

# Cambridge International AS & A Level

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## MATHEMATICS

**9709/11**

Paper 1 Pure Mathematics 1

May/June 2020

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

- 1** The sum of the first nine terms of an arithmetic progression is 117. The sum of the next four terms is 91.

Find the first term and the common difference of the progression.

[4]

[illegible]

- 2** The coefficient of  $\frac{1}{x}$  in the expansion of  $\left(kx + \frac{1}{x}\right)^5 + \left(1 - \frac{2}{x}\right)^8$  is 74.

Find the value of the positive constant  $k$ .

[5]

[illegible]

- 3** Each year the selling price of a diamond necklace increases by 5% of the price the year before. The selling price of the necklace in the year 2000 was \$36 000.

- (a)** Write down an expression for the selling price of the necklace  $n$  years later and hence find the selling price in 2008. [3]

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- (b)** The company that makes the necklace only sells one each year. Find the total amount of money obtained in the ten-year period starting in the year 2000. [2]

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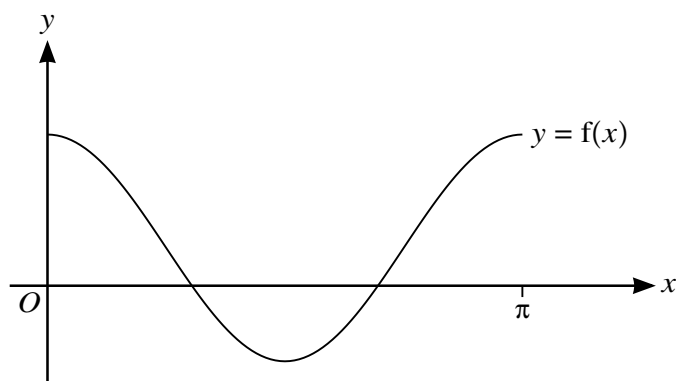
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The diagram shows the graph of  $y = f(x)$ , where  $f(x) = \frac{3}{2} \cos 2x + \frac{1}{2}$  for  $0 \leq x \leq \pi$ .

- (a) State the range of  $f$ . [2]

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A function  $g$  is such that  $g(x) = f(x) + k$ , where  $k$  is a positive constant. The  $x$ -axis is a tangent to the curve  $y = g(x)$ .

- (b) State the value of  $k$  and hence describe fully the transformation that maps the curve  $y = f(x)$  on to  $y = g(x)$ . [2]

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- (c) State the equation of the curve which is the reflection of  $y = f(x)$  in the  $x$ -axis. Give your answer in the form  $y = a \cos 2x + b$ , where  $a$  and  $b$  are constants. [1]

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- 5 The equation of a line is  $y = mx + c$ , where  $m$  and  $c$  are constants, and the equation of a curve is  $xy = 16$ .

(a) Given that the line is a tangent to the curve, express  $m$  in terms of  $c$ . [3]

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(b) Given instead that  $m = -4$ , find the set of values of  $c$  for which the line intersects the curve at two distinct points. [3]

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6 Functions  $f$  and  $g$  are defined for  $x \in \mathbb{R}$  by

$$f : x \mapsto \frac{1}{2}x - a,$$

$$g : x \mapsto 3x + b,$$

where  $a$  and  $b$  are constants.

(a) Given that  $gg(2) = 10$  and  $f^{-1}(2) = 14$ , find the values of  $a$  and  $b$ . [4]

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(b) Using these values of  $a$  and  $b$ , find an expression for  $gf(x)$  in the form  $cx + d$ , where  $c$  and  $d$  are constants. [2]

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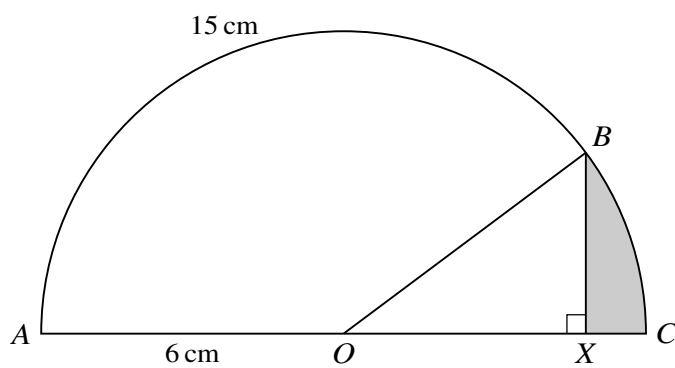
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**7 (a)** Prove the identity  $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} \equiv \frac{2}{\cos \theta}$ . [3]

