
MATHEMATICS

9709/11

Paper 1

May/June 2017

MARK SCHEME

Maximum Mark: 75

Published

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 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

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- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

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Question	Answer	Marks	Guidance
1	$(3-2x)^6$		
	Coeff of $x^2 = 3^4 \times (-2)^2 \times {}_6C_2 = a$ Coeff of $x^3 = 3^3 \times (-2)^3 \times {}_6C_3 = b$	B3,2,1	Mark unsimplified forms. –1 each independent error but powers must be correct. Ignore any ‘x’ present.
	$\frac{a}{b} = -\frac{9}{8}$	B1	OE. Negative sign must appear before or in the numerator
	Total:	4	
2	$\overrightarrow{OA} = \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 2 \\ -6 \\ -7 \end{pmatrix}$		
2(i)	Angle $AOB = 90^\circ \rightarrow 6 + 36 - 7p = 0$	M1	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$ or Pythagoras
	$\rightarrow p = 6$	A1	
	Total:	2	

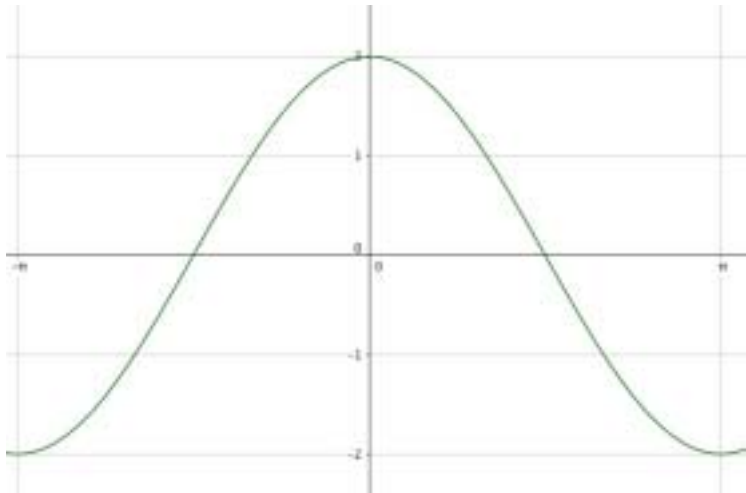
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Question	Answer	Marks	Guidance
2(ii)	$\overrightarrow{OC} = \frac{2}{3} \begin{pmatrix} 3 \\ -6 \\ p \end{pmatrix} = \begin{pmatrix} 2 \\ -4 \\ 4 \end{pmatrix}$	B1 FT	CAO FT on their value of p
	$\overrightarrow{BC} = \mathbf{c} - \mathbf{b} = \begin{pmatrix} 0 \\ 2 \\ 11 \end{pmatrix}$; magnitude = $\sqrt{125}$	M1 M1	Use of $\mathbf{c} - \mathbf{b}$. Allow magnitude of $\mathbf{b} + \mathbf{c}$ or $\mathbf{b} - \mathbf{c}$ Allow first M1 in terms of p
	Unit vector = $\frac{1}{\sqrt{125}} \begin{pmatrix} 0 \\ 2 \\ 11 \end{pmatrix}$	A1	OE Allow $\pm$ and decimal equivalent
3(i)	$\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} \equiv \frac{2}{\sin \theta}$		
	$\frac{(1+c)^2 + s^2}{s(1+c)} = \frac{1+2c+c^2+s^2}{s(1+c)}$	M1	Correct use of fractions
	$= \frac{2+2c}{s(1+c)} = \frac{2(1+c)}{s(1+c)} \rightarrow \frac{2}{s}$	M1 A1	Use of trig identity, A1 needs evidence of cancelling
	Total:	3	
3(ii)	$\frac{2}{s} = \frac{3}{c} \rightarrow t = \frac{2}{3}$	M1	Use part (i) and $t = s \div c$, may restart from given equation
	$\rightarrow \theta = 33.7^\circ$ or 213.7°	A1 A1FT	FT for $180^\circ + 1$ st answer. 2nd A1 lost for extra solns in range
	Total:	3	

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Question	Answer	Marks	Guidance
4(a)	$a = 32, a + 4d = 22, \rightarrow d = -2.5$	B1	
	$a + (n - 1)d = -28 \rightarrow n = 25$	B1	
	$S_{25} = \frac{25}{2}(64 - 2.5 \times 24) = 50$	M1 A1	M1 for correct formula with $n = 24$ or $n = 25$
	Total:	4	
4(b)	$a = 2000, r = 1.025$	B1	$r = 1 + 2.5\%$ ok if used correctly in S_n formula
	$S_{10} = 2000\left(\frac{1.025^{10} - 1}{1.025 - 1}\right) = 22400$ or a value which rounds to this	M1 A1	M1 for correct formula with $n = 9$ or $n = 10$ and their a and r
			SR: correct answer only for $n = 10$ B3 , for $n = 9$, B1 (£19 900)
	Total:	3	

