

CANDIDATE
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MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2017

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



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- This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 2 Relative to an origin O , the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix},$$

and angle $AOB = 90^\circ$.

- (i) Find the value of p .

[2]

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The point C is such that $\overrightarrow{OC} = \frac{2}{3}\overrightarrow{OA}$.

- (ii) Find the unit vector in the direction of $\overrightarrow{BC}$.

[4]

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3 (i) Prove the identity $\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} \equiv \frac{2}{\sin \theta}$. [3]

[illegible]

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- 4 (a)** An arithmetic progression has a first term of 32, a 5th term of 22 and a last term of -28 . Find the sum of all the terms in the progression. [4]

[illegible]

- (b)** Each year a school allocates a sum of money for the library. The amount allocated each year increases by 2.5% of the amount allocated the previous year. In 2005 the school allocated \$2000. Find the total amount allocated in the years 2005 to 2014 inclusive. [3]

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5 The equation of a curve is $y = 2 \cos x$.

- (i) Sketch the graph of $y = 2 \cos x$ for $-\pi \leq x \leq \pi$, stating the coordinates of the point of intersection with the y -axis. [2]

Points P and Q lie on the curve and have x -coordinates of $\frac{1}{3}\pi$ and π respectively.

- (ii) Find the length of PQ correct to 1 decimal place. [2]

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[illegible]

- 6 The horizontal base of a solid prism is an equilateral triangle of side x cm. The sides of the prism are vertical. The height of the prism is h cm and the volume of the prism is 2000 cm^3 .

- (i) Express h in terms of x and show that the total surface area of the prism, $A \text{ cm}^2$, is given by

$$A = \frac{\sqrt{3}}{2}x^2 + \frac{24\,000}{\sqrt{3}}x^{-1}. \quad [3]$$

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- (ii) Given that x can vary, find the value of x for which A has a stationary value. [3]

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- (iii) Determine, showing all necessary working, the nature of this stationary value. [2]

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- 7** A curve for which $\frac{dy}{dx} = 7 - x^2 - 6x$ passes through the point $(3, -10)$.

(i) Find the equation of the curve.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (ii) Express $7 - x^2 - 6x$ in the form $a - (x + b)^2$, where a and b are constants. [2]

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- (iii) Find the set of values of x for which the gradient of the curve is positive. [3]

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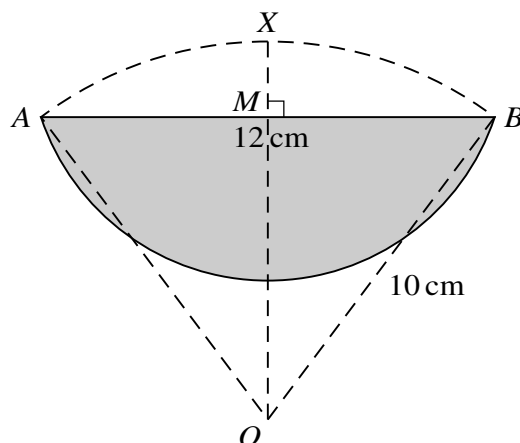
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In the diagram, $OAXB$ is a sector of a circle with centre O and radius 10 cm . The length of the chord AB is 12 cm . The line OX passes through M , the mid-point of AB , and OX is perpendicular to AB . The shaded region is bounded by the chord AB and by the arc of a circle with centre X and radius XA .

- (i) Show that angle AXB is 2.498 radians, correct to 3 decimal places. [3]

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- (ii) Find the perimeter of the shaded region. [3]

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(iii) Find the area of the shaded region.

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- 9 The function f is defined by $f : x \mapsto \frac{2}{3-2x}$ for $x \in \mathbb{R}$, $x \neq \frac{3}{2}$.

(i) Find an expression for $f^{-1}(x)$.

[3]

[illegible]

The function g is defined by $g : x \mapsto 4x + a$ for $x \in \mathbb{R}$, where a is a constant.

- (ii) Find the value of a for which $gf(-1) = 3$. [3]

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- (iii) Find the possible values of a given that the equation $f^{-1}(x) = g^{-1}(x)$ has two equal roots. [4]

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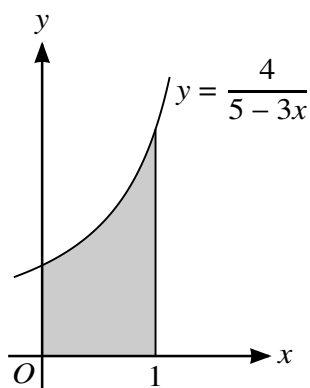
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(i) Find the equation of the normal to the curve at the point where $x = 1$ in the form $y = mx + c$, where m and c are constants. [5]

[illegible]

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MATHEMATICS

9709/12

Paper 1 Pure Mathematics 1 (P1)

May/June 2017

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

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- 1 (i) Find the coefficient of x in the expansion of $\left(2x - \frac{1}{x}\right)^5$. [2]

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- (ii) Hence find the coefficient of x in the expansion of $(1 + 3x^2)\left(2x - \frac{1}{x}\right)^5$. [4]

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- 2 The point A has coordinates $(-2, 6)$. The equation of the perpendicular bisector of the line AB is $2y = 3x + 5$.

