
MATHEMATICS

9709/11

Paper 1

May/June 2016

MARK SCHEME

Maximum Mark: 75

Published

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Page 2	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	11

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 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously ‘correct’ answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.
 - The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.
 - Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
 - For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	11

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR–1	A penalty of MR–1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become ‘follow through $\frac{1}{2}$ ’ marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
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Page 4	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	11

1	$\left(x - \frac{3}{2x}\right)^6$ Term is ${}^6C_3 \times x^3 \times \left(\frac{-3}{2x}\right)^3$ $\rightarrow -67.5$ oe	B1 B1 B1 [3]	B1 for Bin coeff. B1 for rest.
2	$3\sin^2\theta = 4\cos\theta - 1$ Uses $s^2 + c^2 = 1$ $\rightarrow 3c^2 + 4c - 4 (= 0)$ $(\rightarrow c = \frac{2}{3} \text{ or } -2)$ $\rightarrow \theta = 48.2^\circ \text{ or } 311.8^\circ$ 0.841, 5.44 rads, A1 only (0.268 π , 1.73 π)	M1 A1 A1 A1 ^{ft} [4]	Equation in $\cos\theta$ only. All terms on one side of (=) For $360^\circ - 1^{\text{st}}$ answer.
3	$x = \frac{12}{y^2} - 2.$ Vol = $(\pi) \times \int x^2 \, dy$ $\rightarrow \left[\frac{-144}{3y^3} + 4y + \frac{48}{y} \right]$ Limits 1 to 2 used $\rightarrow 22\pi$	M1 3 $\times$ A1 A1 [5]	Ignore omission of π at this stage Attempt at integration Un-simplified only from correct integration
4 (i)	$\frac{dy}{dx} = 2 - 8(3x + 4)^{-1/2}$ $(x = 0, \rightarrow \frac{dy}{dx} = -2)$ $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt} \rightarrow -0.6$	M1A1 [2]	Ignore notation. Must be $\frac{dy}{dx} \times 0.3$
(ii)	$y = \{2x\} \left\{ -\frac{8\sqrt{3x+4}}{\frac{1}{2}} \div 3 \right\} (+c)$ $x = 0, y = \frac{4}{3} \rightarrow c = 12.$	B1 B1 M1 A1 [4]	No need for $+c$. Uses x, y values after $\int$ with c
5 (i)	$A = 2y \times 4x (= 8xy)$ $10y + 12x = 480$ $\rightarrow A = 384x - 9.6x^2$	B1 B1 B1 [3]	answer given

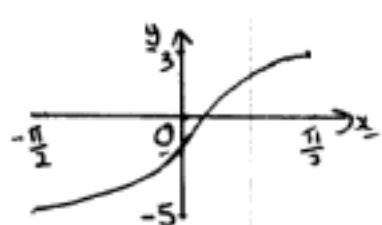
Page 5	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	11

(ii)	$\frac{dA}{dx} = 384 - 19.2x$ $= 0 \text{ when } x = 20$ $\rightarrow x = 20, y = 24.$ Uses $x = -\frac{b}{2a} = \frac{-384}{-19.2} = 20$, M1, A1 $y = 24$, A1 From graph: B1 for $x = 20$, M1, A1 for $y = 24$	B1 M1 A1 [3]	Sets to 0 and attempt to solve oe Might see completion of square Needs both x and y Trial and improvement B3 .
6 (a)	$y = 2x^2 - 4x + 8$ Equates with $y = mx$ and selects a, b, c Uses $b^2 = 4ac$ $\rightarrow m = 4$ or -12 .	M1 M1 A1 [3]	Equate + solution or use of dy/dx Use of discriminant for both.
(b) (i)	$f(x) = x^2 + ax + b$ Eqn of form $(x-1)(x-9)$ $\rightarrow a = -10, b = 9$ (or using 2 sim eqns M1 A1)	M1 A1 [2]	Any valid method allow $(x+1)(x+9)$ for M1 must be stated
(ii)	Calculus or $x = \frac{1}{2}(1+9)$ by symmetry $\rightarrow (5, -16)$	M1 A1 [2]	Any valid method
7 (i)	$CD = r\cos\theta, BD = r - r\sin\theta$ oe Arc $CB = r(\frac{1}{2}\pi - \theta)$ oe $\rightarrow P = r\cos\theta + r - r\sin\theta + r(\frac{1}{2}\pi - \theta)$ oe	B1 B1 B1 B1 ✓ [4]	allow degrees but not for last B1 ✓ sum – assuming trig used
(ii)	Sector = $\frac{1}{2} \cdot 5^2 \cdot (\frac{1}{2}\pi - 0.6)$ (12.135) Triangle = $\frac{1}{2} \cdot 5\cos 0.6 \cdot 5\sin 0.6$ (5.825) $\rightarrow$ Area = 6.31 (or $\frac{1}{4}$ circle – triangle – sector)	M1 M1 A1 [3]	Uses $\frac{1}{2}r^2\theta$ Uses $\frac{1}{2}bh$ with some use of trig.