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Question	Answer	Marks	Guidance
5	$y = 2\cos x$		
5(i)		B1	One whole cycle – starts and finishes at –ve value
		DB1	Smooth curve, flattens at ends and middle. Shows (0, 2).
	Total:	2	
5(ii)	$P(\frac{\pi}{3}, 1) Q(\pi, -2)$		
	$\rightarrow PQ^2 = \left(\frac{2\pi}{3}\right)^2 + 3^2 \rightarrow PQ = 3.7$	M1 A1	Pythagoras (on their coordinates) must be correct, OE.
	Total:	2	

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Question	Answer	Marks	Guidance
5(iii)	Eqn of PQ $y - 1 = -\frac{9}{2\pi}\left(x - \frac{\pi}{3}\right)$	M1	Correct form of line equation or sim equations from their P & Q
	If $y = 0 \rightarrow h = \frac{5\pi}{9}$	A1	AG, condone $x = \frac{5\pi}{9}$
	If $x = 0 \rightarrow k = \frac{5}{2},$	A1	SR: non-exact solutions A1 for both
	Total:	3	
6(i)	Volume = $\left(\frac{1}{2}\right)x^2 \frac{\sqrt{3}}{2}h = 2000 \rightarrow h = \frac{8000}{\sqrt{3}x^2}$	M1	Use of (area of triangle, with attempt at ht) $\times h = 2000$, $h = f(x)$
	$A = 3xh + (2) \times \left(\frac{1}{2}\right) \times x^2 \times \frac{\sqrt{3}}{2}$	M1	Uses 3 rectangles and at least one triangle
	Sub for $h \rightarrow A = \frac{\sqrt{3}}{2}x^2 + \frac{24000}{\sqrt{3}}x^{-1}$	A1	AG
	Total:	3	
6(ii)	$\frac{dA}{dx} = \frac{\sqrt{3}}{2}2x - \frac{24000}{\sqrt{3}}x^{-2}$	B1	CAO, allow decimal equivalent
	$= 0$ when $x^3 = 8000 \rightarrow x = 20$	M1 A1	Sets their $\frac{dA}{dx}$ to 0 and attempt to solve for x
	Total:	3	

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Question	Answer	Marks	Guidance
6(iii)	$\frac{d^2 A}{dx^2} = \frac{\sqrt{3}}{2} 2 + \frac{48000}{\sqrt{3}} x^{-3} > 0$	M1	Any valid method, ignore value of $\frac{d^2 A}{dx^2}$ providing it is positive
	→ Minimum	A1 FT	FT on their x providing it is positive
	Total:	2	
7	$\frac{dy}{dx} = 7 - x^2 - 6x$		
7(i)	$y = 7x - \frac{x^3}{3} - \frac{6x^2}{2} (+c)$	B1	CAO
	Uses $(3, -10) \rightarrow c = 5$	M1 A1	Uses the given point to find c
	Total:	3	
7(ii)	$7 - x^2 - 6x = 16 - (x + 3)^2$	B1 B1	B1 $a = 16$, B1 $b = 3$.
	Total:	2	
7(iii)	$16 - (x + 3)^2 > 0 \rightarrow (x + 3)^2 < 16$, and solve	M1	or factors $(x + 7)(x - 1)$
	End-points $x = 1$ or -7	A1	
	→ $-7 < x < 1$	A1	needs $<$, not $\leq$. (SR $x < 1$ only, or $x > -7$ only B1 i.e. 1/3)
	Total:	3	

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Question	Answer	Marks	Guidance
8(i)	Letting M be midpoint of AB		
	$OM = 8$ (Pythagoras) $\rightarrow XM = 2$	B1	(could find $\sqrt{40}$ and use $\sin^{-1}$ or $\cos^{-1}$)
	$\tan AXM = \frac{6}{2}$ $AXB = 2\tan^{-1}3 = 2.498$	M1 A1	AG Needs $\times 2$ and correct trig for M1
	(Alternative 1: $\sin AOM = \frac{6}{10}$, $AOM = 0.6435$, $AXB = \pi - 0.6435$)		(Alternative 1: Use of isosceles triangles, B1 for AOM, M1,A1 for completion) (Alternative 2: Use of circle theorem, B1 for AOB, M1,A1 for completion)
	Total:	3	
8(ii)	$AX = \sqrt{6^2 + 2^2} = \sqrt{40}$	B1	CAO, could be gained in part (i) or part (iii)
	$\text{Arc } AYB = r\theta = \sqrt{40} \times 2.498$	M1	Allow for incorrect $\sqrt{40}$ (not $r = 6$ or 12 or 10)
	Perimeter = $12 + \text{arc} = 27.8$ cm	A1	
	Total:	3	
8(iii)	area of sector $AXBY = \frac{1}{2} \times (\sqrt{40})^2 \times 2.498$	M1	Use of $\frac{1}{2}r^2\theta$ with their r , (not $r = 6$ or $r = 10$)
	Area of triangle $AXB = \frac{1}{2} \times 12 \times 2$, Subtract these $\rightarrow 38.0$ cm ²	M1 A1	Use of $\frac{1}{2}bh$ and subtraction. Could gain M1 with $r = 10$.
	Total:	3	

