

CANDIDATE
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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

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You may use an HB pencil for any diagrams or graphs.

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Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **21** printed pages and **3** blank pages.



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- 1 The term independent of x in the expansion of $\left(2x + \frac{k}{x}\right)^6$, where k is a constant, is 540.

(i) Find the value of k . [3]

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(ii) For this value of k , find the coefficient of x^2 in the expansion. [2]

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- 2 The line $4y = x + c$, where c is a constant, is a tangent to the curve $y^2 = x + 3$ at the point P on the curve.

(i) Find the value of c .

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(ii) Find the coordinates of P .

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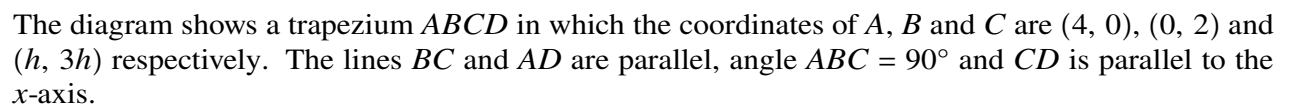
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- 3** A sector of a circle of radius r cm has an area of A cm². Express the perimeter of the sector in terms of r and A . [4]

[illegible]



- [illegible]

[3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

5 The function f is defined by $f(x) = -2x^2 + 12x - 3$ for $x \in \mathbb{R}$.

(i) Express $-2x^2 + 12x - 3$ in the form $-2(x + a)^2 + b$, where a and b are constants. [2]

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(ii) State the greatest value of $f(x)$. [1]

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[3]

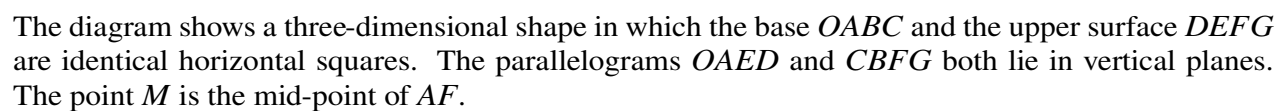
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6 (i) Prove the identity $\left(\frac{1}{\cos x} - \tan x\right)^2 \equiv \frac{1 - \sin x}{1 + \sin x}$. [4]

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(ii) Hence solve the equation $\left(\frac{1}{\cos 2x} - \tan 2x\right)^2 = \frac{1}{3}$ for $0 \leq x \leq \pi$. [3]

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(i) Express each of the vectors $\overrightarrow{AM}$ and $\overrightarrow{GM}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]

[illegible]

[illegible]

- 8** (a) The third and fourth terms of a geometric progression are 48 and 32 respectively. Find the sum to infinity of the progression. [3]

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- (b) Two schemes are proposed for increasing the amount of household waste that is recycled each week.

Scheme *A* is to increase the amount of waste recycled each month by 0.16 tonnes.

Scheme *B* is to increase the amount of waste recycled each month by 6% of the amount recycled in the previous month.

The proposal is to operate the scheme for a period of 24 months. The amount recycled in the first month is 2.5 tonnes.

For each scheme, find the total amount of waste that would be recycled over the 24-month period. [5]

Scheme *A*

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Scheme *B*

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- 9 The function f is defined by $f(x) = 2 - 3 \cos x$ for $0 \leq x \leq 2\pi$.

(i) State the range of f .

[2]

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(ii) Sketch the graph of $y = f(x)$.

[2]

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[illegible]

- 10** A curve for which $\frac{d^2y}{dx^2} = 2x - 5$ has a stationary point at (3, 6).

(i) Find the equation of the curve.

[6]

[illegible]

- (ii) Find the x -coordinate of the other stationary point on the curve. [1]

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- (iii) Determine the nature of each of the stationary points. [2]

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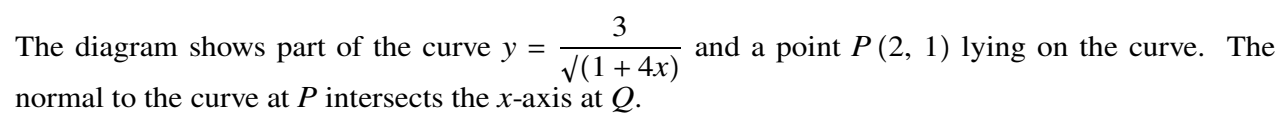
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- This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings present.

[6]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1 (P1)

May/June 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

This document consists of **18** printed pages and **2** blank pages.