[illegible]

**[Turn over**



In the diagram,  $ABC$  is a semicircle with diameter  $AC$ , centre  $O$  and radius 6 cm. The length of the arc  $AB$  is 15 cm. The point  $X$  lies on  $AC$  and  $BX$  is perpendicular to  $AX$ .

Find the perimeter of the shaded region  $BXC$ .

[6]

[illegible]



**9** The equation of a curve is  $y = (3 - 2x)^3 + 24x$ .

(a) Find expressions for  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Find the coordinates of each of the stationary points on the curve.

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(c) Determine the nature of each stationary point.

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- 10** The coordinates of the points  $A$  and  $B$  are  $(-1, -2)$  and  $(7, 4)$  respectively.

**(a)** Find the equation of the circle,  $C$ , for which  $AB$  is a diameter.

[4]

[illegible]

- (b) Find the equation of the tangent,  $T$ , to circle  $C$  at the point  $B$ . [4]

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- (c) Find the equation of the circle which is the reflection of circle  $C$  in the line  $T$ . [3]

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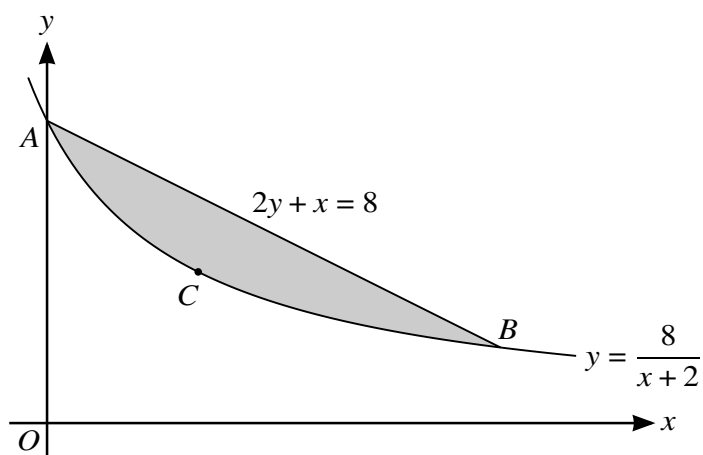
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(a) Find, by calculation, the coordinates of  $A$ ,  $B$  and  $C$ . [6]

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- (b)** Find the volume generated when the shaded region, bounded by the curve and the line, is rotated through  $360^\circ$  about the  $x$ -axis. [6]

[illegible]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



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## MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1

**May/June 2020**

**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

## INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

## INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Blank pages are indicated.

- 1 (a)** Find the coefficient of  $x^2$  in the expansion of  $\left(x - \frac{2}{x}\right)^6$ . [2]

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- (b)** Find the coefficient of  $x^2$  in the expansion of  $(2 + 3x^2)\left(x - \frac{2}{x}\right)^6$ . [3]

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- 2 (a) Express the equation  $3 \cos \theta = 8 \tan \theta$  as a quadratic equation in  $\sin \theta$ . [3]

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- (b) Hence find the acute angle, in degrees, for which  $3 \cos \theta = 8 \tan \theta$ . [2]

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- 3** A weather balloon in the shape of a sphere is being inflated by a pump. The volume of the balloon is increasing at a constant rate of  $600 \text{ cm}^3$  per second. The balloon was empty at the start of pumping.

**(a)** Find the radius of the balloon after 30 seconds. [2]

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**(b)** Find the rate of increase of the radius after 30 seconds. [3]

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- 4** The  $n$ th term of an arithmetic progression is  $\frac{1}{2}(3n - 15)$ .

Find the value of  $n$  for which the sum of the first  $n$  terms is 84.

[5]

[illegible]

5 The function  $f$  is defined for  $x \in \mathbb{R}$  by

$$f : x \mapsto a - 2x,$$

where  $a$  is a constant.