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Question	Answer	Marks	Guidance
9	$f: x \mapsto \frac{2}{3-2x}$ $g: x \mapsto 4x + a$,		
9(i)	$y = \frac{2}{3-2x} \rightarrow y(3-2x) = 2 \rightarrow 3-2x = \frac{2}{y}$	M1	Correct first 2 steps
	$\rightarrow 2x = 3 - \frac{2}{y} \rightarrow f^{-1}(x) = \frac{3}{2} - \frac{1}{x}$	M1 A1	Correct order of operations, any correct form with $f(x)$ or $y =$
	Total:	3	
9(ii)	$gf(-1) = 3$ $f(-1) = \frac{2}{5}$	M1	Correct first step
	$\frac{8}{5} + a = 3 \rightarrow a = \frac{7}{5}$	M1 A1	Forms an equation in a and finds a , OE
			(or $\frac{8}{3-2x} + a = 3$, M1 Sub and solves M1 , A1)
	Total:	3	
9(iii)	$g^{-1}(x) = \frac{x-a}{4} = f^{-1}(x)$	M1	Finding $g^{-1}(x)$ and equating to their $f^{-1}(x)$ even if $a = 7/5$
	$\rightarrow x^2 - x(a+6) + 4 (= 0)$	M1	Use of $b^2 - 4ac$ on a quadratic with a in a coefficient
	Solving $(a+6)^2 = 16$ or $a^2 + 12a + 20 (= 0)$	M1	Solution of a 3 term quadratic
	$\rightarrow a = -2$ or -10	A1	
	Total:	4	

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Question	Answer	Marks	Guidance
10(i)	$\frac{dy}{dx} = \frac{-4}{(5-3x)^2} \times (-3)$	B1 B1	B1 without $\times(-3)$ B1 For $\times(-3)$
	Gradient of tangent = 3, Gradient of normal = $-\frac{1}{3}$	*M1	Use of $m_1 m_2 = -1$ after calculus
	$\rightarrow \text{eqn: } y - 2 = -\frac{1}{3}(x - 1)$	DM1	Correct form of equation, with (1, their y), not (1,0)
	$\rightarrow y = -\frac{1}{3}x + \frac{7}{3}$	A1	This mark needs to have come from $y = 2$, y must be subject
	Total:	5	
10(ii)	$\text{Vol} = \pi \int_0^1 \frac{16}{(5-3x)^2} dx$	M1	Use of $V = \pi \int y^2 dx$ with an attempt at integration
	$\pi \left[\frac{-16}{(5-3x)} \div -3 \right]$	A1 A1	A1 without $(\div -3)$, A1 for $(\div -3)$
	$= \left(\pi \left(\frac{16}{6} - \frac{16}{15} \right) \right) = \frac{8\pi}{5}$ (if limits switched must show – to +)	M1 A1	Use of both correct limits M1
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1(i)	Coefficient of $x = 80(x)$	B2	Correct value must be selected for both marks. SR +80 seen in an expansion gets B1 or –80 gets B1 <u>if selected</u> .
	Total:	2	
1(ii)	Coefficient of $\frac{1}{x} = -40 \left(\frac{1}{x} \right)$	B2	Correct value soi in (ii), if powers unsimplified only allow if selected. SR +40 soi in (ii) gets B1 .
	Coefficient of $x = (1 \times \text{their } 80) + (3 \times \text{their } -40) = -40(x)$	M1 A1	Links the appropriate 2 terms only for M1 .
	Total:	4	
2(i)	Gradient = 1.5 Gradient of perpendicular = $-\frac{2}{3}$	B1	
	Equation of AB is $y - 6 = -\frac{2}{3}(x + 2)$ Or $3y + 2x = 14$ oe	M1 A1	Correct use of straight line equation with a changed gradient and $(-2, 6)$, the $(-(-2))$ must be resolved for the A1 ISW.
			Using $y = mx + c$ gets A1 as soon as c is evaluated.
	Total:	3	
2(ii)	Simultaneous equations $\rightarrow$ Midpoint $(1, 4)$	M1	Attempt at solution of simultaneous equations as far as $x =$, or $y =$.
	Use of midpoint or vectors $\rightarrow B(4, 2)$	M1A1	Any valid method leading to x , or to y .
	Total:	3	