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	11

8	$y = 3x - \frac{4}{x}$ $\frac{dy}{dx} = 3 + \frac{4}{x^2}$ $m \text{ of } AB = 4$ Equate $\rightarrow x = \pm 2$ $\rightarrow C(2, 4) \text{ and } D(-2, -4)$ $\rightarrow M(0, 0) \text{ or stating } M \text{ is the origin}$ $m \text{ of } CD = 2$ Perpendicular gradient $(= -\frac{1}{2})$ $\rightarrow y = -\frac{1}{2}x$	B1 B1 M1 A1 B1 ✓ M1 A1 [7]	Equating + solution. ✓ on their C and D Use of $m_1 m_2 = -1$, must use m_{CD} (not $m = 4$)
9 (a)	$a = 50, ar^2 = 32$ $\rightarrow r = \frac{4}{5}$ (allow $-\frac{4}{5}$ for M mark) $\rightarrow S_{\infty} = 250$	B1 M1 A1 [3]	seen or implied Finding r and use of correct S_{∞} formula Only if $ r < 1$
(b) (i)	$2\sin x, 3\cos x, (\sin x + 2\cos x)$. $3c - 2s = (s + 2c) - 3c$ (or uses $a, a + d, a + 2d$) $\rightarrow 4c = 3s \rightarrow t = \frac{4}{3}$ SC uses $t = \frac{4}{3}$ to show $u_1 = \frac{8}{5}, u_2 = \frac{9}{5}, u_3 = \frac{10}{5}$, B1 only	M1 M1 A1 [3]	Links terms up with AP, needs one expression for d . Arrives at $t = k$. ag
(ii)	$\rightarrow c = \frac{3}{5}, s = \frac{4}{5}$ or calculator $x = 53.1^\circ$ $\rightarrow a = 1.6, d = 0.2$ $\rightarrow S_{20} = 70$	M1 M1 A1 [3]	Correct method for both a and d . (Uses S_n formula)
10 (i)	$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 5 \\ -1 \\ k \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$ $10 - 1 - 2k = 0 \rightarrow k = 4\frac{1}{2}$	M1 A1 [2]	Use of scalar product = 0.

Page 7	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	11

(ii)	$\overrightarrow{AB} = \begin{pmatrix} 3 \\ -2 \\ k+2 \end{pmatrix},$$\overrightarrow{OC} = 7 \text{ (seen or implied)}$$3^2 + (-2)^2 + (k+2)^2 = 49$$\rightarrow k = 4 \text{ or } -8$	B1 B1 M1 A1 [4]	Correct method. Both correct. Condone sign error in $\overrightarrow{AB}$
(iii)	$\overrightarrow{OA} = 3$$\overrightarrow{OD} = 3\overrightarrow{OA} = \begin{pmatrix} 6 \\ 3 \\ -6 \end{pmatrix} \text{ and } \overrightarrow{OE} = 2$$\overrightarrow{OC} = \begin{pmatrix} 4 \\ 12 \\ -6 \end{pmatrix}$$\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = \begin{pmatrix} -2 \\ 9 \\ 0 \end{pmatrix},$$\rightarrow \text{Magnitude of } \sqrt{85}.$	M1 A1 M1 A1 [4]	Scaling from magnitudes/unit vector – oe. Correct vector subtraction.
11 (i)	$f : x \rightarrow 4\sin x - 1 \text{ for } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$$\text{Range } -5 \leq f(x) \leq 3$	B1 B1 [2]	–5 and 3 Correct range
(ii)	$4s - 1 = 0 \rightarrow s = \frac{1}{4} \rightarrow x = 0.253$$x = 0 \rightarrow y = -1$	M1 A1 B1 [3]	Makes $\sin x$ subject. Degrees M1 A0, (14.5°)
(iii)		B1✓ B1 [2]	Shape from their range in (i) Flattens, curve.
(iv)	$\text{range } -\frac{1}{2}\pi \leq f^{-1}(x) \leq \frac{1}{2}\pi$$\text{domain } -5 \leq x \leq 3$$\text{Inverse } f^{-1}(x) = \sin^{-1}\left(\frac{x+1}{4}\right)$	B1 B1✓ M1 A1 [4]	✓ on part (i) (only for 2 numerical values) Correct order of operations

