

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge International Advanced Subsidiary and Advanced Level

MARK SCHEME for the May/June 2015 series

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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Penalties

MR–1	A penalty of MR–1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
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1	θ is obtuse , $\sin \theta = k$ (i) $\cos \theta = -\sqrt{1 - k^2}$ (ii) $\tan \theta = \frac{\sin \theta}{\cos \theta}$ used $\rightarrow \tan \theta = -\frac{k}{\sqrt{1 - k^2}}$ aef (iii) $\sin (\theta + \pi) = -k$	B1 [1] M1 A1✓ [2] B1 [1]	cao Used, attempt at cosine seen in (i) Ft for their cosine as a function of k only, from part (i) cao
2	$y = 2x^2$, $X(-2, 0)$ and $P(p, 0)$ (i) $A = \frac{1}{2} \times (2 + p) \times 2p^2 (= 2p^2 + p^3)$ (ii) $\frac{dA}{dp} = 4p + 3p^2$ $\frac{dA}{dt} = \frac{dA}{dp} \times \frac{dp}{dt} = 0.02 \times 20 = 0.4$ or $\frac{dA}{dt} = 4p \frac{dp}{dt} + 3p^2 \frac{dp}{dt}$	M1 A1 [2] B1 M1 A1 [3]	Attempt at base and height in terms of p and use of $\frac{bh}{2}$ cao any correct method, cao
3	$(1 - x)^2 (1 + 2x)^6$. (i) (a) $(1 - x)^6 = 1 - 6x + 15x^2$ (b) $(1 + 2x)^6 = 1 + 12x + 60x^2$ (ii) Product of (a) and (b) with >1 term $\rightarrow 60 - 72 + 15 = 3$	B2,1 [2] B2,1 [2] M1 DM1A1 [3]	-1 each error -1 each error SC B1 only, in each part, for all 3 correct descending powers SC only one penalty for omission of the '1' in each expansion Must be 2 or more products M1 exactly 3 products. cao, condone $3x^2$

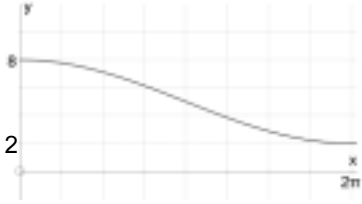
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4	(i)	$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}, \overrightarrow{OB} = \begin{pmatrix} 6 \\ -3 \\ 2 \end{pmatrix}, \overrightarrow{OC} = \begin{pmatrix} k \\ -2k \\ 2k-3 \end{pmatrix}$ $OA \cdot OB = 18 - 8 = 10$ Modulus of $OA = 5$, of $OB = 7$ $\text{Angle } AOB = \cos^{-1}\left(\frac{10}{35}\right) \text{ aef}$ $\rightarrow \frac{10}{35} \text{ or } \frac{2}{7}$	M1 M1 A1 [3]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ All linked with modulus cao, (if angle given, no penalty), correct angle implies correct cosine
	(ii)	$\overrightarrow{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 3 \\ -3 \\ 6 \end{pmatrix}$ $k^2 + 4k^2 + (2k-3)^2 = 9 + 9 + 36$ $\rightarrow 9k^2 - 12k - 45 (= 0)$ $\rightarrow k = 3 \text{ or } k = -\frac{5}{3}$	B1 M1 DM1 A1 [4]	allow for $\mathbf{a} - \mathbf{b}$ Correct use of moduli using their AB obtains 3 term quadratic. cao
5	(i)	$24 = r + r + r\theta$ $\rightarrow \theta = \frac{24 - 2r}{r}$ $A = \frac{1}{2} r^2 \theta = \frac{24r}{2} - r^2 = 12r - r^2. \text{ aef, ag}$	M1 M1A1 [3]	(May not use θ) Attempt at $s = r\theta$ linked with 24 and r Uses A formula with θ as $f(r)$. cao
	(ii)	$(A =) 36 - (r - 6)^2$	B1 B1 [2]	cao
	(iii)	Greatest value of $A = 36$ $(r = 6) \rightarrow \theta = 2$	B1 ✓ B1 [2]	Ft on (ii). cao, may use calculus or the discriminant on $12r - r^2$

