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**MATHEMATICS**

**9709/11**

Paper 1 Pure Mathematics 1 (P1)

**May/June 2016**

**1 hour 45 minutes**

Additional Materials: List of Formulae (MF9)

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**READ THESE INSTRUCTIONS FIRST**

An answer booklet is provided inside this question paper. You should follow the instructions on the front cover of the answer booklet. If you need additional answer paper ask the invigilator for a continuation booklet.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [ ] at the end of each question or part question.

The total number of marks for this paper is 75.

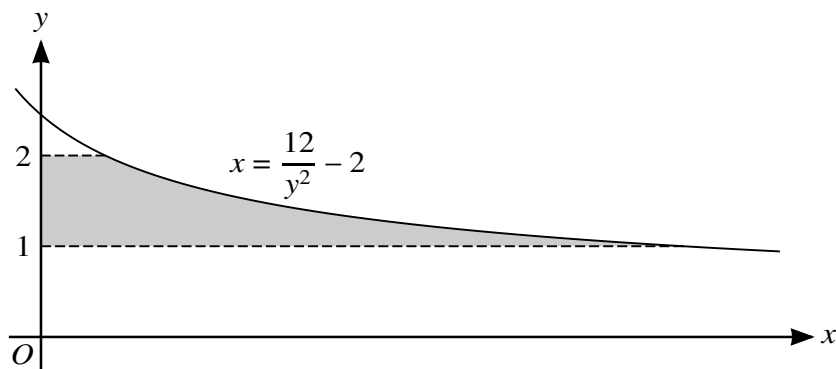
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This document consists of **4** printed pages and **1** insert.

- 1 Find the term independent of  $x$  in the expansion of  $\left(x - \frac{3}{2x}\right)^6$ . [3]

- 2 Solve the equation  $3 \sin^2 \theta = 4 \cos \theta - 1$  for  $0^\circ \leq \theta \leq 360^\circ$ . [4]

3



The diagram shows part of the curve  $x = \frac{12}{y^2} - 2$ . The shaded region is bounded by the curve, the  $y$ -axis and the lines  $y = 1$  and  $y = 2$ . Showing all necessary working, find the volume, in terms of  $\pi$ , when this shaded region is rotated through  $360^\circ$  about the  $y$ -axis. [5]

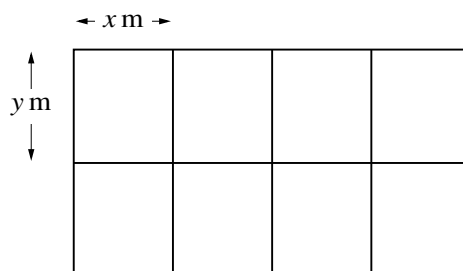
- 4 A curve is such that  $\frac{dy}{dx} = 2 - 8(3x + 4)^{-\frac{1}{2}}$ .

- (i) A point  $P$  moves along the curve in such a way that the  $x$ -coordinate is increasing at a constant rate of 0.3 units per second. Find the rate of change of the  $y$ -coordinate as  $P$  crosses the  $y$ -axis. [2]

The curve intersects the  $y$ -axis where  $y = \frac{4}{3}$ .

- (ii) Find the equation of the curve. [4]

5



A farmer divides a rectangular piece of land into 8 equal-sized rectangular sheep pens as shown in the diagram. Each sheep pen measures  $x$  m by  $y$  m and is fully enclosed by metal fencing. The farmer uses 480 m of fencing.

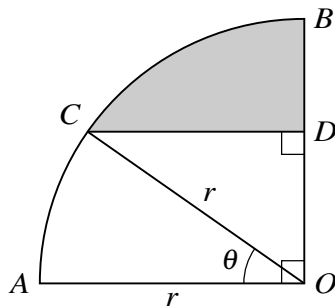
- (i) Show that the total area of land used for the sheep pens,  $A$  m<sup>2</sup>, is given by

$$A = 384x - 9.6x^2. \quad [3]$$

- (ii) Given that  $x$  and  $y$  can vary, find the dimensions of each sheep pen for which the value of  $A$  is a maximum. (There is no need to verify that the value of  $A$  is a maximum.) [3]

