

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2014

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

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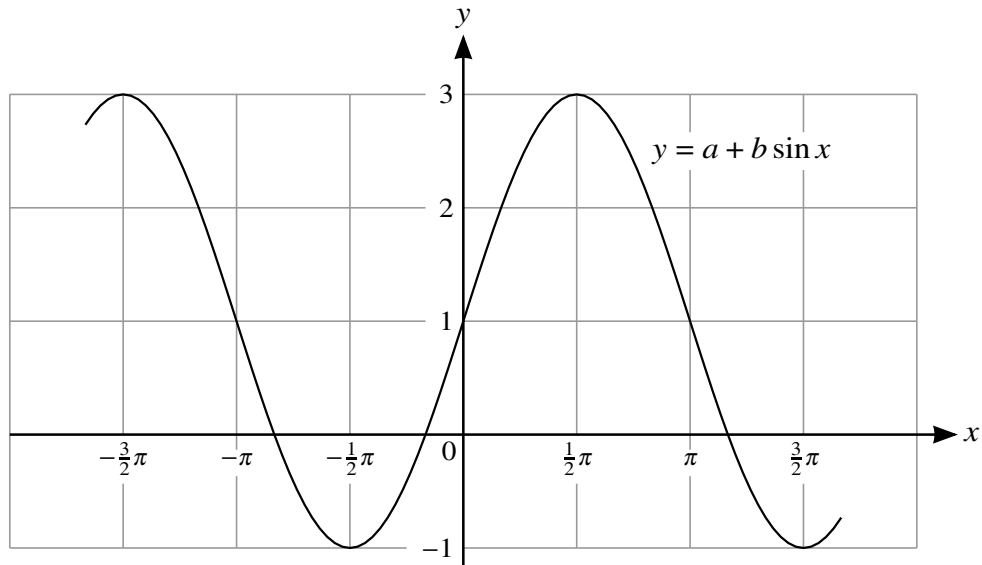
The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.

1



The diagram shows part of the graph of $y = a + b \sin x$. State the values of the constants a and b . [2]

2 (i) Express $4x^2 - 12x$ in the form $(2x + a)^2 + b$. [2]

(ii) Hence, or otherwise, find the set of values of x satisfying $4x^2 - 12x > 7$. [2]

3 Find the term independent of x in the expansion of $\left(4x^3 + \frac{1}{2x}\right)^8$. [4]

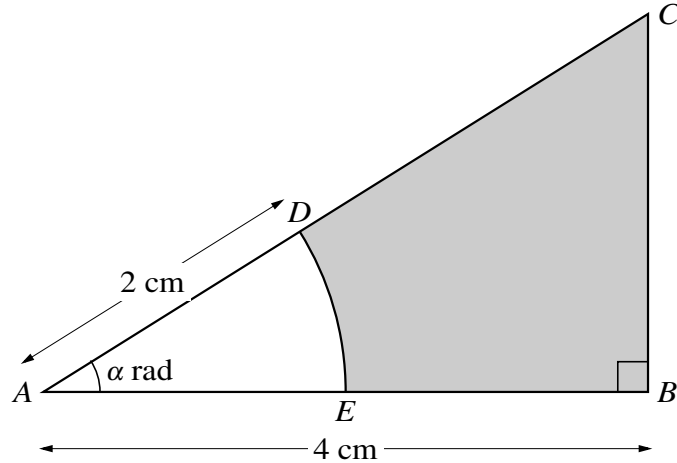
4 A curve has equation $y = \frac{4}{(3x + 1)^2}$. Find the equation of the tangent to the curve at the point where the line $x = -1$ intersects the curve. [5]

5 An arithmetic progression has first term a and common difference d . It is given that the sum of the first 200 terms is 4 times the sum of the first 100 terms.