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6	(i)	$y - 2t = -2(x - 3t)(y + 2x = 8t)$ Set x to 0 $\rightarrow B(0, 8t)$ Set y to 0 $\rightarrow A(4t, 0)$ $\rightarrow \text{Area} = 16t^2$	M1 M1 A1 [3]	Unsimplified or equivalent forms Attempt at both A and B , then using cao
	(ii)	$m = \frac{1}{2}$ $\rightarrow y - 2t = \frac{1}{2}(x - 3t)(2y = x + t)$ Set y to 0 $\rightarrow C(-t, 0)$ Midpoint of CP is (t, t) This lies on the line $y = x$.	B1 M1 A1 A1 [4]	cao Unsimplified or equivalent forms co correctly shown.
7	(a)	$ar^2 = \frac{1}{3}, ar^3 = \frac{2}{9}$ $\rightarrow r = \frac{2}{3}$ aef Substituting $\rightarrow a = \frac{3}{4}$ $\rightarrow S_{\infty} = \frac{\frac{3}{4}}{\frac{1}{3}} = 2\frac{1}{4}$ aef	M1 A1 M1 A1 [4]	Any valid method, seen or implied. Could be answers only. Both a and r Correct formula with $ r < 1$, cao
	(b)	$4a = a + 4d \rightarrow 3a = 4d$ $360 = S_5 = \frac{5}{2}(2a + 4d)$ or $12.5a$ $\rightarrow a = 28.8^\circ$ aef Largest $= a + 4d$ or $4a = 115.2^\circ$ aef	B1 M1 A1 B1 [4]	May be implied in $360 = 5/2(a + 4a)$ Correct S_n formula or sum of 5 terms cao, may be implied (may use degrees or radians)

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8	$f: x \mapsto 5 + 3\cos\left(\frac{1}{2}x\right)$ for $0 \leq x \leq 2\pi$.	B1	Makes $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$
(i)	$5 + 3\cos\left(\frac{1}{2}x\right) = 7$ $\cos\left(\frac{1}{2}x\right) = \frac{2}{3}$ $\frac{1}{2}x = 0.84 \quad x = 1.68$ only, aef (in given range)	M1A1 [3]	Looks up $\cos^{-1}$ first, then $\times 2$
(ii)		B1 B1 [2]	y always +ve, m always –ve. from (0, 8) to (2π, 2) (may be implied)
(iii)	No turning point on graph or 1:1	B1 [1]	cao, independent of graph in (ii)
(iv)	$y = 5 + 3\cos\left(\frac{1}{2}x\right)$ Order; $-5, \div 3, \cos^{-1}, \times 2$ $x = 2\cos^{-1}\left(\frac{x-5}{3}\right)$	M1 M1 A1 [3]	Tries to make x subject. Correct order of operations cao

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9	$y = x^3 + px^2$ (i) $\frac{dy}{dx} = 3x^2 + 2px$ Sets to 0 $\rightarrow x = 0$ or $-\frac{2p}{3}$ $\rightarrow (0, 0)$ or $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right)$	B1 M1 A1 A1 [4]	cao Sets differential to 0 cao cao, first A1 for any correct turning point or any correct pair of x values. 2nd A1 for 2 complete TPs
	(ii) $\frac{d^2y}{dx^2} = 6x + 2p$ At $(0, 0) \rightarrow 2p$ +ve Minimum At $\left(-\frac{2p}{3}, \frac{4p^3}{27}\right) \rightarrow -2p$ -ve Maximum (iii) $y = x^3 + px^2 + px \rightarrow 3x^2 + 2px + p (= 0)$ Uses $b^2 - 4ac$ $\rightarrow 4p^2 - 12p < 0$ $\rightarrow 0 < p < 3$ aef	M1 A1 A1 [3] B1 M1 A1 [3]	Other methods include; clear demonstration of sign change of gradient, clear reference to the shape of the curve www Any correct use of discriminant cao (condone $\leq$)