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3(i)	$\text{LHS} = \left(\frac{1}{c} - \frac{s}{c} \right)^2$	M1	Eliminates tan by replacing with $\frac{\sin}{\cos}$ leading to a function of sin and/or cos only.
	$= \frac{(1-s)^2}{1-s^2}$	M1	Uses $s^2 + c^2 = 1$ leading to a function of sin only.
	$= \frac{(1-s)(1-s)}{(1-s)(1+s)} = \frac{1-\sin\theta}{1+\sin\theta}$	A1	AG. Must show use of factors for A1 .
	Total:	3	
3(ii)	Uses part (i) $\rightarrow 2 - 2s = 1 + s$		
	$\rightarrow s = \frac{1}{3}$	M1	Uses part (i) to obtain $s = k$
	$\theta = 19.5^\circ$ or 160.5°	A1A1 FT	FT from error in 19.5° Allow 0.340° (0.3398°) & $2.80(2)$ or $0.108\pi^\circ$ & $0.892\pi^\circ$ for A1 only. Extra answers in the range lose the second A1 if gained for 160.5° .
	Total:	3	
4(i)	$(AB) = 2r\sin\theta \text{ (or } r\sqrt{2-2\cos 2\theta} \text{ or } \frac{r\sin 2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)})$	B1	Allow unsimplified throughout eg $r + r$, $\frac{2\theta}{2}$ etc
	$(\text{Arc } AB) = 2r\theta$	B1	
	$(P) = 2r + 2r\theta + 2r\sin\theta \text{ (or } r\sqrt{2-2\cos 2\theta} \text{ or } \frac{r\sin 2\theta}{\sin\left(\frac{\pi}{2}-\theta\right)})$	B1	
	Total:	3	

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Question	Answer	Marks	Guidance
4(ii)	Area sector $AOB = (\frac{1}{2} r^2 2\theta) \frac{25\pi}{6}$ or 13.1	B1	Use of segment formula gives 2.26 B1B1
	Area triangle $AOB = (\frac{1}{2} \times 2r \sin \theta \times r \cos \theta$ or $\frac{1}{2} \times r^2 \sin 2\theta$) $\frac{25\sqrt{3}}{4}$ or 10.8	B1	
	Area rectangle $ABCD = (r \times 2r \sin \theta) 25$	B1	
	(Area =) Either $25 - (25\pi/6 - 25\sqrt{3}/4)$ or 22.7	B1	Correct final answer gets B4 .
	Total:	4	
5(i)	Crosses x -axis at (6, 0)	B1	$x = 6$ is sufficient.
	$\frac{dy}{dx} = (0+) -12(2-x)^{-2} \times (-1)$	B2,1,0	–1 for each incorrect term of the three or addition of + C.
	Tangent $y = \frac{3}{4}(x-6)$ or $4y = 3x - 18$	M1 A1	Must use dy/dx , $x =$ their 6 but not $x = 0$ (which gives $m = 3$), and correct form of line equation.
			Using $y = mx + c$ gets A1 as soon as c is evaluated.
	Total:	5	
5(ii)	If $x = 4$, $dy/dx = 3$		
	$\frac{dy}{dt} = 3 \times 0.04 = 0.12$	M1 A1FT	M1 for (“their m ” from $\frac{dy}{dx}$ and $x = 4$) $\times 0.04$. Be aware: use of $x = 0$ gives the correct answer but gets M0 .
	Total:	2	

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Question	Answer	Marks	Guidance
6	$\text{Vol} = \pi \int (5-x)^2 dx - \pi \int \frac{16}{x^2} dx$	M1*	Use of volume formula at least once, condone omission of π and limits and dx .
		DM1	Subtracting volumes somewhere must be <u>after</u> squaring.
	$\int (5-x)^2 dx = \frac{(5-x)^3}{3} \div -1$	B1 B1	B1 Without $\div (-1)$. B1 for $\div (-1)$
	(or $25x - 10x^2/2 + 1/3x^3$)	(B2,1,0)	-1 for each incorrect term
	$\int \frac{16}{x^2} dx = -\frac{16}{x}$	B1	
	Use of limits 1 and 4 in an integrated expression and subtracted.	DM1	Must have used “ y^2 ” at least once. Need to see values substituted.
	$\rightarrow 9\pi$ or 28.3	A1	
	Total:	7	
7(a)	$(S_n =) \frac{n}{2} [32 + (n-1)8]$ and 20000	M1	M1 correct formula used with d from $16 + d = 24$
		A1	A1 for correct expression linked to 20000.
	$\rightarrow n^2 + 3n - 5000 (<, =, > 0)$	DM1	Simplification to a three term quadratic.
	$\rightarrow (n = 69.2) \rightarrow 70$ terms needed.	A1	Condone use of 20001 throughout. Correct answer from trial and improvement gets 4/4.
	Total:	4	