(i) Find d in terms of a . [3]

(ii) Find the 100th term in terms of a . [2]

6



The diagram shows triangle ABC in which AB is perpendicular to BC . The length of AB is 4 cm and angle CAB is α radians. The arc DE with centre A and radius 2 cm meets AC at D and AB at E . Find, in terms of α ,

- (i) the area of the shaded region, [3]
- (ii) the perimeter of the shaded region. [3]

- 7 The coordinates of points A and B are $(a, 2)$ and $(3, b)$ respectively, where a and b are constants. The distance AB is $\sqrt{125}$ units and the gradient of the line AB is 2. Find the possible values of a and of b . [6]

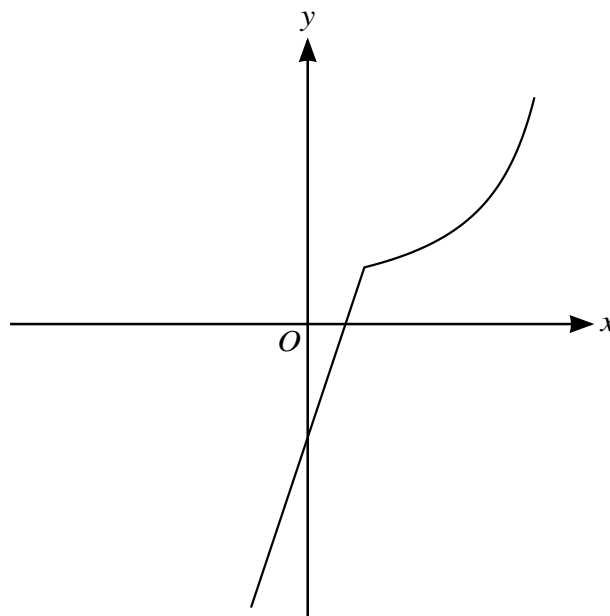
- 8 Relative to an origin O , the position vectors of points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} 3p \\ 4 \\ p^2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} -p \\ -1 \\ p^2 \end{pmatrix}.$$

- (i) Find the values of p for which angle AOB is 90° . [3]
- (ii) For the case where $p = 3$, find the unit vector in the direction of $\overrightarrow{BA}$. [3]

- 9 (i) Prove the identity $\frac{\sin \theta}{1 - \cos \theta} - \frac{1}{\sin \theta} \equiv \frac{1}{\tan \theta}$. [4]
- (ii) Hence solve the equation $\frac{\sin \theta}{1 - \cos \theta} - \frac{1}{\sin \theta} = 4 \tan \theta$ for $0^\circ < \theta < 180^\circ$. [3]

[Questions 10, 11 and 12 are printed on the next page.]



The diagram shows the function f defined for $-1 \leq x \leq 4$, where

$$f(x) = \begin{cases} 3x - 2 & \text{for } -1 \leq x \leq 1, \\ \frac{4}{5-x} & \text{for } 1 < x \leq 4. \end{cases}$$

- (i) State the range of f . [1]
- (ii) Copy the diagram and on your copy sketch the graph of $y = f^{-1}(x)$. [2]
- (iii) Obtain expressions to define the function f^{-1} , giving also the set of values for which each expression is valid. [6]
- 11** A line has equation $y = 2x + c$ and a curve has equation $y = 8 - 2x - x^2$.
- (i) For the case where the line is a tangent to the curve, find the value of the constant c . [3]
- (ii) For the case where $c = 11$, find the x -coordinates of the points of intersection of the line and the curve. Find also, by integration, the area of the region between the line and the curve. [7]
- 12** A curve is such that $\frac{dy}{dx} = x^{\frac{1}{2}} - x^{-\frac{1}{2}}$. The curve passes through the point $(4, \frac{2}{3})$.
- (i) Find the equation of the curve. [4]
- (ii) Find $\frac{d^2y}{dx^2}$. [2]
- (iii) Find the coordinates of the stationary point and determine its nature. [5]