- 1** Find the coefficient of x in the expansion of $\left(\frac{2}{x} - 3x\right)^5$. [3]

[illegible]

- 2** Two points A and B have coordinates $(1, 3)$ and $(9, -1)$ respectively. The perpendicular bisector of AB intersects the y -axis at the point C . Find the coordinates of C . [5]

[illegible]

3 A curve is such that $\frac{dy}{dx} = x^3 - \frac{4}{x^2}$. The point $P(2, 9)$ lies on the curve.

- (i) A point moves on the curve in such a way that the x -coordinate is decreasing at a constant rate of 0.05 units per second. Find the rate of change of the y -coordinate when the point is at P . [2]

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- (ii) Find the equation of the curve. [3]

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4 Angle x is such that $\sin x = a + b$ and $\cos x = a - b$, where a and b are constants.

(i) Show that $a^2 + b^2$ has a constant value for all values of x . [3]

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(ii) In the case where $\tan x = 2$, express a in terms of b . [2]

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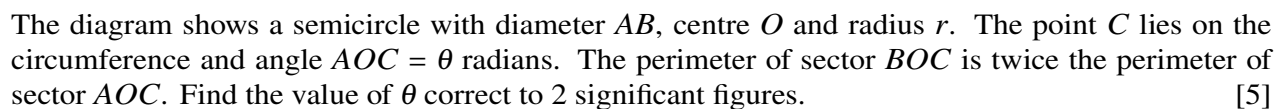
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The diagram shows a semicircle with diameter AB , centre O and radius r . The point C lies on the circumference and angle $AOC = \theta$ radians. The perimeter of sector BOC is twice the perimeter of sector AOC . Find the value of θ correct to 2 significant figures. [5]

[illegible]

- 6 The equation of a curve is $y = 3 \cos 2x$ and the equation of a line is $2y + \frac{3x}{\pi} = 5$.

(i) State the smallest and largest values of y for both the curve and the line for $0 \leq x \leq 2\pi$. [3]

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(ii) Sketch, on the same diagram, the graphs of $y = 3 \cos 2x$ and $2y + \frac{3x}{\pi} = 5$ for $0 \leq x \leq 2\pi$. [3]

(iii) State the number of solutions of the equation $6 \cos 2x = 5 - \frac{3x}{\pi}$ for $0 \leq x \leq 2\pi$. [1]

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7 Functions f and g are defined by

$$f : x \mapsto 3x - 2, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{2x+3}{x-1}, \quad x \in \mathbb{R}, x \neq 1.$$

(ii) Obtain expressions for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined.

[4]

[illegible]

[Turn over

- 8 The position vectors of points A and B , relative to an origin O , are given by

$$\overrightarrow{OA} = \begin{pmatrix} 6 \\ -2 \\ -6 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ k \\ -3 \end{pmatrix},$$

where k is a constant.

- (i) Find the value of k for which angle AOB is 90° . [2]

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- (ii) Find the values of k for which the lengths of OA and OB are equal. [2]

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The point C is such that $\overrightarrow{AC} = 2\overrightarrow{CB}$.

(iii) In the case where $k = 4$, find the unit vector in the direction of $\overrightarrow{OC}$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 9** The curve C_1 has equation $y = x^2 - 4x + 7$. The curve C_2 has equation $y^2 = 4x + k$, where k is a constant. The tangent to C_1 at the point where $x = 3$ is also the tangent to C_2 at the point P . Find the value of k and the coordinates of P . [8]

[illegible]

- 10 (a)** In an arithmetic progression, the sum of the first ten terms is equal to the sum of the next five terms. The first term is a .

(i) Show that the common difference of the progression is $\frac{1}{3}a$. [4]

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(ii) Given that the tenth term is 36 more than the fourth term, find the value of a . [2]

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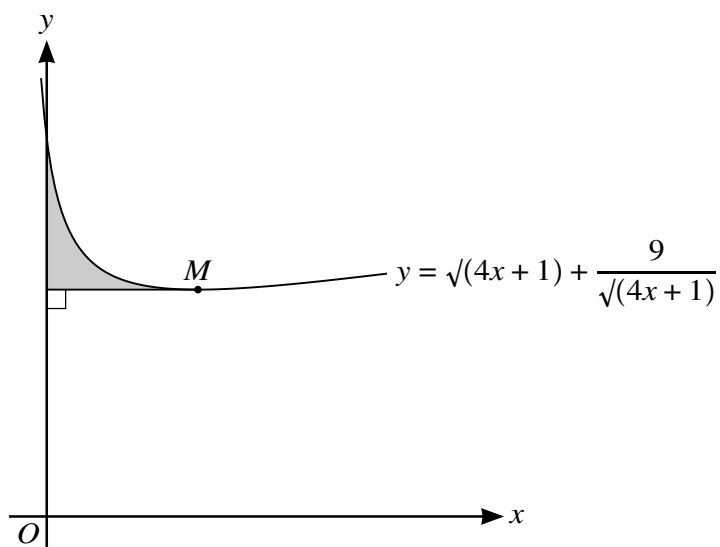
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- (b)** The sum to infinity of a geometric progression is 9 times the sum of the first four terms. Given that the first term is 12, find the value of the fifth term. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.