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	Cambridge International AS/A Level – May/June 2016	9709	12

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	Cambridge International AS/A Level – May/June 2016	9709	12

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2	$f'(x) = \frac{8}{(5 - 2x)^2}$ $f(x) = \frac{8(5 - 2x)^{-1}}{-1} \div -2 (+c)$ Uses $x = 2, y = 7,$ $c = 3$	B1 B1 M1 A1 [4]	Correct without ($\div$ by -2) An attempt at integration ($\div$ by -2) Substitution of correct values into an integral to find c
3	$\overrightarrow{OA} = 2\mathbf{i} - 5\mathbf{j} - 2\mathbf{k}$ and $\overrightarrow{OB} = 4\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}.$ $\overrightarrow{AB} = 2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ or $\overrightarrow{AC} = 4\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}$ $\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC} = 6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}$ OR $\begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} = \begin{pmatrix} x - 4 \\ y + 4 \\ z - 2 \end{pmatrix},$ $\overrightarrow{OC} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$ OR $\overrightarrow{OB} - \overrightarrow{OA} = \overrightarrow{OC} - \overrightarrow{OB}$ $\therefore \overrightarrow{OC} = 2\overrightarrow{OB} - \overrightarrow{OA}$ $= \begin{pmatrix} 8 \\ -8 \\ 4 \end{pmatrix} - \begin{pmatrix} 2 \\ -5 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ -3 \\ 6 \end{pmatrix}$ Unit vector = (Their $\overrightarrow{OC}$) $\div$ (Mod their $\overrightarrow{OC}$) $= (6\mathbf{i} - 3\mathbf{j} + 6\mathbf{k}) \div 9$	B1 M1 B1 M1 B1 M1 M1 A1 [4]	correct method for $\overrightarrow{OC}$ Divides by their mod of their $\overrightarrow{OC}$ Correct unsimplified expression

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	12

4	(i)	$\left(x - \frac{2}{x}\right)^6$ Term is ${}_6C_3 \times (-2)^3 = (-)160$ -160	B1 B1 [2]	±160 seen anywhere
	(ii)	$\left(2 + \frac{3}{x^2}\right)\left(x - \frac{2}{x}\right)^6$ Term in $x^2 = {}_6C_2(-2)^2 x^2$ = 60 (x^2) Term independent of x: = $2 \times (\text{their } -160) + 3 \times (\text{their } 60)$ -140	B1 B1 M1 A1 [4]	±60 seen anywhere Using 2 products correctly
5	(i)	$\tan\left(\frac{\pi}{3}\right) = \frac{AC}{2x} \text{ or } \cos\left(\frac{\pi}{3}\right) = \sin\frac{\pi}{6} = \frac{2x}{AB}$ → $AC = 2\sqrt{3}x$ or $AB = 4x$ $AM = \sqrt{13x^2}, \sqrt{13}x, 3.61x$	B1 M1A1 [3]	Either trig ratio Complete method.
	(ii)	$\tan(\hat{MAC}) = \frac{x}{\text{Their } AC}$ $\theta = \frac{1}{6}\pi - \tan^{-1} \frac{1}{2\sqrt{3}}$ AG	M1 A1 [2]	“Their AC ” must be $f(x)$, $(\hat{MAC}) \neq \theta$. Justifies $\frac{\pi}{6}$ and links MAC & θ
6	(i)	$PT = r \tan \alpha$ $QT = OT - OQ = \frac{r}{\cos \alpha} - r$ or $\sqrt{r^2 + r^2 \tan^2 \alpha} - r$ Perimeter = sum of the 3 parts including $r\alpha$	B1 B1 B1 [3]	
	(ii)	Area of triangle = $\frac{1}{2} \times 10 \times 10 \tan \frac{\pi}{3}$ Area of sector = $\frac{1}{2} \times 10^2 \times \frac{1}{3}\pi$ Shaded region has area 34 (2sf)	M1 M1 A1 [3]	Correct formula used, $50\sqrt{3}, 86.6$ Correct formula used, $\frac{50\pi}{3}, 52.36$