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MATHEMATICS

9709/12

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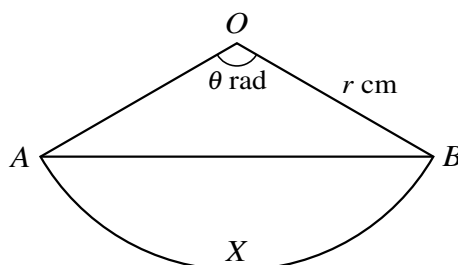
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- 1 Find the coordinates of the point at which the perpendicular bisector of the line joining (2, 7) to (10, 3) meets the x -axis. [5]
- 2 Find the coefficient of x^2 in the expansion of $(1 + x^2) \left(\frac{x}{2} - \frac{4}{x} \right)^6$. [5]
- 3 The reflex angle θ is such that $\cos \theta = k$, where $0 < k < 1$.
- (i) Find an expression, in terms of k , for
- (a) $\sin \theta$, [2]
- (b) $\tan \theta$. [1]
- (ii) Explain why $\sin 2\theta$ is negative for $0 < k < 1$. [2]

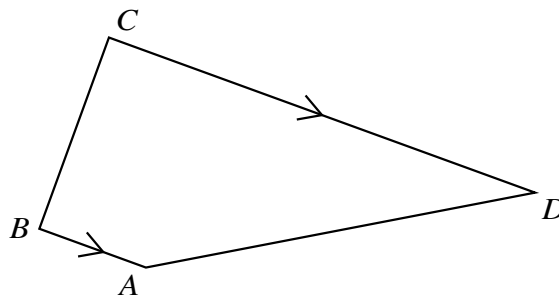
4



The diagram shows a sector of a circle with radius r cm and centre O . The chord AB divides the sector into a triangle AOB and a segment AXB . Angle AOB is θ radians.

- (i) In the case where the areas of the triangle AOB and the segment AXB are equal, find the value of the constant p for which $\theta = p \sin \theta$. [2]
- (ii) In the case where $r = 8$ and $\theta = 2.4$, find the perimeter of the segment AXB . [3]
- 5 (i) Prove the identity $\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} \equiv \tan \theta$. [4]
- (ii) Solve the equation $\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} + 2 = 0$ for $0^\circ \leq \theta \leq 360^\circ$. [3]
- 6 The 1st, 2nd and 3rd terms of a geometric progression are the 1st, 9th and 21st terms respectively of an arithmetic progression. The 1st term of each progression is 8 and the common ratio of the geometric progression is r , where $r \neq 1$. Find
- (i) the value of r , [4]
- (ii) the 4th term of each progression. [3]

7



The diagram shows a trapezium $ABCD$ in which BA is parallel to CD . The position vectors of A , B and C relative to an origin O are given by

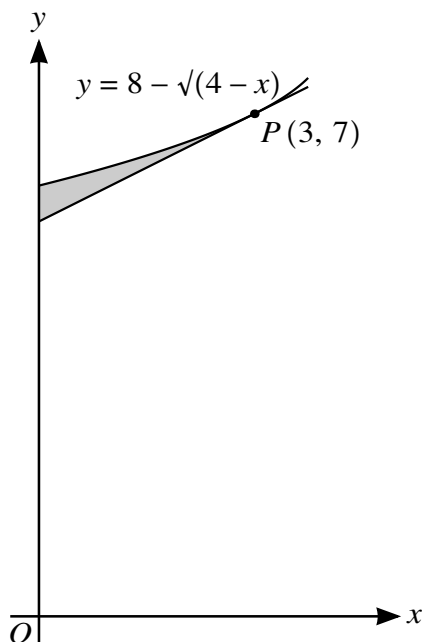
$$\vec{OA} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}, \quad \vec{OB} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} \quad \text{and} \quad \vec{OC} = \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}.$$

(i) Use a scalar product to show that AB is perpendicular to BC . [3]

(ii) Given that the length of CD is 12 units, find the position vector of D . [4]

- 8 The equation of a curve is such that $\frac{d^2y}{dx^2} = 2x - 1$. Given that the curve has a minimum point at $(3, -10)$, find the coordinates of the maximum point. [8]

9



The diagram shows part of the curve $y = 8 - \sqrt{4 - x}$ and the tangent to the curve at $P(3, 7)$.

