frac{2}{3}}$ WWW or $\pm 1.41, \pm 0.816$ WWW
		5	

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Question	Answer	Marks	Guidance
8(a)	$[fg(x) = 1/(2x+1)^2 - 1]$	B1	SOI
	$1/(2x+1)^2 - 1 = 3$ leading to $4(2x+1)^2 = 1$ or $\frac{1}{(2x+1)} = [\pm]2$ or $16x^2 + 16x + 3 = 0$	M1	Setting $fg(x) = 3$ and reaching a stage before $2x+1 = \pm\frac{1}{2}$ or reaching a 3 term quadratic in x
	$2x+1 = \pm\frac{1}{2}$ or $2x+1 = -\frac{1}{2}$ or $(4x+1)(4x+3) [= 0]$	A1	Or formula or completing square on quadratic
	$x = -\frac{3}{4}$ only	A1	
	Alternative method for Question 8(a)		
	$x^2 - 1 = 3$	M1	
	$g(x) = -2$	A1	
	$\frac{1}{(2x+1)} = -2$	M1	
	$x = -\frac{3}{4}$ only	A1	
		4	

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Question	Answer	Marks	Guidance
8(b)	$y = \frac{1}{(2x+1)^2} - 1$ leading to $(2x+1)^2 = \frac{1}{y+1}$ leading to $2x+1 = [\pm] \frac{1}{\sqrt{y+1}}$	*M1	Obtain $2x+1$ or $2y+1$ as the subject
	$x = [\pm] \frac{1}{2\sqrt{y+1}} - \frac{1}{2}$	DM1	Make x (or y) the subject
	$-\frac{1}{2\sqrt{x+1}} - \frac{1}{2}$	A1	OE e.g. $-\frac{\sqrt{x+1}}{2x+2} - \frac{1}{2}, -\left(\sqrt{\frac{-x}{4x+4} + \frac{1}{4}} + \frac{1}{2}\right)$
		3	

Question	Answer	Marks	Guidance
9(a)	$ar = \frac{24}{100} \times \frac{a}{1-r}$	M1	Form an equation using a numerical form of the percentage and correct formula for u_2 and S_∞
	$100r^2 - 100r + 24 [= 0]$	A1	OE. All 3 terms on one side of an equation.
	$(20r-8)(5r-3)[=0] \rightarrow r = \frac{2}{5}, \frac{3}{5}$	A1	Dependent on factors or formula seen from their quadratic.
		3	

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Question	Answer	Marks	Guidance
9(b)	$3 \times \{(a + 4d)\} = \{2(a + 1) + 11(d + 1)\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $a + d = 13$	A1	
	$\left[\frac{5}{2}\right] \times 3\{(2a + 4d)\} = \left[\frac{5}{2}\right] \times 2\{4(a + 1) + 4(d + 1)\}$	*M1	SOI Attempt to cross multiply with contents of at least one { } correct
	Simplifies to $-a + 2d = 8$	A1	
	Solve 2 linear equations simultaneously	DM1	Elimination or substitution expected
	$d = 7, a = 6$	A1	SC B1 for $a=6, d=7$ without complete working
		6	

Question	Answer	Marks	Guidance
10(a)	Gradient of $AB = -\frac{3}{5}$, gradient of $BC = \frac{5}{3}$ or lengths of all 3 sides or vectors	M1	Attempting to find required gradients, sides or vectors
	$m_{ab}m_{bc} = -1$ or Pythagoras or $\overrightarrow{AB} \cdot \overrightarrow{BC} = 0$ or $\cos ABC = 0$ from cosine rule	A1	WWW
		2	
10(b)	Centre = mid-point of $AC = (2,4)$	B1	
		1	

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Question	Answer	Marks	Guidance
10(c)	$(x - \text{their } x_c)^2 + (y - \text{their } y_c)^2 [= r^2] \text{ or } (\text{their } x_c - x)^2 + (\text{their } y_c - y)^2 = [r^2]$	M1	Use of circle equation with <i>their</i> centre
	$(x - 2)^2 + (y - 4)^2 = 17$	A1	Accept $x^2 - 4x + y^2 - 8y + 3 = 0$ OE
		2	
10(d)	$\left(\frac{x+3}{2}, \frac{y+0}{2}\right) = (2, 4)$ or $\mathbf{BE} = 2\mathbf{BD} = 2\begin{pmatrix} -1 \\ 4 \end{pmatrix}$ Or Equation of BE is $y = -4(x - 3)$ or $y - 4 = -4(x - 2)$ leading to $y = -4x + 12$ Substitute equation of BE into circle and form a 3-term quadratic.	M1	Use of mid-point formula, vectors, steps on a diagram May be seen to find x coordinate at E
	$(x, y) = (1, 8)$ or $\mathbf{OE} = \begin{pmatrix} 3 \\ 0 \end{pmatrix} + \begin{pmatrix} -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 1 \\ 8 \end{pmatrix}$	A1	$E = (1, 8)$ Accept without working for both marks SC B2
	Gradient of BD , m , $= -4$ or gradient $AC = \frac{1}{4} =$ gradient of tangent	B1	Or gradient of $BE = -4$
	Equation of tangent is $y - 8 = \frac{1}{4}(x - 1)$ OE	M1 A1	For M1, equation through <i>their</i> E or $(1, 8)$ (not, A , B or C) and with gradient $\frac{-1}{\text{their } -4}$
		5	

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Question	Answer	Marks	Guidance
11(a)	$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} - \frac{1}{2}k^2x^{-3/2}$	B1 B1	Allow any correct unsimplified form
	$\frac{1}{2}x^{-1/2} - \frac{1}{2}k^2x^{-3/2} = 0$ leading to $\frac{1}{2}x^{-1/2} = \frac{1}{2}k^2x^{-3/2}$	M1	OE. Set to zero and one correct algebraic step towards the solutions. $\frac{dy}{dx}$ must only have 2 terms.
	$(k^2, 2k)$	A1	
		4	
11(b)	When $x = 4k^2$, $\frac{dy}{dx} = \left[\frac{1}{4k} - \frac{1}{16k} \right] \frac{3}{16k}$	B1	OE
	$y = \left[2k + k^2 \times \frac{1}{2k} \right] = \frac{5k}{2}$	B1	OE. Accept $2k + \frac{k}{2}$
	Equation of tangent is $y - \frac{5k}{2} = \frac{3}{16k}(x - 4k^2)$ or $y = mx + c \rightarrow \frac{5k}{2} = \frac{3}{16k}(4k^2) + c$	M1	Use of line equation with <i>their</i> gradient and $(4k^2, \text{their } y)$,
	When $x = 0$, $y = \left[\frac{5k}{2} - \frac{3k}{4} \right] \frac{7k}{4}$ or from $y = mx + c$, $c = \frac{7k}{4}$	A1	OE
		4	

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Question	Answer	Marks	Guidance
11(c)	$\int \left(x^{\frac{1}{2}} + k^2 x^{-\frac{1}{2}} \right) dx = \frac{2x^{\frac{3}{2}}}{3} + 2k^2 x^{\frac{1}{2}}$	B1	Any unsimplified form
	$\left(\frac{16k^3}{3} + 4k^3 \right) - \left(\frac{9k^3}{4} + 3k^3 \right)$	M1	Apply limits $\frac{9}{4}k^2 \rightarrow 4k^2$ to an integration of y . M0 if volume attempted.
	$\frac{49k^3}{12}$	A1	OE. Accept $4.08 k^3$
		3	