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Question	Answer	Marks	Guidance
7(b)	$a = 6, \frac{a}{1-r} = 18 \rightarrow r = \frac{2}{3}$	M1A1	Correct S_{∞} formula used to find r .
	New progression $a = 36, r = \frac{4}{9}$ oe	M1	Obtain new values for a and r by any valid method.
	New $S_{\infty} = \frac{36}{1-\frac{4}{9}} \rightarrow 64.8$ or $\frac{324}{5}$ oe	A1	(Be aware that $r = -\frac{2}{3}$ leads to 64.8 but can only score M marks)
	Total:	4	
8(i)	Uses scalar product correctly: $3 \times 6 + 2 \times 6 + (-4) \times 3 = 18$	M1	Use of dot product with $\overrightarrow{OA}$ or $\overrightarrow{AO}$ & $\overrightarrow{OB}$ or $\overrightarrow{BO}$ only.
	$ \overrightarrow{OA} = \sqrt{29}, \overrightarrow{OB} = 9$	M1	Correct method for any one of $ \overrightarrow{OA} , \overrightarrow{AO} , \overrightarrow{OB} $ or $ \overrightarrow{BO} $.
	$\sqrt{29} \times 9 \times \cos AOB = 18$	M1	All linked correctly.
	$\rightarrow AOB = 68.2^{\circ}$ or 1.19°	A1	Multiples of π are acceptable (e.g. $0.379\pi^{\circ}$)
	Total:	4	
8(ii)	$\overrightarrow{AB} = 3\mathbf{i} + 4\mathbf{j} + (3+2p)\mathbf{k}$	*M1	For use of $\overrightarrow{OB} - \overrightarrow{OA}$, allow with $p = 2$
	Comparing “j”	DM1	For comparing, $\overrightarrow{OC}$ must contain p & q . Can be implied by $\overrightarrow{AB} = 2\overrightarrow{OC}$.
	$\rightarrow p = 2\frac{1}{2}$ and $q = 4$	A1 A1	Accuracy marks only available if $\overrightarrow{AB}$ is correct.
	Total:	4	

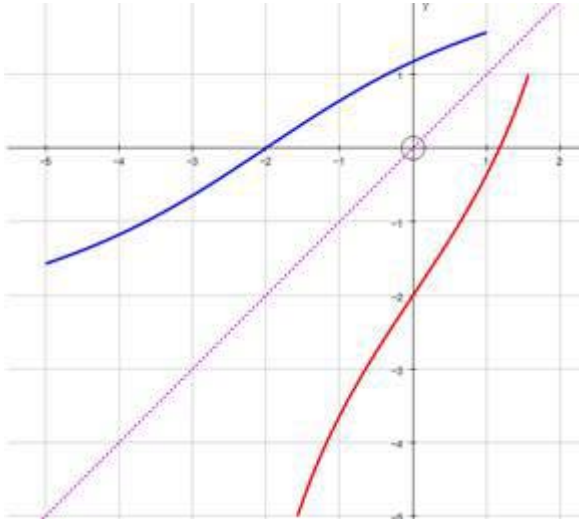
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Question	Answer	Marks	Guidance
9(i)	$\frac{dy}{dx} = 4x^{-\frac{1}{2}} - 2$	B1	Accept unsimplified.
	$= 0$ when $\sqrt{x} = 2$		
	$x = 4, y = 8$	B1B1	
	Total:	3	
9(ii)	$\frac{d^2y}{dx^2} = -2x^{-\frac{3}{2}}$	B1FT	FT providing –ve power of x
	$\left(\frac{d^2y}{dx^2} = -\frac{1}{4}\right) \rightarrow \text{Maximum}$	B1	Correct $\frac{d^2y}{dx^2}$ and $x=4$ in (i) are required. Followed by “< 0 or negative” is sufficient” but $\frac{d^2y}{dx^2}$ must be correct if evaluated.
	Total:	2	
9(iii)	<i>EITHER:</i> Recognises a quadratic in $\sqrt{x}$	(M1)	Eg $\sqrt{x} = u \rightarrow 2u^2 - 8u + 6 = 0$
	1 and 3 as solutions to this equation	A1	
	$\rightarrow x = 9, x = 1.$	A1)	

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Question	Answer	Marks	Guidance
	OR: Rearranges then squares	(M1)	$\sqrt{x}$ needs to be isolated before squaring both sides.
	$\rightarrow x^2 - 10x + 9 = 0$ oe	A1	
	$\rightarrow x = 9, x = 1.$	A1)	Both correct by trial and improvement gets 3/3
	Total:	3	
9(iv)	$k > 8$	B1	
	Total:	1	
10(i)	$3 \tan\left(\frac{1}{2}x\right) = -2 \rightarrow \tan\left(\frac{1}{2}x\right) = -\frac{2}{3}$	M1	Attempt to obtain $\tan\left(\frac{1}{2}x\right) = k$ from $3 \tan\left(\frac{1}{2}x\right) + 2 = 0$
	$\frac{1}{2}x = -0.6$ (-0.588) $\rightarrow x = -1.2$	M1 A1	$\tan^{-1} k$. Seeing $\frac{1}{2}x = -33.69^\circ$ or $x = -67.4^\circ$ implies M1M1 .
			Extra answers between -1.57 & 1.57 lose the A1 . Multiples of π are acceptable (eg -0.374π)
	Total:	3	
10(ii)	$\frac{y+2}{3} = \tan\left(\frac{1}{2}x\right)$	M1	Attempt at isolating $\tan(\frac{1}{2}x)$
	$\rightarrow f^{-1}(x) = 2 \tan^{-1}\left(\frac{x+2}{3}\right)$	M1 A1	Inverse tan followed by $\times 2$. Must be function of x for A1 .
	$-5, 1$	B1 B1	Values stated B1 for -5, B1 for 1.
	Total:	5	