- 6 (a) Find the values of the constant  $m$  for which the line  $y = mx$  is a tangent to the curve  $y = 2x^2 - 4x + 8$ . [3]
- (b) The function  $f$  is defined for  $x \in \mathbb{R}$  by  $f(x) = x^2 + ax + b$ , where  $a$  and  $b$  are constants. The solutions of the equation  $f(x) = 0$  are  $x = 1$  and  $x = 9$ . Find
- (i) the values of  $a$  and  $b$ , [2]
- (ii) the coordinates of the vertex of the curve  $y = f(x)$ . [2]

7



In the diagram,  $AOB$  is a quarter circle with centre  $O$  and radius  $r$ . The point  $C$  lies on the arc  $AB$  and the point  $D$  lies on  $OB$ . The line  $CD$  is parallel to  $AO$  and angle  $AOC = \theta$  radians.

- (i) Express the perimeter of the shaded region in terms of  $r$ ,  $\theta$  and  $\pi$ . [4]
- (ii) For the case where  $r = 5$  cm and  $\theta = 0.6$ , find the area of the shaded region. [3]
- 8 A curve has equation  $y = 3x - \frac{4}{x}$  and passes through the points  $A(1, -1)$  and  $B(4, 11)$ . At each of the points  $C$  and  $D$  on the curve, the tangent is parallel to  $AB$ . Find the equation of the perpendicular bisector of  $CD$ . [7]
- 9 (a) The first term of a geometric progression in which all the terms are positive is 50. The third term is 32. Find the sum to infinity of the progression. [3]
- (b) The first three terms of an arithmetic progression are  $2 \sin x$ ,  $3 \cos x$  and  $(\sin x + 2 \cos x)$  respectively, where  $x$  is an acute angle.
- (i) Show that  $\tan x = \frac{4}{3}$ . [3]
- (ii) Find the sum of the first twenty terms of the progression. [3]

[Questions 10 and 11 are printed on the next page.]

**10** Relative to an origin  $O$ , the position vectors of points  $A$ ,  $B$  and  $C$  are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$$

respectively, where  $k$  is a constant.

(i) Find the value of  $k$  in the case where angle  $AOB = 90^\circ$ . [2]

(ii) Find the possible values of  $k$  for which the lengths of  $AB$  and  $OC$  are equal. [4]

The point  $D$  is such that  $\overrightarrow{OD}$  is in the same direction as  $\overrightarrow{OA}$  and has magnitude 9 units. The point  $E$  is such that  $\overrightarrow{OE}$  is in the same direction as  $\overrightarrow{OC}$  and has magnitude 14 units.

(iii) Find the magnitude of  $\overrightarrow{DE}$  in the form  $\sqrt{n}$  where  $n$  is an integer. [4]

**11** The function  $f$  is defined by  $f : x \mapsto 4 \sin x - 1$  for  $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$ .

(i) State the range of  $f$ . [2]

(ii) Find the coordinates of the points at which the curve  $y = f(x)$  intersects the coordinate axes. [3]

(iii) Sketch the graph of  $y = f(x)$ . [2]

(iv) Obtain an expression for  $f^{-1}(x)$ , stating both the domain and range of  $f^{-1}$ . [4]

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**MATHEMATICS**

**9709/12**

Paper 1 Pure Mathematics 1 (P1)

**May/June 2016**

**1 hour 45 minutes**

Additional Materials: List of Formulae (MF9)

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The total number of marks for this paper is 75.

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This document consists of **4** printed pages and **1** insert.