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	12

7	(i)	$\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \equiv \frac{4}{\sin \theta \tan \theta}$ $\text{LHS} = \frac{1 + 2c + c^2 - (1 - 2c + c^2)}{(1 - c)(1 + c)}$ $= \frac{4c}{1 - c^2}$ $= \frac{4c}{s^2}$ $= \frac{4}{ts} \text{ AG}$	M1 A1 A1 A1 [4]	Attempt at combining fractions. A1 for numerator. A1 denominator Essential step for award of A1
	(ii)	$\sin \theta \left(\frac{1 + \cos \theta}{1 - \cos \theta} - \frac{1 - \cos \theta}{1 + \cos \theta} \right) = 3.$ $\rightarrow s \times \frac{4}{ts} = 3 \left(\rightarrow t = \frac{4}{3} \right)$ $\theta = 53.1^\circ \text{ and } 233.1^\circ$	M1 A1 A1 $\checkmark^h$ [3]	Uses part (i) to eliminate “s” correctly. $\checkmark^h$ for $180^\circ + 1^{\text{st}}$ answer.
8	(i)	$A(0, 7), B(8, 3)$ and $C(3k, k)$ m of AB is $-\frac{1}{2}$ oe. Eqn of AB is $y = -\frac{1}{2}x + 7$ Let $x = 3k, y = k$ $k = 2.8$ oe OR $\frac{7 - k}{0 - 3k} = \frac{3 - k}{8 - 3k}$ $\rightarrow 20k = 56 \rightarrow k = 2.8$ OR $\frac{7 - k}{0 - 3k} = \frac{7 - 3}{0 - 8}$ $\rightarrow 20k = 56 \rightarrow k = 2.8$	B1 M1 M1 A1 M1A1 DM1A1 M1A1 DM1A1 [4]	Using A, B or C to get an equation Using C or A, B in the equation Using A, B & C to equate gradients Simplifies to a linear or 3 term quadratic = 0. Using A, B and C to equate gradients Simplifies to a linear or 3 term quadratic = 0.

Page 7	Mark Scheme	Syllabus	Paper
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(ii)	M(4, 5) Perpendicular gradient = 2. Perp bisector has eqn $y - 5 = 2(x - 4)$ Let $x = 3k, y = k$ $k = \frac{3}{5}$ oe OR $(0 - 3k)^2 + (7 - k)^2 = (8 - 3k)^2 + (3 - k)^2$ $-14k + 49 = 73 - 54k \rightarrow 40k = 24 \rightarrow k = 0.6$	B1 M1 M1 A1 M1A1 DM1A1 [4]	anywhere in (ii) Use of $m_1 m_2 = -1$ soi Forming eqn using their M and their “perpendicular m” Use of Pythagoras. Simplifies to a linear or 3 term quadratic = 0.
9 (i) (a) (b) (ii)	$a + (n - 1)d = 10 + 29 \times 2$ = 68 $\frac{1}{2}n(20 + 2(n - 1)) = 2000$ or 0 $\rightarrow 2n^2 + 18n - 4000 = 0$ oe (n=) 41 $r = 1.1$, oe Uses $S_{30} = \frac{10(1.1^{30} - 1)}{1.1 - 1}$ (= 1645) Percentage lost = $\frac{2000 - 1645}{2000} \times 100$ = 17.75	M1 A1 [2] M1 A1 A1 [3] B1 M1 DM1 A1 [4]	Use of nth term of an AP with $a = \pm 10, d = \pm 2, n = 30$ or 29 Condone – 68 $\rightarrow$ 68 Use of S_n formula for an AP with $a = \pm 10, d = \pm 2$ and equated to either 0 or 2000. Correct 3 term quadratic = 0. e.g. $\frac{11}{10}, 110\%$ Use of S_n formula for a GP, $a = \pm 10, n = 30$. Fully correct method for % left with “their 1645” allow 17.7 or 17.8.
10 (i)	$y = \frac{8}{x} + 2x$. $\frac{dy}{dx} = -8x^{-2} + 2$ $\frac{d^2y}{dx^2} = 16x^{-3}$ $\int y^2 dx = -64x^{-1}$ oe + $32x$ oe + $\frac{4x^3}{3}$ oe (+c)	B1 B1 3 × B1 [5]	unsimplified ok unsimplified ok B1 for each term – unsimplified ok