(i) Find expressions for $\frac{dy}{dx}$ and $\int y \, dx$. [5]

(ii) Find the equation of the tangent to the curve at P in the form $y = mx + c$. [2]

(iii) Find, showing all necessary working, the area of the shaded region. [4]

10 Functions f and g are defined by

$$f : x \mapsto 2x - 3, \quad x \in \mathbb{R},$$

$$g : x \mapsto x^2 + 4x, \quad x \in \mathbb{R}.$$

(i) Solve the equation $ff(x) = 11$. [2]

(ii) Find the range of g . [2]

(iii) Find the set of values of x for which $g(x) > 12$. [3]

(iv) Find the value of the constant p for which the equation $gf(x) = p$ has two equal roots. [3]

Function h is defined by $h : x \mapsto x^2 + 4x$ for $x \geq k$, and it is given that h has an inverse.

(v) State the smallest possible value of k . [1]

(vi) Find an expression for $h^{-1}(x)$. [4]

MATHEMATICS

9709/13

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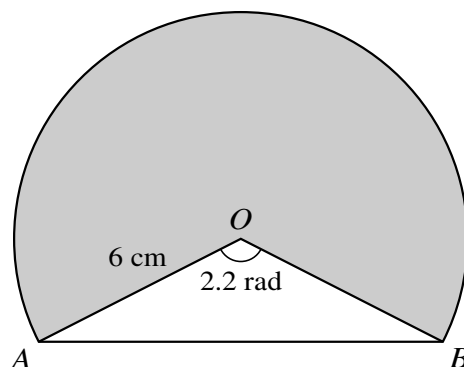
- 1 Find the coefficient of x in the expansion of $\left(x^2 - \frac{2}{x}\right)^5$. [3]

- 2 The first term in a progression is 36 and the second term is 32.

(i) Given that the progression is geometric, find the sum to infinity. [2]

(ii) Given instead that the progression is arithmetic, find the number of terms in the progression if the sum of all the terms is 0. [3]

3



The diagram shows part of a circle with centre O and radius 6 cm . The chord AB is such that angle $AOB = 2.2$ radians. Calculate

(i) the perimeter of the shaded region, [3]

(ii) the ratio of the area of the shaded region to the area of the triangle AOB , giving your answer in the form $k : 1$. [3]

- 4 (i) Prove the identity $\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$. [3]

(ii) Hence solve the equation $\frac{\tan x + 1}{\sin x \tan x + \cos x} = 3 \sin x - 2 \cos x$ for $0 \leq x \leq 2\pi$. [3]

- 5 A function f is such that $f(x) = \frac{15}{2x+3}$ for $0 \leq x \leq 6$.

(i) Find an expression for $f'(x)$ and use your result to explain why f has an inverse. [3]

(ii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1} . [4]

- 6 A curve is such that $\frac{dy}{dx} = \frac{12}{\sqrt{(4x+a)}}$, where a is a constant. The point $P(2, 14)$ lies on the curve and the normal to the curve at P is $3y + x = 44$.