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Question	Answer	Marks	Guidance
10(iii)		B1 B1 B1	A tan graph through the first, third and fourth quadrants. (B1) An invtan graph through the first, second and third quadrants. (B1) Two curves clearly symmetrical about $y = x$ either by sight or by exact end points. Line not required. Approximately in correct domain and range. (Not intersecting.) (B1) Labels on axes not required.
	Total:	3	

MATHEMATICS

9709/13

Paper 1

May/June 2017

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2017 series for most Cambridge IGCSE[®], Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF/OE Any Equivalent Form (of answer is equally acceptable) / Or Equivalent

AG Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)

CAO Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)

CWO Correct Working Only – often written by a ‘fortuitous’ answer

ISW Ignore Subsequent Working

SOI Seen or implied

SR Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1 A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.

PA –1 This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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Question	Answer	Marks	Guidance
1	$7C1 \times 2^6 \times a(x), 7C2 \times 2^5 \times [a(x)]^2$	B1 B1	SOI Can be part of expansion. Condone ax^2 only if followed by a^2 . ALT $2^7 [1 + ax/2]^7 \rightarrow 7C1 [a(x)/2] = 7C2 [a(x)/2]^2$
	$a = \frac{7 \times 2^6}{21 \times 2^5} = \frac{2}{3}$	B1	Ignore extra soln $a = 0$. Allow $a = 0.667$. Do not allow an extra x in the answer
	Total:	3	

Question	Answer	Marks	Guidance
2(i)	$S = \frac{r^2 - 3r + 2}{1 - r}$	M1	
	$S = \frac{(r-1)(r-2)}{1-r} = \frac{-(1-r)(r-2)}{1-r} = 2-r$ OR $\frac{(1-r)(2-r)}{1-r} = 2-r$ OE	A1	AG Factors must be shown. Expressions requiring minus sign taken out must be shown
	Total:	2	
2(ii)	Single range $1 < S < 3$ or $(1, 3)$	B2	Accept $1 < 2 - r < 3$. Correct range but with $S = 2$ omitted scores SR B1 $1 \leq S \leq 3$ scores SR B1 . $[S > 1 \text{ and } S < 3]$ scores SR B1 .
	Total:	2	

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Question	Answer	Marks	Guidance
3	EITHER Elim y to form 3-term quad eqn in $x^{1/3}$ (or u or y or even x)	(M1)	Expect $x^{2/3} - x^{1/3} - 2 (= 0)$ or $u^2 - u - 2 (= 0)$ etc.
	$x^{1/3}$ (or u or y or x) = 2, -1	*A1	Both required. But <u>x</u> = 2, -1 and not then cubed or cube rooted scores A0
	Cube solution(s)	DM1	Expect $x = 8, -1$. Both required
	(8, 3), (-1, 0)	A1)	
	OR Elim x to form quadratic equation in y	(M1)	Expect $y + 1 = (y - 1)^2$
	$y^2 - 3y = 0$	*A1	
	Attempt solution	DM1	Expect $y = 3, 0$
	(8, 3), (-1, 0)	A1)	
	Total:	4	

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Question	Answer	Marks	Guidance
4(i)	$\overrightarrow{OB} - \overrightarrow{OA} (= \overrightarrow{AB}) = \begin{pmatrix} 5 \\ 4 \\ -3 \end{pmatrix} - \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix}$	B1	
	$\overrightarrow{OP} = \begin{pmatrix} 5 \\ 1 \\ 3 \end{pmatrix} + \frac{1}{3} \begin{pmatrix} 0 \\ 3 \\ -6 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 1 \end{pmatrix}$	M1 A1	If $\overrightarrow{OP}$ not scored in (i) can score SR B1 if seen correct in (ii). Other equivalent methods possible
	Total:	3	
4(ii)	Distance $OP = \sqrt{5^2 + 2^2 + 1^2} = \sqrt{30}$ or 5.48	B1 FT	FT on <i>their</i> $\overrightarrow{OP}$ from (i)
	Total:	1	
4(iii)	Attempt $\overrightarrow{AB} \cdot \overrightarrow{OP}$. Can score as part of $\overrightarrow{AB} \cdot \overrightarrow{OP} = (AB)(OP)\cos\theta$ Rare ALT: Pythagoras $ \overrightarrow{OP} ^2 + \overrightarrow{AP} ^2 = 5 + 30 = \overrightarrow{OA} ^2$	M1	Allow any combination of $\overrightarrow{AB} \cdot \overrightarrow{PO}$ etc. and also if $\overrightarrow{AP}$ or $\overrightarrow{PB}$ used instead of $\overrightarrow{AB}$ giving $2-2 = 0$ & $4-4 = 0$ respectively. Allow notation $\times$ instead of $\cdot$.
	$(0 + 6 - 6) = 0$ hence perpendicular. (Accept 90°)	A1 FT	If result not zero then 'Not perpendicular' can score A1FT if value is 'correct' for <i>their</i> values of $\overrightarrow{AB}, \overrightarrow{OP}$ etc. from (i).
	Total:	2	