Page 8	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	12

(ii)	sets $\frac{dy}{dx}$ to 0 $\rightarrow x = \pm 2$ $\rightarrow M(2, 8)$ Other turning point is $(-2, -8)$	M1 A1 A1	Sets to 0 and attempts to solve Any pair of correct values A1 Second pair of values A1
	If $x = -2$, $\frac{d^2y}{dx^2} < 0$ $\therefore$ Maximum	M1 A1 [5]	Using their $\frac{d^2y}{dx^2}$ if kx^{-3} and $x < 0$
(iii)	Vol = $\pi \times$ [part (i)] from 1 to 2 $\frac{220\pi}{3}, 73.3\pi, 230$	M1 A1 [2]	Evidence of using limits 1&2 in their integral of y^2 (ignore π)
11	$f: x \mapsto 6x - x^2 - 5$		
(i)	$6x - x^2 - 5 \leq 3$ $\rightarrow x^2 - 6x + 8 \geq 0$ $\rightarrow x = 2, x = 4$ $x \leq 2, x \geq 4$ condone $<$ and/or $>$	M1 A1 A1 [3]	$\pm(6x - x^2 - 8) =, \leq, \geq 0$ and attempts to solve Needs both values whether $=2, <2, >2$ Accept all recognisable notation.
(ii)	Equate $mx + c$ and $6x - x^2 - 5$ Use of “ $b^2 - 4ac$ ” $4c = m^2 - 12m + 16$. AG OR $\frac{dy}{dx} = 6 - 2x = m \rightarrow x = \left(\frac{6-m}{2}\right)$ $m\left(\frac{6-m}{2}\right) + c = 6\left(\frac{6-m}{2}\right) - \left(\frac{6-m}{2}\right)^2 - 5$ $4c = m^2 - 12m + 16$. AG	M1 DM1 A1 M1 M1 A1 [3]	Equates, sets to 0. Use of discriminant with values of a, b, c independent of x . $= (0)$ must appear before last line. Equates $\frac{dy}{dx}$ to m and rearrange Equates $mx + c$ and $6x - x^2 - 5$ and substitutes for x
(iii)	$6x - x^2 - 5 = 4 - (x - 3)^2$	B1 B1 [2]	4 B1 – $(x - 3)^2$ B1
(iv)	$k = 3$.	B1 [✓] [1]	[✓] for “ b ”.
(v)	$g^{-1}(x) = \sqrt{4-x} + 3$	M1 A1 [2]	Correct order of operations. $\pm\sqrt{4-x} + 3$ M1A0 $\sqrt{x-4} + 3$ M1A0 $\sqrt{4-y} + 3$ M1A0

MATHEMATICS

9709/13

Paper 1

May/June 2016

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge will not enter into discussions about these mark schemes.

Cambridge is publishing the mark schemes for the May/June 2016 series for most Cambridge IGCSE[®], Cambridge International A and AS Level components and some Cambridge O Level components.



Page 2	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	13

Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\Rightarrow$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	13