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10	$y = \frac{8}{\sqrt{3x+4}}$		
	(i) $\frac{dy}{dx} = \frac{-4}{(3x+4)^{\frac{3}{2}}} \times 3$ aef $\rightarrow m_{(x=0)} = -\frac{3}{2}$ Perpendicular $m_{(x=0)} = \frac{2}{3}$ Eqn of normal $y - 4 = \frac{2}{3}(x - 0)$ Meets $x = 4$ at $B \left(4, \frac{20}{3}\right)$ (ii) $\int \frac{8}{\sqrt{3x+4}} dx = \frac{8\sqrt{3x+4}}{\frac{1}{2}} \div 3$ Limits from 0 to 4 $\rightarrow$ Area $P = \frac{32}{3}$ Area $Q =$ Trapezium $- P$ Area of Trapezium = $\frac{1}{2} \left(4 + \frac{20}{3} \right) \times 4 = \frac{64}{3}$ $\rightarrow$ Areas of P and Q are both $\frac{32}{3}$	B1 B1 M1 M1 A1 [5] B1 B1 M1 A1 M1 A1 [6]	Without the “$\times 3$” For “$\times 3$” even if 1st B mark lost. Use of $m_1 m_2 = -1$ after attempting to find $\frac{dy}{dx}_{(x=0)}$ Unsimplified line equation cao Without “$\div 3$”. For “$\div 3$” Correct use of correct limits. cao Correct method for area of trapezium All correct.

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1	$f'(x) = 5 - 2x^2$ and $(3, 5)$ $f(x) = 5x - \frac{2x^3}{3} (+c)$ Uses $(3, 5)$ $\rightarrow c = 8$	B1 M1 A1 [3]	For integral Uses the point in an integral co
2	Radius of semicircle = $\frac{1}{2}AB = r\sin\theta$ Area of semicircle = $\frac{1}{2}\pi r^2\sin^2\theta = A_1$ Shaded area = semicircle – segment $= A_1 - \frac{1}{2}r^2 2\theta + \frac{1}{2}r^2\sin 2\theta$	B1 B1✓ B1B1 [4]	aef Uses $\frac{1}{2}\pi r^2$ with $r = f(\theta)$ B1 (–sector), B1 for + (triangle)
3 (i)	$(2-x)^6$ Coeff of x^2 is 240 Coeff of x^3 is $-20 \times 8 = -160$	B1 B2,1 [3]	co B1 for +160
(ii)	$(3x+1)(2-x)^6$ Product needs exactly 2 terms $\rightarrow 720 - 160 = 560$	M1 A1✓ [2]	3 × their 240 + their -160 ✓ for candidate's answers.
4	$u = 2x(y-x)$ and $x+3y=12$, $u = 2x\left(\frac{12-x}{3} - x\right)$ $= 8x - \frac{8x^2}{3}$ $\frac{du}{dx} = 8 - \frac{16x}{3}$ $= 0$ when $x = 1\frac{1}{2}$ $\rightarrow (y = 3\frac{1}{2})$ $\rightarrow u = 6$	M1 A1 M1 A1 A1 [5]	Expresses u in terms of x Differentiate candidate's quadratic, sets to 0 + attempt to find x , or other valid method Complete method that leads to u Co
5 (i)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$ Divides top and bottom by $\cos\theta$ $\rightarrow \frac{t-1}{t+1}$	B1 [1]	Answer given.
(ii)	$\frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{1}{6}\tan\theta$ $\rightarrow \frac{t-1}{t+1} = \frac{t}{6}$ $\rightarrow t^2 - 5t + 6 = 0$ $\rightarrow t = 2$ or $t = 3$ $\rightarrow \theta = 63.4^\circ$ or 71.6°	B1 M1 A1 A1 [4]	Using the identity. Forms a 3 term quadratic with terms all on same side. co co