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Question	Answer	Marks	Guidance
5(i)	$\frac{2\sin\theta + \cos\theta}{\sin\theta + \cos\theta} = \frac{2\sin\theta}{\cos\theta}$	M1	Replace $\tan\theta$ by $\sin\theta / \cos\theta$
	$2\sin\theta\cos\theta + \cos^2\theta = 2\sin^2\theta + 2\sin\theta\cos\theta \Rightarrow c^2 = 2s^2$	M1 A1	Mult by $c(s + c)$ or making this a common denom.. For A1 simplification to AG without error or omission must be seen.
	Total:	3	
5(ii)	$\tan^2\theta = 1/2$ or $\cos^2\theta = 2/3$ or $\sin^2\theta = 1/3$	B1	Use $\tan\theta = s/c$ or $c^2 + s^2 = 1$ and simplify to one of these results
	$\theta = 35.3^\circ$ or 144.7°	B1 B1 FT	FT for 180 – other solution. SR B1 for radians 0.615, 2.53 (0.196 π , 0.804 π) Extra solutions in range amongst solutions of which 2 are correct gets B1B0
	Total:	3	

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Question	Answer	Marks	Guidance
6	Gradient of normal is $-1/3 \rightarrow$ gradient of tangent is 3 SOI	B1 B1 FT	FT from <i>their</i> gradient of normal.
	$dy/dx = 2x - 5 = 3$	M1	Differentiate and set = <i>their</i> 3 (numerical).
	$x = 4$	*A1	
	Sub $x = 4$ into line $\rightarrow y = 7$ & sub <i>their</i> (4, 7) into curve	DM1	OR sub $x = 4$ into curve $\rightarrow y = k - 4$ and sub <i>their</i> (4, $k - 4$) into line OR other valid methods deriving a linear equation in k (e.g. equating curve with either normal or tangent and sub $x = 4$).
	$k = 11$	A1	
	Total:	6	

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Question	Answer	Marks	Guidance
7(i)	$\sin ABC = 8/10 \rightarrow ABC = 0.927(3)$	B1	Or $\cos = 6/10$ or $\tan = 8/6$. Accept 0.295π .
	Total:	1	
7(ii)	$AB = 6$ (Pythagoras) $\rightarrow \Delta BCD = 8 \times 6 = 48.0$	M1A1	OR $8 \times 10 \sin 0.6435$ or $\frac{1}{2} \times 10 \times 10 \sin((2) \times 0.927) = 48.24$ or 40 or 80 gets M1A0
	Area sector $BCD = \frac{1}{2} \times 10^2 \times (2) \times \text{their } 0.9273$	*M1	Expect $92.7(3)$. 46.4 gets M1
	Area segment = $92.7(3) - 48$	*A1	Expect $44.7(3)$. Might not appear until final calculation.
	Area semi-circle – segment = $\frac{1}{2} \times \pi \times 8^2 - \text{their}(92.7 - 48)$	DM1	Dep. on previous M1A1 OR $\pi \times 8^2 - (\frac{1}{2} \times \pi \times 8^2 + \text{their } 44.7)$.
	Shaded area = $55.8 - 56.0$	A1	
	Total:	6	

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Question	Answer	Marks	Guidance
8(i)	$(b-1)/(a+1)=2$	M1	OR Equation of AP is $y-1=2(x+1) \rightarrow y=2x+3$
	$b=2a+3$ CAO	A1	Sub $x=a, y=b \rightarrow b=2a+3$
	Total:	2	
8(ii)	$AB^2 = 11^2 + 2^2 = 125$ oe	B1	Accept $AB = \sqrt{125}$
	$(a+1)^2 + (b-1)^2 = 125$	B1 FT	FT on <i>their</i> 125.
	$(a+1)^2 + (2a+2)^2 = 125$	M1	Sub from part (i) $\rightarrow$ quadratic eqn in a (or possibly in $b \rightarrow b^2 - 2b - 99 = 0$)
	$(5)(a^2 + 2a - 24) = 0 \rightarrow \text{eg } (a-4)(a+6) = 0$	M1	Simplify and attempt to solve
	$a = 4$ or -6	A1	
	$b = 11$ or -9	A1	OR (4, 11), (-6, -9) If A0A0 , SR1 for either (4, 11) or (-6, -9)
	Total:	6	