(a) Express  $ff(x)$  and  $f^{-1}(x)$  in terms of  $a$  and  $x$ . [4]

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(b) Given that  $ff(x) = f^{-1}(x)$ , find  $x$  in terms of  $a$ . [2]

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**6** The equation of a curve is  $y = 2x^2 + kx + k - 1$ , where  $k$  is a constant.

- (a)** Given that the line  $y = 2x + 3$  is a tangent to the curve, find the value of  $k$ . [3]

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It is now given that  $k = 2$ .

- (b)** Express the equation of the curve in the form  $y = 2(x + a)^2 + b$ , where  $a$  and  $b$  are constants, and hence state the coordinates of the vertex of the curve. [3]

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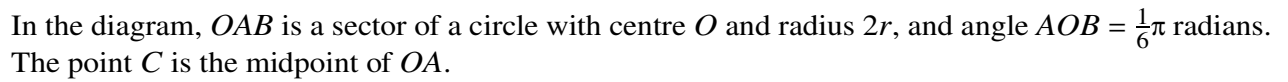
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- This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Find the exact perimeter of the shaded region.

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(c) Find the exact area of the shaded region.

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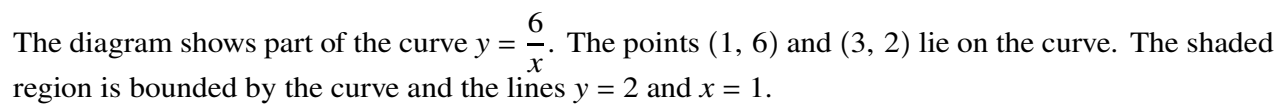
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- [illegible]

- (b)** The tangent to the curve at a point  $X$  is parallel to the line  $y + 2x = 0$ . Show that  $X$  lies on the line  $y = 2x$ . [3]

[illegible]

- 9 Functions  $f$  and  $g$  are such that

$$f(x) = 2 - 3 \sin 2x \quad \text{for } 0 \leq x \leq \pi,$$

$$g(x) = -2f(x) \quad \text{for } 0 \leq x \leq \pi.$$

- (a) State the ranges of  $f$  and  $g$ .

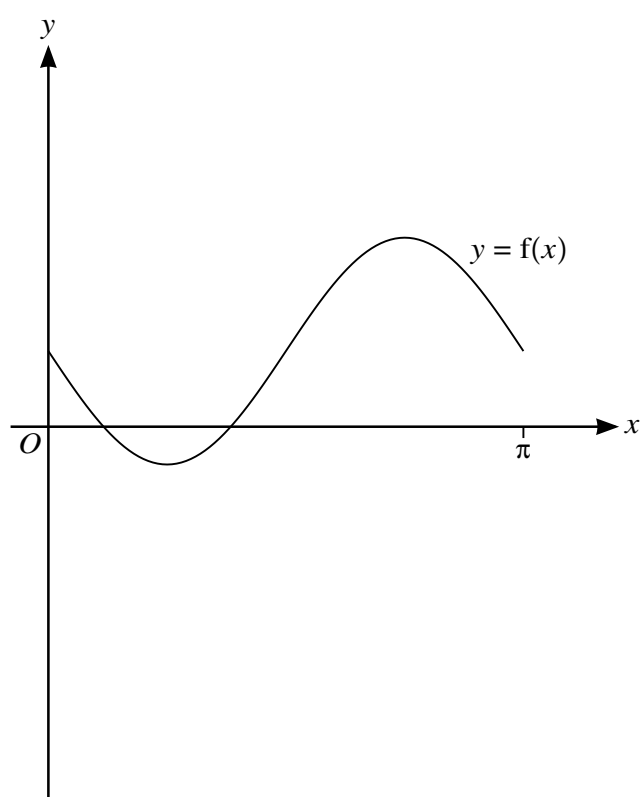
[3]

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The diagram below shows the graph of  $y = f(x)$ .



- (b) Sketch, on this diagram, the graph of  $y = g(x)$ .

[2]

The function  $h$  is such that

$$h(x) = g(x + \pi) \quad \text{for } -\pi \leq x \leq 0.$$

- (c) Describe fully a sequence of transformations that maps the curve  $y = f(x)$  on to  $y = h(x)$ .