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	13

1	$5C2\left(\frac{1}{x}\right)^3(3x^2)^2$ $10(\times 1) \times 3^2$ $90(x)$	B1 B1 B1 [3]	Can be seen in expansion Identified as leading to answer
2	$(\pi) \int (x^3 + 1) dx$ $(\pi) \left[\frac{x^4}{4} + x \right]$ 6π or 18.8	M1 A1 DM1A1 [4]	Attempt to resolve y^2 and attempt to integrate Applying limits 0 and 2. (Limits reversed: Allow M mark and allow A mark if final answer is 6π)
3 (i)	$6 + k = 2 \rightarrow k = -4$	B1 [1]	
(ii)	$(y) = \frac{6x^3}{3} - \frac{4}{-2}x^{-2} (+c)$ $9 = 2 + 2 + c$ c must be present $(y) = 2x^3 + 2x^{-2} + 5$	B1B1 M1 A1 [4]	fit on <i>their</i> k . Accept $+\frac{k}{-2}x^{-2}$ Sub (1,9) with numerical k . Dep on attempt $\int$ Equation needs to be seen Sub (2, 3) $\rightarrow c = -13\frac{1}{2}$ scores M1A0
4	$r = \frac{3+2d}{3}$ or $\frac{3+12d}{3+2d}$ or $r^2 = \frac{3+12d}{3}$ $(3+2d)^2 = 3(3+12d)$ oe OR sub $2d = 3r - 3$ $(4)d(d-6) = 0$ OR $3r^2 = 18r - 15 \rightarrow (r-1)(r-5)$ $d = 6$ $r = 5$	B1 M1 DM1 A1 A1 [5]	1 correct equation in r and d only is sufficient Eliminate r or d using valid method Attempt to simplify and solve quadratic Ignore $d = 0$ or $r = 1$ Do not allow -5 or ± 5

Page 5	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	13

5	$\frac{dy}{dx} = [8] + [-2] [(2x-1)^{-2}]$ $= 0 \rightarrow 4(2x-1)^2 = 1$ oe eg $16x^2 - 16x + 3 = 0$ $x = \frac{1}{4}$ and $\frac{3}{4}$ $\frac{d^2y}{dx^2} = 8(2x-1)^{-3}$ When $x = \frac{1}{4}$, $\frac{d^2y}{dx^2} (= -64)$ and/or < 0 MAX When $x = \frac{3}{4}$, $\frac{d^2y}{dx^2} (= 64)$ and/or > 0 MIN	B2,1,0 M1 A1 B1 * DB1 DB1 [7]	Set to zero, simplify and attempt to solve soi Needs both x values. Ignore y values fit to $k(2x-1)^{-3}$ where $k > 0$ Alt. methods for last 3 marks (values either side of $1/4$ & $3/4$) must indicate <u>which</u> x -values and cannot use $x = 1/2$. (M1A1A1)
6	$BAC = \sin^{-1}(3/5)$ or $\cos^{-1}(4/5)$ or $\tan^{-1}(3/4)$ $ABC = \sin^{-1}(4/5)$ or $\cos^{-1}(3/5)$ or $\tan^{-1}(4/3)$ $ACB = \pi/2$ (Allow 90°) Shaded area = ΔABC – sectors ($AEF + BEG + CFG$) $\Delta ABC = \frac{1}{2} \times 4 \times 3$ oe Sum sectors = $\frac{1}{2} [3^2 0.6435] +$ $2^2 0.9273 + 1^2 1.5708]$ OR $\frac{\pi}{360} [3^2 36.8(7) + 2^2 53.1(3) + 1^2 90]$ $6 - 5.536 = 0.464$	B1 B1 B1 M1 B1 M1 A1 [7]	Accept $36.8(7)^\circ$ Accept $53.1(3)^\circ$
7	$\frac{dy}{dx} = 2x - 5x^{1/2} + 5$ $\frac{dy}{dx} = 2$ $2x - 5x^{1/2} + 5 = 2$ $2x - 5x^{1/2} + 3 (= 0)$ or equivalent 3-term quadratic Attempt to solve for $x^{1/2}$ e.g. $(2x^{1/2} - 3)(x^{1/2} - 1) = 0$ $x^{1/2} = 3/2$ and 1 $x = 9/4$ and 1	B1 B1 M1 A1 DM1 A1 A1 [7]	Equate their dy/dx to <i>their</i> 2 or $1/2$. Dep. on 3-term quadratic ALT $5x^{1/2} = 2x + 3 \rightarrow 25x = (2x + 3)^2$ $4x^2 - 13x + 9 (= 0)$ $x = 9/4$ and 1