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Question	Answer	Marks	Guidance
9(i)	$(3x-1)^2 + 5$	B1B1B1	First 2 marks dependent on correct $(ax+b)^2$ form. OR $a=3$, $b=-1$, $c=5$ e.g. from equating coefs
	Total:	3	
9(ii)	Smallest value of p is $1/3$ seen. (Independent of (i))	B1	Allow $p \geq 1/3$ or $p = 1/3$ or $1/3$ seen. But not in terms of x .
	Total:	1	
9(iii)	$y = (3x-1)^2 + 5 \Rightarrow 3x-1 = (\pm)\sqrt{y-5}$	B1 FT	OR $y = 9\left(x - \frac{1}{3}\right)^2 + 5 \Rightarrow (y-5)/9 = \left(x - \frac{1}{3}\right)^2$ (Fresh start)
	$x = (\pm)^{1/3}\sqrt{y-5} + 1/3$ OE	B1 FT	Both starts require 2 operations for each mark. FT for <i>their</i> values from part (i)
	$f^{-1}(x) = 1/3\sqrt{x-5} + 1/3$ OE domain is $x \geq \text{their}5$	B1B1 FT	Must be a function of x and $\pm$ removed. Domain must be in terms of x . Note: $\sqrt{y-5}$ expressed as $\sqrt{y} - \sqrt{5}$ scores Max B0B0B0B1 [See below for general instructions for different starts]
	Total:	4	
9(iv)	$q < 5$ CAO	B1	
	Total:	1	
Alt 9(iii) For start $(ax-b)^2 + c$ or $a(x-b)^2 + c$ ($a \neq 0$) ft for their a, b, c For start $(x-b)^2 + c$ ft but award only B1 for 3 correct operations For start $a(bx-c)^2 + d$ ft but award B1 for first2 operations correct and B1 for the next 3 operations correct			

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Question	Answer	Marks	Guidance
10(a)(i)	Attempt to integrate $V = (\pi) \int (y+1) dy$	M1	Use of h in integral e.g. $\int (h+1) = \frac{1}{2}h^2 + h$ is M0 . Use of $\int y^2 dx$ is M0
	$= (\pi) \left[\frac{y^2}{2} + y \right]$	A1	
	$= \pi \left[\frac{h^2}{2} + h \right]$	A1	AG . Must be from clear use of limits $0 \rightarrow h$ somewhere.
	Total:	3	
10(ii)	$\int (y+1)^{1/2} dy$ ALT $6 - \int (x^2 - 1) dx$	M1	Correct variable and attempt to integrate
	$\frac{2}{3} (y+1)^{3/2}$ oe ALT $6 - (\frac{1}{3}x^3 - x)$ CAO	*A1	Result of integration must be shown
	$\frac{2}{3} [8-1]$ ALT $6 - \left[\left(\frac{8}{3} - 1 \right) - \left(\frac{1}{3} - 1 \right) \right]$	DM1	Calculation seen with limits $0 \rightarrow 3$ for y . For ALT, limits are $1 \rightarrow 2$ and rectangle.
	14/3 ALT $6 - 4/3 = 14/3$	A1	16/3 from $\frac{2}{3} \times 8$ gets DM1A0 provided work is correct up to applying limits.
	Total:	4	

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Question	Answer	Marks	Guidance
10(b)	Clear attempt to differentiate wrt h	M1	Expect $\frac{dV}{dh} = \pi(h+1)$. Allow $h+1$. Allow h .
	Derivative = 4π SOI	*A1	
	$\frac{2}{\text{their derivative}}$. Can be in terms of h	DM1	
	$\frac{2}{4\pi}$ or $\frac{1}{2\pi}$ or 0.159	A1	
	Total:	4	

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Question	Answer	Marks	Guidance
11(i)	$f'(x) = [(4x+1)^{1/2} \div \frac{1}{2}] [\div 4] (+c)$	B1 B1	Expect $\frac{1}{2}(4x+1)^{1/2} (+c)$
	$f'(2) = 0 \Rightarrow \frac{3}{2} + c = 0 \Rightarrow c = -\frac{3}{2}$ (Sufficient)	B1 FT	Expect $\frac{1}{2}(4x+1)^{1/2} - \frac{3}{2}$. FT on <i>their</i> $f'(x) = k(4x+1)^{1/2} + c$. (i.e. $c = -3k$)
	Total:	3	
11(ii)	$f''(0) = 1$ SOI	B1	
	$f'(0) = 1/2 - 1\frac{1}{2} = -1$ SOI	B1 FT	Substitute $x = 0$ into <i>their</i> $f'(x)$ but must not involve c otherwise B0B0
	$f(0) = -3$	B1 FT	FT for 3 terms in AP. FT for 3rd B1 dep on 1st B1 . Award marks for the AP method only.
	Total:	3	
11(iii)	$f(x) = \left[\frac{1}{2}(4x+1)^{3/2} \div 3/2 \div 4 \right] - [1\frac{1}{2}x] (+k)$	B1 FT B1 FT	Expect $(1/12)(4x+1)^{3/2} - 1\frac{1}{2}x (+k)$. FT from <i>their</i> $f'(x)$ but c numerical.
	$-3 = 1/12 - 0 + k \Rightarrow k = -37/12$ CAO	M1A1	Sub $x = 0, y = \text{their } f(0)$ into <i>their</i> $f(x)$. Dep on cx & k present (c numerical)
	Minimum value = $f(2) = \frac{27}{12} - 3 - \frac{37}{12} = -\frac{23}{6}$ or -3.83	A1	
	Total:	5	