(i) Find the equation of AB .

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(ii) Find the coordinates of B .

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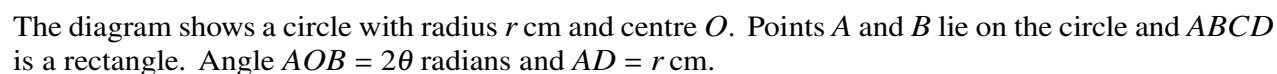
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3 (i) Prove the identity $\left(\frac{1}{\cos \theta} - \tan \theta\right) \equiv \frac{1 - \sin \theta}{1 + \sin \theta}$. [3]

[illegible]

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- (ii)** In the case where $r = 5$ and $\theta = \frac{1}{6}\pi$, find the area of the shaded region. [4]

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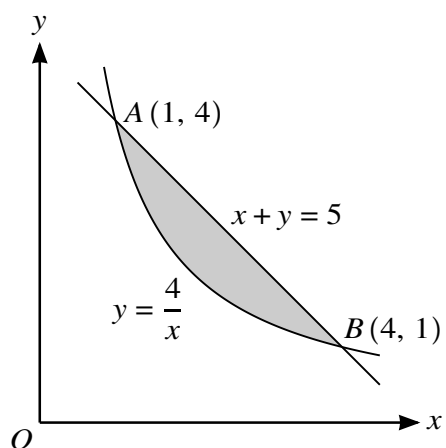
5 A curve has equation $y = 3 + \frac{12}{2-x}$.

(i) Find the equation of the tangent to the curve at the point where the curve crosses the x -axis. [5]

[illegible]

- (ii) A point moves along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.04 units per second. Find the rate of change of the y -coordinate when $x = 4$. [2]

[illegible]



The diagram shows the straight line $x + y = 5$ intersecting the curve $y = \frac{4}{x}$ at the points $A(1, 4)$ and $B(4, 1)$. Find, showing all necessary working, the volume obtained when the shaded region is rotated through 360° about the x -axis. [7]

[illegible]

- 7 (a) The first two terms of an arithmetic progression are 16 and 24. Find the least number of terms of the progression which must be taken for their sum to exceed 20 000. [4]

[illegible]

- (b) A geometric progression has a first term of 6 and a sum to infinity of 18. A new geometric progression is formed by squaring each of the terms of the original progression. Find the sum to infinity of the new progression. [4]

[illegible]

- 8** Relative to an origin O , the position vectors of three points A , B and C are given by

$$\overrightarrow{OA} = 3\mathbf{i} + p\mathbf{j} - 2p\mathbf{k}, \quad \overrightarrow{OB} = 6\mathbf{i} + (p+4)\mathbf{j} + 3\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = (p-1)\mathbf{i} + 2\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

- (i) In the case where $p = 2$, use a scalar product to find angle AOB . [4]

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[illegible]

9 The equation of a curve is $y = 8\sqrt{x} - 2x$.

(i) Find the coordinates of the stationary point of the curve. [3]

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(ii) Find an expression for $\frac{d^2y}{dx^2}$ and hence, or otherwise, determine the nature of the stationary point. [2]

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(iii) Find the values of x at which the line $y = 6$ meets the curve.

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(iv) State the set of values of k for which the line $y = k$ does not meet the curve.

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10 The function f is defined by $f(x) = 3 \tan\left(\frac{1}{2}x\right) - 2$, for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$.

(i) Solve the equation $f(x) + 4 = 0$, giving your answer correct to 1 decimal place. [3]

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(ii) Find an expression for $f^{-1}(x)$ and find the domain of f^{-1} . [5]

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(iii) Sketch, on the same diagram, the graphs of $y = f(x)$ and $y = f^{-1}(x)$.

[3]

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MATHEMATICS

9709/13

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- 1** The coefficients of x and x^2 in the expansion of $(2 + ax)^7$ are equal. Find the value of the non-zero constant a . [3]

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- 2 The common ratio of a geometric progression is r . The first term of the progression is $(r^2 - 3r + 2)$ and the sum to infinity is S .