- 1 Functions  $f$  and  $g$  are defined by

$$f : x \mapsto 10 - 3x, \quad x \in \mathbb{R},$$

$$g : x \mapsto \frac{10}{3 - 2x}, \quad x \in \mathbb{R}, x \neq \frac{3}{2}.$$

Solve the equation  $ff(x) = gf(2)$ . [3]

- 2 A curve is such that  $\frac{dy}{dx} = \frac{8}{(5 - 2x)^2}$ . Given that the curve passes through  $(2, 7)$ , find the equation of the curve. [4]

- 3 Relative to an origin  $O$ , the position vectors of points  $A$  and  $B$  are given by

$$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$$

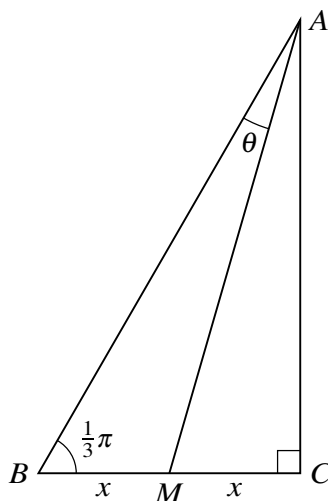
The point  $C$  is such that  $\overrightarrow{AB} = \overrightarrow{BC}$ . Find the unit vector in the direction of  $\overrightarrow{OC}$ . [4]

- 4 Find the term that is independent of  $x$  in the expansion of

(i)  $\left(x - \frac{2}{x}\right)^6$ , [2]

(ii)  $\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$ . [4]

5

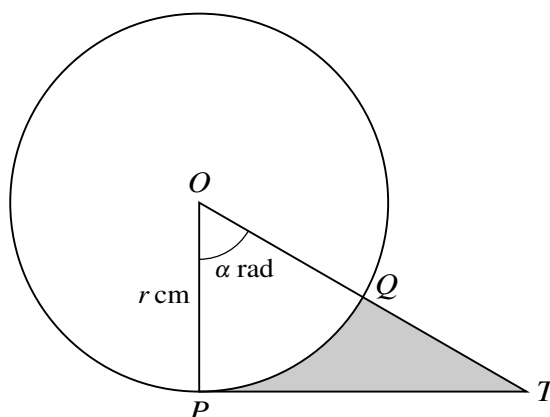


In the diagram, triangle  $ABC$  is right-angled at  $C$  and  $M$  is the mid-point of  $BC$ . It is given that angle  $ABC = \frac{1}{3}\pi$  radians and angle  $BAM = \theta$  radians. Denoting the lengths of  $BM$  and  $MC$  by  $x$ ,

(i) find  $AM$  in terms of  $x$ , [3]

(ii) show that  $\theta = \frac{1}{6}\pi - \tan^{-1}\left(\frac{1}{2\sqrt{3}}\right)$ . [2]

6



The diagram shows a circle with radius  $r$  cm and centre  $O$ . The line  $PT$  is the tangent to the circle at  $P$  and angle  $POT = \alpha$  radians. The line  $OT$  meets the circle at  $Q$ .

(i) Express the perimeter of the shaded region  $PQT$  in terms of  $r$  and  $\alpha$ . [3]

(ii) In the case where  $\alpha = \frac{1}{3}\pi$  and  $r = 10$ , find the area of the shaded region correct to 2 significant figures. [3]

7 (i) Prove the identity  $\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \equiv \frac{4}{\sin \theta \tan \theta}$ . [4]

(ii) Hence solve, for  $0^\circ < \theta < 360^\circ$ , the equation

$$\sin \theta \left( \frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \right) = 3. \quad [3]$$

8 Three points have coordinates  $A(0, 7)$ ,  $B(8, 3)$  and  $C(3k, k)$ . Find the value of the constant  $k$  for which

(i)  $C$  lies on the line that passes through  $A$  and  $B$ , [4]

(ii)  $C$  lies on the perpendicular bisector of  $AB$ . [4]

9 A water tank holds 2000 litres when full. A small hole in the base is gradually getting bigger so that each day a greater amount of water is lost.

(i) On the first day after filling, 10 litres of water are lost and this increases by 2 litres each day.