[3]

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10 The equation of a curve is  $y = 54x - (2x - 7)^3$ .

- (a) Find  $\frac{dy}{dx}$  and  $\frac{d^2y}{dx^2}$ . [4]

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- (b) Find the coordinates of each of the stationary points on the curve. [3]

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- (c) Determine the nature of each of the stationary points. [2]

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**11** The equation of a circle with centre  $C$  is  $x^2 + y^2 - 8x + 4y - 5 = 0$ .

- (a)** Find the radius of the circle and the coordinates of  $C$ . [3]

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The point  $P(1, 2)$  lies on the circle.

- (b)** Show that the equation of the tangent to the circle at  $P$  is  $4y = 3x + 5$ . [3]

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The point  $Q$  also lies on the circle and  $PQ$  is parallel to the  $x$ -axis.

- (c) Write down the coordinates of  $Q$ . [2]

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The tangents to the circle at  $P$  and  $Q$  meet at  $T$ .

- (d) Find the coordinates of  $T$ . [3]

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[illegible]

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## MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1

May/June 2020

**1 hour 50 minutes**

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You will need: List of formulae (MF19)

## INSTRUCTIONS

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## INFORMATION

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- The number of marks for each question or part question is shown in brackets [ ].

This document has **20** pages. Blank pages are indicated.

- 1** Find the set of values of  $m$  for which the line with equation  $y = mx + 1$  and the curve with equation  $y = 3x^2 + 2x + 4$  intersect at two distinct points. [4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** The equation of a curve is such that  $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 3x^{-\frac{1}{2}}$ . It is given that the point (4, 7) lies on the curve.

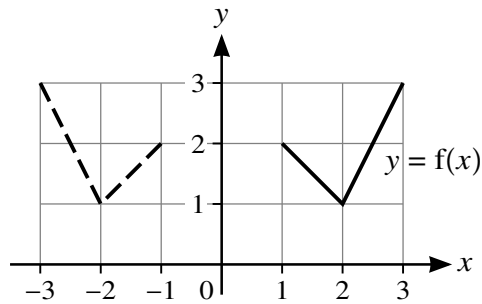
Find the equation of the curve.

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3 In each of parts (a), (b) and (c), the graph shown with solid lines has equation  $y = f(x)$ . The graph shown with broken lines is a transformation of  $y = f(x)$ .

(a)

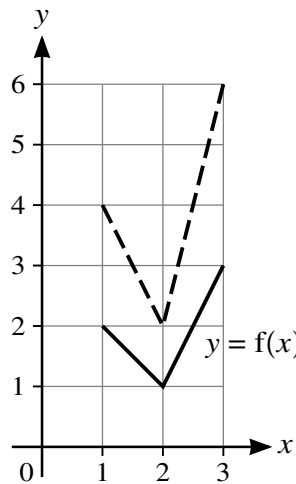


State, in terms of  $f$ , the equation of the graph shown with broken lines.

[1]

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(b)

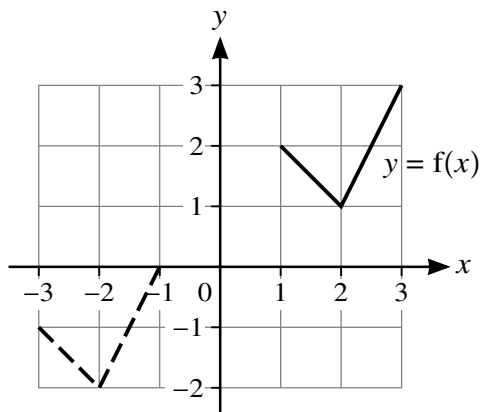


State, in terms of  $f$ , the equation of the graph shown with broken lines.

[1]

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(c)



State, in terms of  $f$ , the equation of the graph shown with broken lines.