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6	$h = 60(1 - \cos kt)$		
	(i) Max h when $\cos = -1 \rightarrow 120$	B1 [1]	Co
	(ii) $h = 0$ and $t = 30$, or $h = 120$ and $t = 15$ $\rightarrow \cos 30k = 1$ or $\cos 15k = -1$ $\rightarrow 30k = 2\pi$ or $15k = \pi$ $\rightarrow k = \frac{2\pi}{30} = \frac{\pi}{15}$	M1 A1 [2]	Substituting a correct pair of values into the equation. co ag
	(iii) $90 = 60(1 - \cos kt)$ $\rightarrow \cos kt = \frac{-30}{60} = -0.5$ $\rightarrow kt = \frac{2\pi}{3}$ or $\rightarrow kt = \frac{4\pi}{3}$ $\rightarrow$ Either $t = 10$ or 20 or both $\rightarrow t = 10$ minutes	B1 B1 [3]	co – but there must be evidence of correct subtraction.
7	$A(4, 6), B(10, 2).$		
	(i) $M = (7, 4)$ m of $AB = -\frac{2}{3}$ m of perpendicular $= \frac{3}{2}$ $\rightarrow y - 4 = \frac{3}{2}(x - 7)$	B1 B1 M1 A1 [4]	co co Use of $m_1 m_2 = -1$ & their midpoint in the equation of a line. co
	(ii) Eqn of line parallel to AB through $(3, 11)$ $\rightarrow y - 11 = -\frac{2}{3}(x - 3)$ Sim eqns $\rightarrow C(9, 7)$	M1 DM1A1 [3]	Needs to use m of AB Must be using their correct lines. Co
8	(a) 1st, 2nd, n th are 56, 53 and -22 $a = 56, d = -3$ $-22 = 56 + (n - 1)(-3)$ $\rightarrow n = 27$ $S_{27} = \frac{27}{2}(112 + 26(-3))$ $\rightarrow 459$	M1 A1 M1 A1 [4]	Uses correct u_n formula. co Needs positive integer n Co
	(b) 1 st , 2 nd , 3 rd are $2k + 6, 2k$ and $k + 2$.		
	(i) Either $\frac{2k}{2k+6} = \frac{k+2}{2k}$ or uses a, r and eliminates $\rightarrow 2k^2 - 10k - 12 = 0$ $\rightarrow k = 6$	M1 DM1 A1 [3]	Correct method for equation in k . Forms quad. or cubic equation with no brackets or fractions. Co

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(ii)	$S_{\infty} = \frac{a}{1-r}$ with $r = \frac{2k}{2k+6}$ or $\frac{k+2}{2k}$ ($=\frac{2}{3}$) $\rightarrow 54$	M1 A1 [2]	Needs attempt at a and r and S_{∞} Co
9	$\vec{OA} = 2\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}$ and $\vec{OB} = 3\mathbf{i} + \mathbf{j} + 4\mathbf{k}$ (ii) $\vec{OA} \cdot \vec{OB} = 6 + 4 + 16 = 26$ $ \vec{OA} = \sqrt{36}, \vec{OB} = \sqrt{26}$ $\cos AOB = \frac{26}{6\sqrt{26}}$ $\rightarrow 31.8^{\circ}$ (ii) $\vec{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}$ $\vec{OC} = \begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix} + 2\vec{AB}$ or $\begin{pmatrix} 3 \\ 1 \\ 4 \end{pmatrix} + \vec{AB}$ $\vec{OC} = \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$ Unit vector $\div$ modulus $\rightarrow \frac{1}{6} \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix}$ (iii) $ \vec{OC} = 6, \vec{OA} = 6$	M1 M1 M1 A1 [4] B1 M1 M1 A1 [4] B1 [1]	Must be numerical at some stage Product of 2 moduli All linked correctly co Correct link $\div$ by modulus. co co