(i) Show that $S = 2 - r$. [2]

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(ii) Find the set of possible values that S can take. [2]

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- 3** Find the coordinates of the points of intersection of the curve $y = x^{\frac{2}{3}} - 1$ with the curve $y = x^{\frac{1}{3}} + 1$. [4]

[illegible]

- 4 Relative to an origin O , the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix}.$$

The point P lies on AB and is such that $\overrightarrow{AP} = \frac{1}{3}\overrightarrow{AB}$.

- (i) Find the position vector of P . [3]

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- (ii) Find the distance OP . [1]

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- (iii) Determine whether OP is perpendicular to AB . Justify your answer. [2]

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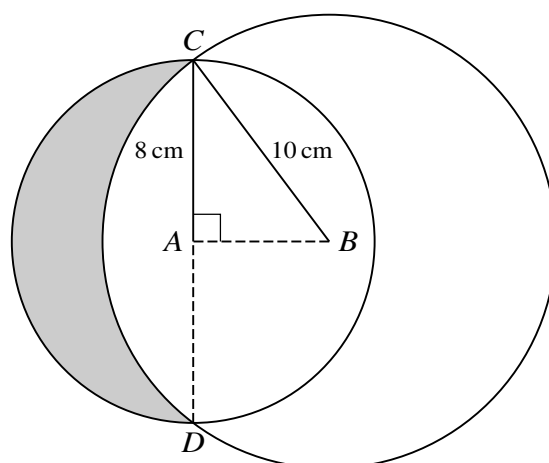
5 (i) Show that the equation $\frac{2 \sin \theta + \cos \theta}{\sin \theta + \cos \theta} = 2 \tan \theta$ may be expressed as $\cos^2 \theta = 2 \sin^2 \theta$. [3]

[illegible]

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- 6** The line $3y + x = 25$ is a normal to the curve $y = x^2 - 5x + k$. Find the value of the constant k . [6]

[illegible]



The diagram shows two circles with centres A and B having radii 8 cm and 10 cm respectively. The two circles intersect at C and D where CAD is a straight line and AB is perpendicular to CD .

(i) Find angle ABC in radians.

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(ii) Find the area of the shaded region.

[6]

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- 8 $A(-1, 1)$ and $P(a, b)$ are two points, where a and b are constants. The gradient of AP is 2.

(i) Find an expression for b in terms of a .

[2]

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- (ii) $B(10, -1)$ is a third point such that $AP = AB$. Calculate the coordinates of the possible positions of P .

[6]

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- 9** (i) Express $9x^2 - 6x + 6$ in the form $(ax + b)^2 + c$, where a , b and c are constants. [3]

[illegible]

The function f is defined by $f(x) = 9x^2 - 6x + 6$ for $x \geq p$, where p is a constant.

- (ii) State the smallest value of p for which f is a one-one function. [1]

- (iii) For this value of p , obtain an expression for $f^{-1}(x)$, and state the domain of f^{-1} . [4]

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- (iv) State the set of values of q for which the equation $f(x) = q$ has no solution. [1]

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10 (a)

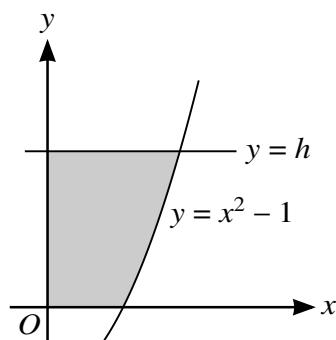


Fig. 1

Fig. 1 shows part of the curve $y = x^2 - 1$ and the line $y = h$, where h is a constant.

- (i) The shaded region is rotated through 360° about the **y-axis**. Show that the volume of revolution, V , is given by $V = \pi\left(\frac{1}{2}h^2 + h\right)$. [3]

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- (ii) Find, showing all necessary working, the area of the shaded region when $h = 3$. [4]

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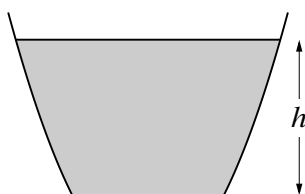
**Fig. 2**

Fig. 2 shows a cross-section of a bowl containing water. When the height of the water level is h cm, the volume, $V \text{ cm}^3$, of water is given by $V = \pi(\frac{1}{2}h^2 + h)$. Water is poured into the bowl at a constant rate of $2 \text{ cm}^3 \text{ s}^{-1}$. Find the rate, in cm s^{-1} , at which the height of the water level is increasing when the height of the water level is 3 cm. [4]

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- 11** The function f is defined for $x \geq 0$. It is given that f has a minimum value when $x = 2$ and that $f''(x) = (4x + 1)^{-\frac{1}{2}}$.

(i) Find $f'(x)$. [3]

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It is now given that $f''(0)$, $f'(0)$ and $f(0)$ are the first three terms respectively of an arithmetic progression.

(ii) Find the value of $f(0)$. [3]

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[5]

[illegible]

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