The diagram shows part of the curve $y = \sqrt{4x + 1} + \frac{9}{\sqrt{4x + 1}}$ and the minimum point M .

- (i) Find expressions for $\frac{dy}{dx}$ and $\int y \, dx$.

[6]

[illegible]

(ii) Find the coordinates of M .

[3]

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The shaded region is bounded by the curve, the y -axis and the line through M parallel to the x -axis.

(iii) Find, showing all necessary working, the area of the shaded region.

[3]

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[illegible]

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MATHEMATICS

9709/13

Paper 1 Pure Mathematics 1 (P1)

May/June 2019

1 hour 45 minutes

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This document consists of **19** printed pages and **1** blank page.



1 The function f is defined by $f(x) = x^2 - 4x + 8$ for $x \in \mathbb{R}$.

(i) Express $x^2 - 4x + 8$ in the form $(x - a)^2 + b$. [2]

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(ii) Hence find the set of values of x for which $f(x) < 9$, giving your answer in exact form. [3]

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- 2 (i) In the binomial expansion of $\left(2x - \frac{1}{2x}\right)^5$, the first three terms are $32x^5 - 40x^3 + 20x$. Find the remaining three terms of the expansion. [3]

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- (ii) Hence find the coefficient of x in the expansion of $(1 + 4x^2)\left(2x - \frac{1}{2x}\right)^5$. [2]

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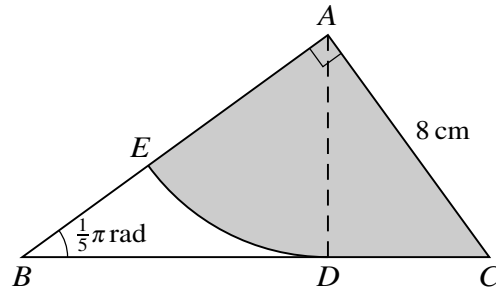
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The diagram shows triangle ABC which is right-angled at A . Angle $ABC = \frac{1}{5}\pi$ radians and $AC = 8$ cm. The points D and E lie on BC and BA respectively. The sector ADE is part of a circle with centre A and is such that BDC is the tangent to the arc DE at D .

- (i) Find the length of AD . [3]

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- (ii) Find the area of the shaded region. [3]

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- 4 The function f is defined by $f(x) = \frac{48}{x-1}$ for $3 \leq x \leq 7$. The function g is defined by $g(x) = 2x - 4$ for $a \leq x \leq b$, where a and b are constants.

- (i) Find the greatest value of a and the least value of b which will permit the formation of the composite function gf . [2]

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It is now given that the conditions for the formation of gf are satisfied.

- (ii) Find an expression for $gf(x)$. [1]

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- (iii) Find an expression for $(gf)^{-1}(x)$. [2]

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- 5 Two heavyweight boxers decide that they would be more successful if they competed in a lower weight class. For each boxer this would require a total weight loss of 13 kg. At the end of week 1 they have each recorded a weight loss of 1 kg and they both find that in each of the following weeks their weight loss is slightly less than the week before.

Boxer A's weight loss in week 2 is 0.98 kg. It is given that his weekly weight loss follows an arithmetic progression.

- (i) Write down an expression for his total weight loss after x weeks. [1]

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- (ii) He reaches his 13 kg target during week n . Use your answer to part (i) to find the value of n . [2]

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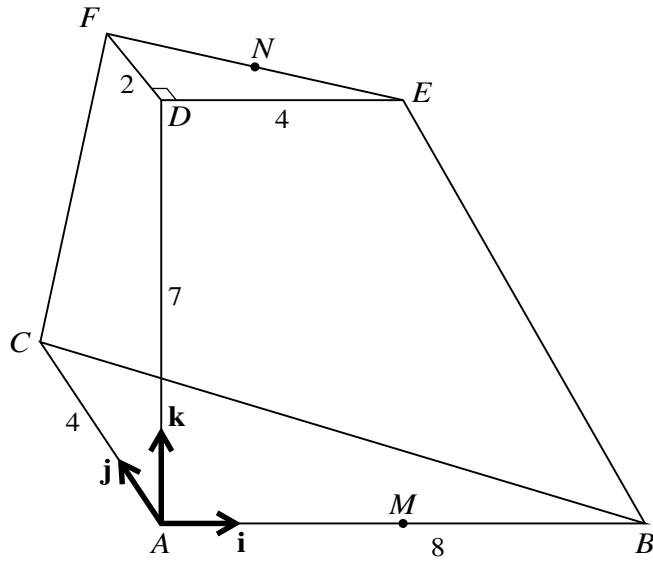
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Boxer B 's weight loss in week 2 is 0.92 kg and it is given that his weekly weight loss follows a geometric progression.