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10	$y = \frac{4}{2x-1}$	B1	Correct without the $\div 2$
	(i) $\int \frac{16}{(2x-1)^2} dx = \frac{-16}{2x-1} \div 2$ Vol = $\pi \left[\frac{-8}{2x-1} \right]$ with limits 1 and 2 $\rightarrow \frac{16\pi}{3}$ (ii) $m = \frac{1}{2} m$ of tangent = -2 $\frac{dy}{dx} = \frac{-4}{(2x-1)^2} \times 2$ Equating their $\frac{dy}{dx}$ to -2 $\rightarrow x = \frac{3}{2}$ or $-\frac{1}{2}$ ($y = 2$ or -2) $\rightarrow c = \frac{5}{2}$ or $-\frac{7}{2}$	B1 M1 A1 [4] M1 B1 B1 DM1 A1 A1 [6]	For the $\div 2$ even if first B1 is lost Use of limits in a changed expression. co Use of $m_1 m_2 = -1$ Correct without the $\times 2$ For the $\times 2$ even if first B1 is lost co co
11	$f: x \mapsto 2x^2 - 6x + 5$		
	(i) $2x^2 - 6x + 5 - p = 0$ has no real roots Uses $b^2 - 4ac \rightarrow 36 - 8(5 - p)$ Sets to 0 $\rightarrow p < \frac{1}{2}$	M1 DM1 A1 [3]	Sets to 0 with p on LHS. Uses discriminant. co – must be “<”, not “$\leq$”.
	(ii) $2x^2 - 6x + 5 = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$	3 $\times$ B1 [3]	co
	(iii) Range of g $\frac{1}{2} \leq g(x) \leq 13$ $h: x \mapsto 2x^2 - 6x + 5$ for $k \leq x \leq 4$	B1 ∇ B1 [2]	∇ on (ii) co from sub of $x = 4$
	(iv) Smallest $k = \frac{3}{2}$	B1 ∇ [1]	∇ on (ii)
	(v) $h(x) = 2\left(x - \frac{3}{2}\right)^2 + \frac{1}{2}$ Order of operations $\pm \frac{1}{2}, \div 2, \sqrt{}, \pm \frac{3}{2}$ $\rightarrow$ Inverse = $\frac{3}{2} + \sqrt{\left(\frac{x}{2} - \frac{1}{4}\right)}$	M1 DM1 A1 [3]	Using comp square form to try and get x as subject or y if transposed. Order must be correct co (without $\pm$)

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Mark Scheme Notes

Marks are of the following three types:

M Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.

A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\Rightarrow$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through ✓” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1	$2(x-3)^2 - 11$	B1B1B1 [3]	For 2, $(x-3)^2$, -11 . Or $a=2$, $b=3$, $c=11$
2	$\left[\frac{(2x+1)^{\frac{3}{2}}}{\frac{3}{2}} \right] [\div 2] \quad (+c)$ $7 = 9 + c$ $y = \frac{(2x+1)^{\frac{3}{2}}}{3} - 2 \quad \text{or unsimplified}$	B1B1 M1 A1 [4]	Attempt subst $x=4$, $y=7$. c must be there. Dep. on attempt at integration. $c = -2$ sufficient
3 (i)	$a^5 - 5a^4x + 10a^3x^2 - 10a^2x^3 + \dots$	B2,1,0 [2]	Ok full expansion (ignore extra terms) Descending: Ok if full expansion but max B1 for 4 terms
(ii)	$(1-ax)(10a^3x^2 - 10a^2x^3) = (x^3)(-10a^4 - 10a^2)$ $-10a^4 - 10a^2 = -200$ $a^2 = 4$ ignore $a^2 = -5$ $a = \pm 2$ cao	M1 A1✓ M1 A1 [4]	Attempt to find coeff. of x^3 from 2 terms Ft from <i>their</i> $10a^3$, $-10a^2$ from part (i) Attempt soln. for a^2 from 3-term quad. in a^2 Ignore any imaginary solutions
4 (i)	$\tan \theta = 1/3$ $\theta = 18.4^\circ$ only	M1 A1 [2]	Ignore solns. outside range $0 \rightarrow 180$
(ii)	$\tan 2x = (\pm)1/\sqrt{3}$ Must be sq. root soi $(x) = 15$ $(x) = \text{any correct second value (75, 105, 165)}$ $(x) = \text{cao}$	M1 A1 A1✓ A1 [4]	$\sin 2x = (\pm)1/2$ or $\cos 2x = (\pm)\sqrt{3}/2$ using $c^2 + s^2 = 1$. Not $\tan x = (\pm)\frac{1}{\sqrt{3}}$ etc. ft for $(90 \pm \text{their } 15)$ or $(180 - \text{their } 15)$ All four correct. Extra solns in range 1
5 (i)	$\overrightarrow{AB} = \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} 6 \\ 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 5 \\ -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 4 \end{pmatrix}$ $\overrightarrow{AB} \cdot \overrightarrow{BC} = 2 - 6 + 4 \quad \text{oe must be seen} = 0$ hence $ABC = 90^\circ$	B1 B1 M1 A1 [4]	Or $\overrightarrow{BA}, \overrightarrow{CB}$. Allow any combination. Ignore labels. Could be part of calculation for angle ABC AG Alt methods Pythag, Cosine Rule