Page 6	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	13

8	(i)	$3\sin^2 x - \cos^2 x + \cos x = 0$ Use $s^2 = 1 - c^2$ and simplify to 3-term quad $\cos x = -3/4$ and 1 $x = 2.42$ (allow 0.77π) or 0 (extra in range max 1)	M1 M1 A1 A1A1 [5]	Multiply by $\cos x$ Expect $4c^2 - c - 3 = 0$ SC1 for 0.723 (or 0.23π), π following $4c^2 + c - 3 = 0$
	(ii)	$2x = 2\pi - \text{their } 2.42$ or $360 - 138.6$ $x = 1.21$ (0.385π), 1.93 ($0.614/5\pi$), 0, π (3.14) (extra max 1)	B1 ^h B1B1 [3]	Expect $2x = 3.86$ Any 2 correct B1. Remaining 2 correct B1. SCB1 for all 69.3, 110.7, 0, 180 (degrees) SCB1 for .361, $\pi/2$, 2.78 after $4c^2 + c - 3 = 0$
9	(i)	$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = \begin{pmatrix} -1 \\ 2 \\ p+4 \end{pmatrix}$ $\mathbf{CB} = \mathbf{OB} - \mathbf{OC} = \begin{pmatrix} -4 \\ 5 \\ p-2 \end{pmatrix}$ $1+4+(p+4)^2 = 16+25+(p-2)^2$ $p = 2$	B1 B1 M1 A1 [4]	Ignore labels. Allow BA or BC Use of $x_1x_2 + y_1y_2 + z_1z_2$
	(ii)	$\mathbf{AB} \cdot \mathbf{CB} = 4+10-5 = 9$ $ \mathbf{AB} = \sqrt{1+4+25} = \sqrt{30}$, $ \mathbf{CB} = \sqrt{16+25+1} = \sqrt{42}$ $\cos ABC = \frac{9}{\sqrt{30}\sqrt{42}}$ or $\frac{9}{6\sqrt{35}}$ $ABC = 75.3^\circ$ or 1.31rads (ignore reflex angle 285°)	M1 M1 M1 A1 [4]	Product of moduli Allow one of AB , CB reversed - but award A0
10	(i)	$2(ax^2 + b) + 3 = 6x^2 - 21$ $a = 3$, $b = -12$	M1 A1A1 [3]	
	(ii)	$3x^2 - 12 \geq 0$ or $6x^2 - 21 \geq 3$ $x \leq -2$ i.e. (max) $q = -2$	M1 A1 [2]	Allow $=$ or $\leq$ or $>$ or $<$. Ft from <i>their</i> a , b Must be in terms of q (eg $q \leq -2$)
	(iii)	$y \geq 6(-3)^2 - 21 \Rightarrow \text{range is } (y) \geq 33$	B1 [1]	Do not allow $y > 33$. Accept all other notations e.g. $[33, \infty)$ or $[33, \infty]$

Page 7	Mark Scheme	Syllabus	Paper
	Cambridge International AS/A Level – May/June 2016	9709	13

(iv)	$y = 6x^2 - 21 \Rightarrow x = (\pm) \sqrt{\frac{y+21}{6}}$ $(fg)^{-1}(x) = -\sqrt{\frac{x+21}{6}}$ Domain is $x \geq 33$	M1 A1 B1 ^h [3]	Allow $y = \dots$. Must be a function of x ft from <i>their</i> part (iii) but x essential
11 (i)	$AB^2 = 6^2 + 7^2 = 85, BC^2 = 2^2 + 9^2 = 85$ ($\rightarrow$ isosceles) $AC^2 = 8^2 + 2^2 = 68$ $M = (2, -2)$ or $BM^2 = (\sqrt{85})^2 - (\frac{1}{2}\sqrt{68})^2$ $BM = \sqrt{2^2 + 8^2} = \sqrt{68}$ or $\sqrt{85 - 17} = \sqrt{68}$ Area $\triangle ABC = \frac{1}{2}\sqrt{68}\sqrt{68} = 34$	B1B1 B1 B1 B1 [6]	Or $AB = BC = \sqrt{85}$ etc Where M is mid-point of AC
(ii)	Gradient of $AB = 7/6$ Equation of AB is $y + 1 = \frac{7}{6}(x + 2)$ Gradient of $CD = -6/7$ Equation of CD is $y + 3 = \frac{-6}{7}(x - 6)$ Sim Eqns $2 = \frac{-6}{7}x + \frac{36}{7} - \frac{7}{6}x - \frac{14}{6}$ $x = \frac{34}{85} = \frac{2}{5}$ oe	B1 M1 M1 M1 A1 [6]	Or $y - 6 = \frac{7}{6}(x - 4)$