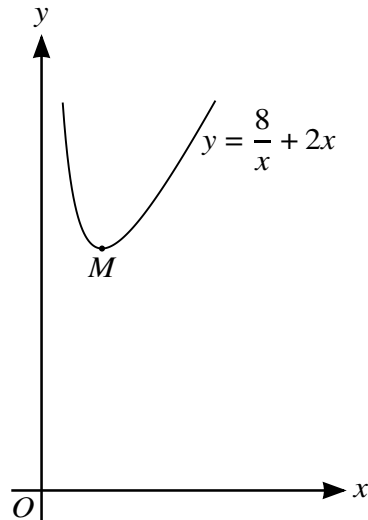
(a) How many litres will be lost on the 30th day after filling? [2]

(b) The tank becomes empty during the  $n$ th day after filling. Find the value of  $n$ . [3]

(ii) Assume instead that 10 litres of water are lost on the first day and that the amount of water lost increases by 10% on each succeeding day. Find what percentage of the original 2000 litres is left in the tank at the end of the 30th day after filling. [4]

[Questions 10 and 11 are printed on the next page.]

10



The diagram shows the part of the curve  $y = \frac{8}{x} + 2x$  for  $x > 0$ , and the minimum point  $M$ .

(i) Find expressions for  $\frac{dy}{dx}$ ,  $\frac{d^2y}{dx^2}$  and  $\int y^2 dx$ . [5]

(ii) Find the coordinates of  $M$  and determine the coordinates and nature of the stationary point on the part of the curve for which  $x < 0$ . [5]

(iii) Find the volume obtained when the region bounded by the curve, the  $x$ -axis and the lines  $x = 1$  and  $x = 2$  is rotated through  $360^\circ$  about the  $x$ -axis. [2]

11 The function  $f$  is defined by  $f : x \mapsto 6x - x^2 - 5$  for  $x \in \mathbb{R}$ .

(i) Find the set of values of  $x$  for which  $f(x) \leq 3$ . [3]

(ii) Given that the line  $y = mx + c$  is a tangent to the curve  $y = f(x)$ , show that  $4c = m^2 - 12m + 16$ . [3]

The function  $g$  is defined by  $g : x \mapsto 6x - x^2 - 5$  for  $x \geq k$ , where  $k$  is a constant.

(iii) Express  $6x - x^2 - 5$  in the form  $a - (x - b)^2$ , where  $a$  and  $b$  are constants. [2]

(iv) State the smallest value of  $k$  for which  $g$  has an inverse. [1]

(v) For this value of  $k$ , find an expression for  $g^{-1}(x)$ . [2]

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**MATHEMATICS**

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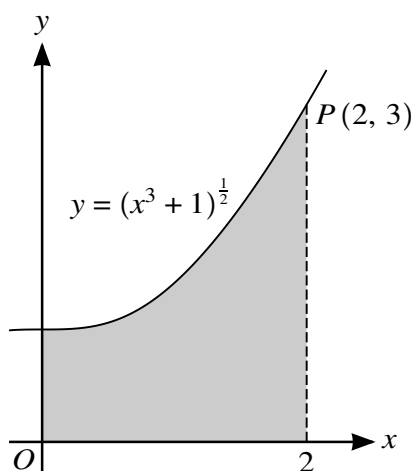
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This document consists of **4** printed pages and **1** insert.

- 1 Find the coefficient of  $x$  in the expansion of  $\left(\frac{1}{x} + 3x^2\right)^5$ . [3]

2



The diagram shows part of the curve  $y = (x^3 + 1)^{\frac{1}{2}}$  and the point  $P(2, 3)$  lying on the curve. Find, showing all necessary working, the volume obtained when the shaded region is rotated through  $360^\circ$  about the  $x$ -axis. [4]

- 3 A curve is such that  $\frac{dy}{dx} = 6x^2 + \frac{k}{x^3}$  and passes through the point  $P(1, 9)$ . The gradient of the curve at  $P$  is 2.