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(ii)	$ \overrightarrow{AB} = \sqrt{14}, \overrightarrow{BC} = \sqrt{21}$ oe $\text{Area} = \frac{1}{2}\sqrt{14}\sqrt{21}$ 8.6 oe	B1 M1 A1 [3]	At least one correct Reasonable attempt at vectors and their magnitudes Allow $\frac{7\sqrt{6}}{2}$
6 (i)	Attempt to find $(f^{-1})^{-1}$ $2xy = 1 - 5x$ or $\frac{1}{2x} = y + \frac{5}{2}$ Allow 1 sign error $x = \frac{1}{2y+5}$ oe Allow 1 sign error (total) $(f(x)) = \frac{1}{2x+5}$ for $x \geq -\frac{9}{4}$ (Allow $-\frac{9}{4} \leq x \leq \infty$)	M1 A1 A1 A1 B1 [5]	Or with x/y transposed. Or with x/y transposed. Allow $x = \frac{\frac{1}{2}}{y + \frac{5}{2}}$. Allow $\frac{\frac{1}{2}}{x + \frac{5}{2}}$. Condone $x > \frac{-9}{4}, (\frac{-9}{4}, \infty)$ (etc.)
(ii)	$f^{-1}\left(\frac{1}{x}\right) = \frac{1 - \frac{5}{x}}{\frac{2}{x}}$ $\frac{x-5}{2}$ or $\frac{1}{2}x - \frac{5}{2}$	M1 A1 [2]	Reasonable attempt to find $f^{-1}\left(\frac{1}{x}\right)$.
7 (i)	$(9-p)^2 + (3p)^2 = 169$ $10p^2 - 18p - 88 (=0)$ oe $p = 4$ or $-11/5$ oe	M1 A1 A1 [3]	Or $\sqrt{\quad} = 13$ 3-term quad
(ii)	Gradient of given line $= -\frac{2}{3}$ Hence gradient of $AB = \frac{3}{2}$ $\frac{3}{2} = \frac{3p}{9-p}$ oe eg $\left(\frac{-2}{3}\right)\left(\frac{3p}{9-p}\right) = 1$ (includes previous M1) $p = 3$	B1 M1 M1 A1 [4]	Attempt using $m_1m_2 = -1$ Or vectors $\begin{pmatrix} 9-p \\ 3p \end{pmatrix} \cdot \begin{pmatrix} 3 \\ -2 \end{pmatrix}$
8 (i)	$-(x+1)^{-2} - 2(x+1)^{-3}$	M1A1 A1 [3]	M1 for recognisable attempt at differentn. Allow $\frac{-x^2 - 4x - 3}{(x+1)^4}$ from Q rule. (A2,1,0)