[2]

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- 4 (a) Expand  $(1 + a)^5$  in ascending powers of  $a$  up to and including the term in  $a^3$ . [1]

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- (b) Hence expand  $[1 + (x + x^2)]^5$  in ascending powers of  $x$  up to and including the term in  $x^3$ , simplifying your answer. [3]

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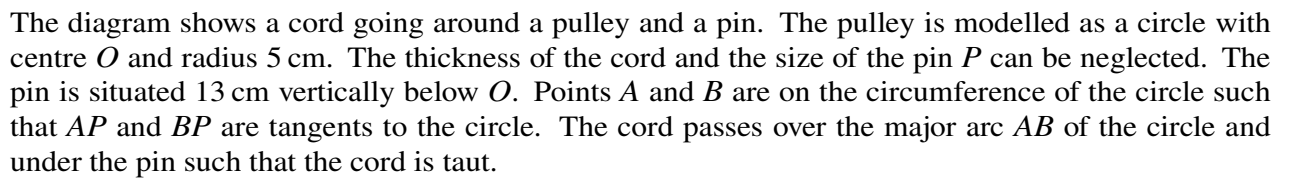
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[6]

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- 6 A point  $P$  is moving along a curve in such a way that the  $x$ -coordinate of  $P$  is increasing at a constant rate of 2 units per minute. The equation of the curve is  $y = (5x - 1)^{\frac{1}{2}}$ .

(a) Find the rate at which the  $y$ -coordinate is increasing when  $x = 1$ . [4]

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[illegible]

7 (a) Show that  $\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} \equiv \frac{2}{\sin \theta \cos \theta}$ . [4]

[illegible]

**[Turn over**

- 8** The first term of a progression is  $\sin^2 \theta$ , where  $0 < \theta < \frac{1}{2}\pi$ . The second term of the progression is  $\sin^2 \theta \cos^2 \theta$ .
- (a)** Given that the progression is geometric, find the sum to infinity. [3]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

It is now given instead that the progression is arithmetic.

- (b) (i) Find the common difference of the progression in terms of  $\sin \theta$ . [3]

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- (ii) Find the sum of the first 16 terms when  $\theta = \frac{1}{3}\pi$ . [3]

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- 9 The functions  $f$  and  $g$  are defined by

$$f(x) = x^2 - 4x + 3 \quad \text{for } x > c, \text{ where } c \text{ is a constant,}$$

$$g(x) = \frac{1}{x+1} \quad \text{for } x > -1.$$

- (a) Express  $f(x)$  in the form  $(x - a)^2 + b$ . [2]

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It is given that  $f$  is a one-one function.

- (b) State the smallest possible value of  $c$ . [1]

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It is now given that  $c = 5$ .

- (c) Find an expression for  $f^{-1}(x)$  and state the domain of  $f^{-1}$ . [3]

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- (d) Find an expression for  $gf(x)$  and state the range of  $gf$ . [3]

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- 10 (a)** The coordinates of two points  $A$  and  $B$  are  $(-7, 3)$  and  $(5, 11)$  respectively.

Show that the equation of the perpendicular bisector of  $AB$  is  $3x + 2y = 11$ .

[4]

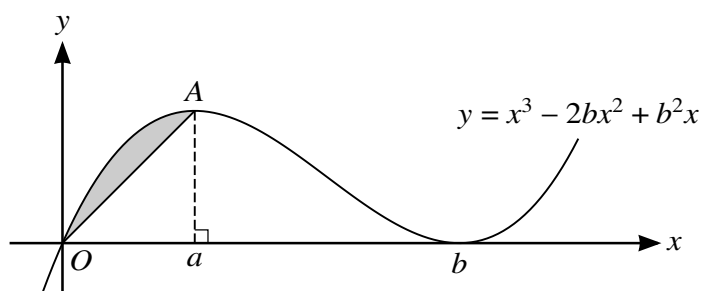
This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- (b)** A circle passes through  $A$  and  $B$  and its centre lies on the line  $12x - 5y = 70$ .

Find an equation of the circle.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



The diagram shows part of the curve with equation  $y = x^3 - 2bx^2 + b^2x$  and the line  $OA$ , where  $A$  is the maximum point on the curve. The  $x$ -coordinate of  $A$  is  $a$  and the curve has a minimum point at  $(b, 0)$ , where  $a$  and  $b$  are positive constants.

- (a) Show that  $b = 3a$ . [4]

[illegible]

- (b)** Show that the area of the shaded region between the line and the curve is  $ka^4$ , where  $k$  is a fraction to be found. [7]

[illegible]

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