- (iii) Calculate his total weight loss after 20 weeks and show that he can never reach his target. [4]

[illegible]

6



The diagram shows a solid figure $ABCDEF$ in which the horizontal base ABC is a triangle right-angled at A . The lengths of AB and AC are 8 units and 4 units respectively and M is the mid-point of AB . The point D is 7 units vertically above A . Triangle DEF lies in a horizontal plane with DE , DF and FE parallel to AB , AC and CB respectively and N is the mid-point of FE . The lengths of DE and DF are 4 units and 2 units respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ respectively.

- (i) Find $\overrightarrow{MF}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

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- (ii) Find $\overrightarrow{FN}$ in terms of $\mathbf{i}$ and $\mathbf{j}$. [1]

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- (iii) Find $\overrightarrow{MN}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [1]

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(iv) Use a scalar product to find angle FMN .

[4]

[illegible]

- 7** The coordinates of two points A and B are $(1, 3)$ and $(9, -1)$ respectively and D is the mid-point of AB . A point C has coordinates (x, y) , where x and y are variables.

(i) State the coordinates of D . [1]

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(ii) It is given that $CD^2 = 20$. Write down an equation relating x and y . [1]

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(iii) It is given that AC and BC are equal in length. Find an equation relating x and y and show that it can be simplified to $y = 2x - 9$. [3]

[illegible]

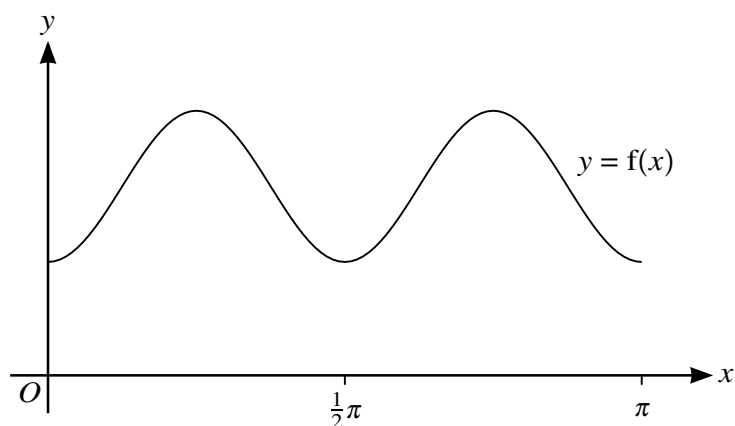
- (iv)** Using the results from parts **(ii)** and **(iii)**, and showing all necessary working, find the possible coordinates of C . [4]

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- 8** A curve is such that $\frac{dy}{dx} = 3x^2 + ax + b$. The curve has stationary points at $(-1, 2)$ and $(3, k)$. Find the values of the constants a, b and k . [8]

[illegible]

9



The function $f : x \mapsto p \sin^2 2x + q$ is defined for $0 \leq x \leq \pi$, where p and q are positive constants. The diagram shows the graph of $y = f(x)$.

- (i) In terms of p and q , state the range of f . [2]

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- (ii) State the number of solutions of the following equations.

- (a) $f(x) = p + q$ [1]

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- (b) $f(x) = q$ [1]

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- (c) $f(x) = \frac{1}{2}p + q$ [1]

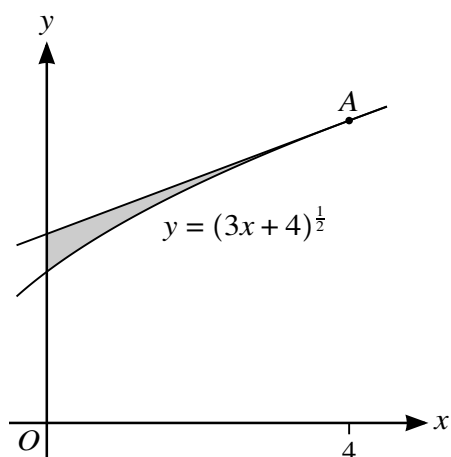
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- (iii) For the case where $p = 3$ and $q = 2$, solve the equation $f(x) = 4$, showing all necessary working. [5]

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(i) Find the equation of the tangent to the curve at A.

[5]

[illegible]

- (ii)** Find, showing all necessary working, the area of the shaded region.

[5]

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[Question 10 (iii) is printed on the next page.]

- (iii) A point is moving along the curve. At the point P the y -coordinate is increasing at half the rate at which the x -coordinate is increasing. Find the x -coordinate of P . [3]

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