(i) Show that $a = 8$. [3]

(ii) Find the equation of the curve. [4]

- 7 The position vectors of points A , B and C relative to an origin O are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ -1 \\ 7 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix}.$$

(i) Show that angle $BAC = \cos^{-1}(\frac{1}{3})$. [5]

(ii) Use the result in part (i) to find the exact value of the area of triangle ABC . [3]

- 8 (i) Express $2x^2 - 10x + 8$ in the form $a(x + b)^2 + c$, where a , b and c are constants, and use your answer to state the minimum value of $2x^2 - 10x + 8$. [4]

(ii) Find the set of values of k for which the equation $2x^2 - 10x + 8 = kx$ has no real roots. [4]

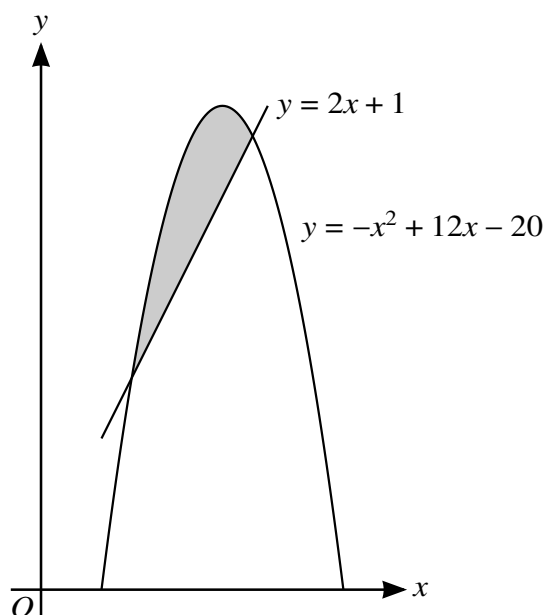
- 9 The base of a cuboid has sides of length x cm and $3x$ cm. The volume of the cuboid is 288 cm^3 .

(i) Show that the total surface area of the cuboid, $A \text{ cm}^2$, is given by

$$A = 6x^2 + \frac{768}{x}. \quad [3]$$

(ii) Given that x can vary, find the stationary value of A and determine its nature. [5]

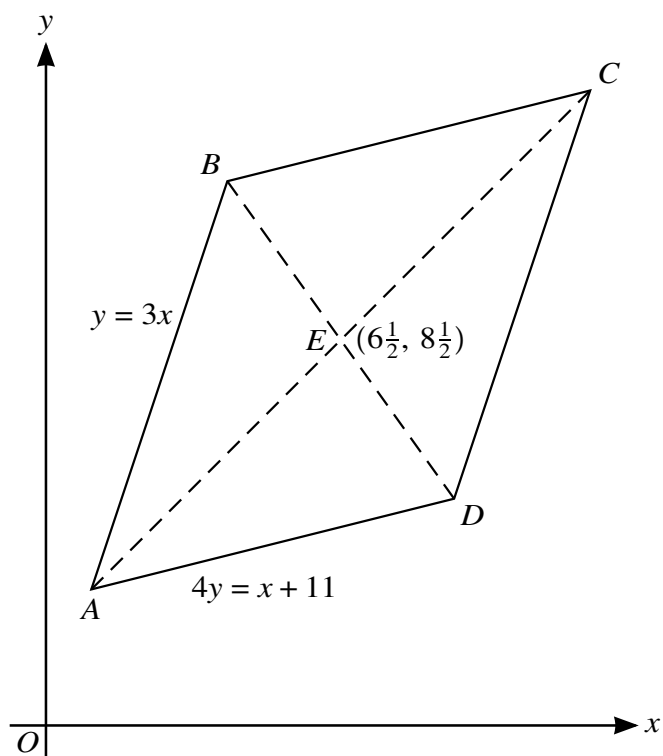
10



The diagram shows the curve $y = -x^2 + 12x - 20$ and the line $y = 2x + 1$. Find, showing all necessary working, the area of the shaded region. [8]

[Question 11 is printed on the next page.]

11



The diagram shows a parallelogram $ABCD$, in which the equation of AB is $y = 3x$ and the equation of AD is $4y = x + 11$. The diagonals AC and BD meet at the point $E(6\frac{1}{2}, 8\frac{1}{2})$. Find, by calculation, the coordinates of A , B , C and D . [9]

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