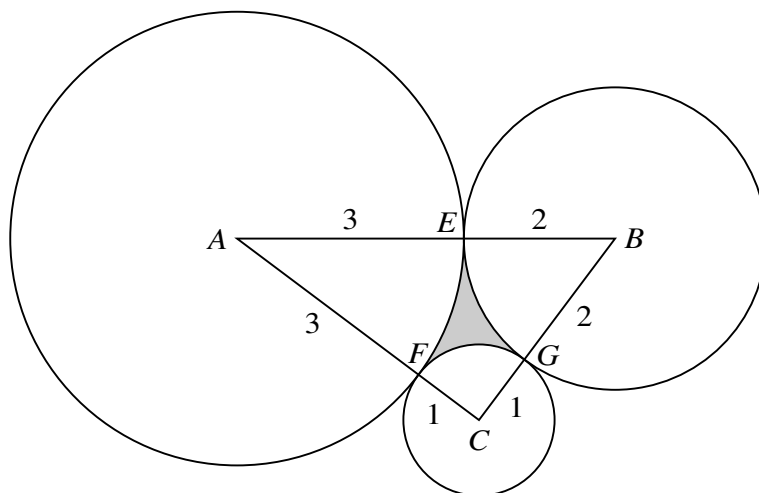
(i) Find the value of the constant  $k$ . [1]

(ii) Find the equation of the curve. [4]

- 4 The 1st, 3rd and 13th terms of an arithmetic progression are also the 1st, 2nd and 3rd terms respectively of a geometric progression. The first term of each progression is 3. Find the common difference of the arithmetic progression and the common ratio of the geometric progression. [5]

- 5 A curve has equation  $y = 8x + (2x - 1)^{-1}$ . Find the values of  $x$  at which the curve has a stationary point and determine the nature of each stationary point, justifying your answers. [7]

6



The diagram shows triangle  $ABC$  where  $AB = 5$  cm,  $AC = 4$  cm and  $BC = 3$  cm. Three circles with centres at  $A$ ,  $B$  and  $C$  have radii 3 cm, 2 cm and 1 cm respectively. The circles touch each other at points  $E$ ,  $F$  and  $G$ , lying on  $AB$ ,  $AC$  and  $BC$  respectively. Find the area of the shaded region  $EFG$ .

[7]

- 7 The point  $P(x, y)$  is moving along the curve  $y = x^2 - \frac{10}{3}x^{\frac{3}{2}} + 5x$  in such a way that the rate of change of  $y$  is constant. Find the values of  $x$  at the points at which the rate of change of  $x$  is equal to half the rate of change of  $y$ .

[7]

- 8 (i) Show that  $3 \sin x \tan x - \cos x + 1 = 0$  can be written as a quadratic equation in  $\cos x$  and hence solve the equation  $3 \sin x \tan x - \cos x + 1 = 0$  for  $0 \leq x \leq \pi$ .

[5]

- (ii) Find the solutions to the equation  $3 \sin 2x \tan 2x - \cos 2x + 1 = 0$  for  $0 \leq x \leq \pi$ .

[3]

- 9 The position vectors of  $A$ ,  $B$  and  $C$  relative to an origin  $O$  are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 5 \\ p \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix},$$

where  $p$  is a constant.

- (i) Find the value of  $p$  for which the lengths of  $AB$  and  $CB$  are equal.

[4]

- (ii) For the case where  $p = 1$ , use a scalar product to find angle  $ABC$ .

[4]

[Questions 10 and 11 are printed on the next page.]

- 10** The function  $f$  is such that  $f(x) = 2x + 3$  for  $x \geq 0$ . The function  $g$  is such that  $g(x) = ax^2 + b$  for  $x \leq q$ , where  $a$ ,  $b$  and  $q$  are constants. The function  $fg$  is such that  $fg(x) = 6x^2 - 21$  for  $x \leq q$ .

(i) Find the values of  $a$  and  $b$ . [3]

(ii) Find the greatest possible value of  $q$ . [2]

It is now given that  $q = -3$ .

(iii) Find the range of  $fg$ . [1]

(iv) Find an expression for  $(fg)^{-1}(x)$  and state the domain of  $(fg)^{-1}$ . [3]

- 11** Triangle  $ABC$  has vertices at  $A(-2, -1)$ ,  $B(4, 6)$  and  $C(6, -3)$ .

(i) Show that triangle  $ABC$  is isosceles and find the exact area of this triangle. [6]

(ii) The point  $D$  is the point on  $AB$  such that  $CD$  is perpendicular to  $AB$ . Calculate the  $x$ -coordinate of  $D$ . [6]

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