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(ii)	$f'(x) < 0$ hence decreasing	B1 [1]	Dep. on <i>their</i> (i) < 0 for $x > 1$
(iii)	$\frac{-1}{(x+1)^2} - \frac{2}{(x+1)^3} = 0 \text{ or } \frac{-x^2 - 4x - 3}{(x+1)^4} = 0$ $\frac{-(x+1) - 2}{(x+1)^3} = 0 \rightarrow -x - 1 - 2 = 0 \text{ or}$ $-x^2 - 4x - 3 = 0$ $x = -3, y = -1/4$	M1* M1 Dep* A1A1 [4]	Set $\frac{dy}{dx}$ to 0 OR mult by $(x+1)^3$ or $(x+1)^5$ (i.e. $\times$ mult) $\times$ multn $\rightarrow -(x+1)^3 - 2(x+1)^2 = 0$ $(-3, -1/4)$ www scores 4/4
9 (a)	$2222/17 (=131 \text{ or } 130.7)$ $131 \times 17 (=2227)$ $-2222 + 2227 = 5$	M1 M1 A1 [3]	Ignore signs. Allow $2239/17 \rightarrow 131.7$ or 132 Ignore signs. Use 131. 5 www gets 3/3
(b)	$r = \frac{2 \cos \theta}{\sqrt{3}}$ soi oe $(-1 <) \frac{2 \cos \theta}{\sqrt{3}} < 1$ or $(0 <) \frac{2 \cos \theta}{\sqrt{3}} < 1$ soi $\pi/6, 5\pi/6$ soi (but dep. on M1) $\pi/6 < \theta < 5\pi/6$ cao	B1 M1 ^h A1A1 A1 [5]	Ft on <i>their</i> r . Ignore a 2nd inequality on LHS Allow $30^\circ, 150^\circ$. Accept $\leq$
10 (i)	$\frac{dy}{dx} = 6 - 6x$ At $x = 2$, gradient $= -6$ soi $y - 9 = -6(x - 2)$ oe Expect $y = -6x + 21$ When $y = 0$, $x = 3\frac{1}{2}$ cao	B1 B1 ^h M1 A1 [4]	Line through $(2, 9)$ and with gradient <i>their</i> -6
(ii)	Area under curve: $\int 9 + 6x - 3x^2 dx = 9x + 3x^2 - x^3$ $(27 + 27 - 27) - (18 + 12 - 8)$ Area under tangent: $\frac{1}{2} \times \frac{3}{2} \times 9 (= \frac{27}{4})$ Area required $\frac{27}{4} - 5 = \frac{7}{4}$	B2,1,0 M1 B1 ^h A1 [5]	Allow unsimplified terms Apply limits 2,3. Expect 5 OR $\int_2^{7/2} (-6x + 21) dx (\rightarrow \frac{27}{4})$. Ft on <i>their</i> $-6x + 21$ and/or <i>their</i> $7/2$.

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11 (i)	$OC = r \cos \alpha$ or $AC = r \sin \alpha$ or oe soi (Area $\triangle OAC \Rightarrow \frac{1}{2}r^2 \sin \alpha \cos \alpha$ $\frac{1}{2}r^2 \sin \alpha \cos \alpha = \frac{1}{2} \times \frac{1}{2}r^2 \alpha$ oe $\sin \alpha \cos \alpha = \frac{1}{2} \alpha$	M1 A1 M1 A1 [4]	Or e.g. $\frac{1}{2}r^2 \alpha - \frac{1}{2}r^2 \cos \alpha \sin \alpha = \frac{1}{4}r^2 \alpha$ $\frac{1}{2}r^2 \alpha - \frac{1}{2}r^2 \cos \alpha \sin \alpha = \frac{1}{2}r^2 \cos \alpha \sin \alpha$ AG
(ii)	Perimeter $\triangle OAC = r + r \sin \alpha + r \cos \alpha = 2.4(0)r$ Perim. $ACB = r\alpha + r \sin \alpha + r - r \cos \alpha = 2.18r$ or $2.17r$ Ratio $= \frac{2.4(0)}{2.18 \text{ or } 2.17} : 1 = 1.1 : 1$	M1A1 M1A1 A1 [5]	Allow with r a number. 2.0164 gets M1A0 Allow with r a number. 0.9644 gets M1A0 Allow 2.2 www. Use of $\cos = 0.6$, $\sin = 0.8$, $\alpha = 0.9$ is PA 1
(iii)	54.3° cao	B1 [1]	