

MARK SCHEME for the May/June 2014 series

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

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B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol ∇ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
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Penalties

MR–1	A penalty of MR–1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
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1	$a = 1, \quad b = 2$	B1B1 [2]	Or $1 + 2 \sin x$
2	(i) $(2x-3)^2 - 9$ (ii) $2x-3 > 4 \quad 2x-3 < -4$ $x > 3\frac{1}{2}$ (or) $x < -\frac{1}{2}$ cao Allow $-\frac{1}{2} > x > 3\frac{1}{2}$ OR $4x^2 - 12x - 7 \rightarrow (2x-7)(2x+1)$ $x > 3\frac{1}{2}$ (or) $x < -\frac{1}{2}$ cao Allow $-\frac{1}{2} > x > 3\frac{1}{2}$	B1B1 [2] M1 A1 M1 A1 [2]	For -3 and -9 At least one of these statements Allow 'and' $3\frac{1}{2}, -\frac{1}{2}$ soi scores first M1 Attempt to solve 3-term quadratic Allow 'and' $3\frac{1}{2}, -\frac{1}{2}$ soi scores first M1
3	$[{}^8C_6 \text{ or } 28] \times [16 \text{ or } 4^2] (x^6) \times \left[\frac{1}{(64 \text{ or } 2^6)(x^6)} \right]$ 7	B1B1B1 B1 [4]	Seen in expansion ok. Allow ${}^8C_2$ Identified as answer
4	$\frac{dy}{dx} = [-2 \times 4(3x+1)^{-3}] \times [3]$ When $x = -1, \frac{dy}{dx} = 3$ When $x = -1, y = 1$ soi $y - 1 = 3(x + 1) \quad (\rightarrow y = 3x + 4)$	B1B1 B1 B1 B1 ✓ [5]	$[-2 \times 4u^{-3}] \times [3]$ is B0B1 unless resolved Ft on <i>their</i> '3' only (not $-\frac{1}{3}$). Dep on diffn
5	(i) $200/2(2a + 199d) = 4 \times 100/2(2a + 99d)$ $d = 2a$ cao (ii) $a + 99d = a + 99 \times 2a$ $199a$ cao	M1A1 A1 [3] M1 A1 [2]	Correct formula used (once) M1, correct eqn A1 Sub. <i>their</i> part(i) into correct formula
6	(i) area $\Delta = \frac{1}{2} \times 4 \times 4 \tan \alpha$ oe soi Area sector $= \frac{1}{2} \times 2^2 \alpha$ oe soi Shaded area $= 8 \tan \alpha - 2\alpha$ cao (ii) $DC = \frac{4}{\cos \alpha} - 2$ oe soi Arc $DE = 2\alpha$ soi anywhere provided clear Perimeter $= \frac{4}{\cos \alpha} + 4 \tan \alpha + 2\alpha$ cao	B1 B1 B1 [3] B1 B1 B1 [3]	$4 \tan \alpha = \sqrt{16 / \cos^2 \alpha - 16}$. (Can also score in answer) Accept θ throughout Little/no working – accept terms in answer $\frac{4}{\cos \alpha} = \sqrt{16 + 16 \tan^2 \alpha}$. Can score in answer Little/no working – accept terms in answer

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7 $(a-3)^2 + (2-b)^2 = 125$ oe $\frac{2-b}{a-3} = 2$ oe $(a-3)^2 + (2a-6)^2 = 125$ (sub for a or b) $(5)(a+2)(a-8) (=0)$ Attempt factorise/solve $a = -2$ or 8, $b = 12$ or -8	B1 B1 M1 M1 A1A1 [6]	Or $1/4(2-b)^2 + (2-b)^2 = 125$ Or $(5)(b-12)(b+8) (=0)$ Answers (no working) after 2 correct eqns score SCB1B1 for each correct pair (a, b)
8 (i) $OA \cdot OB = -3p^2 - 4 + p^4$ soi $(p^2 + 1)(p^2 - 4) = 0$ oe e.g. with substitution $p = \pm 2$ and no other real solutions (ii) $\overrightarrow{BA} = \begin{pmatrix} 9 \\ 4 \\ 9 \end{pmatrix} - \begin{pmatrix} -3 \\ -1 \\ 9 \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix}$ $\overrightarrow{BA} = \sqrt{12^2 + 5^2} = 13$ and division by <i>their</i> 13 Unit vector $= \frac{1}{13} \begin{pmatrix} 12 \\ 5 \\ 0 \end{pmatrix}$ cao	M1 M1 A1 [3] M1 M1 A1 [3]	Put $= 0$ (soi) and attempt to solve Reversed subtraction can score M1M1A0
9 (i) $LHS \equiv \frac{\sin^2 \theta - (1 - \cos \theta)}{(1 - \cos \theta) \sin \theta}$ cao $\equiv \frac{1 - \cos^2 \theta - 1 + \cos \theta}{(1 - \cos \theta) \sin \theta}$ $\equiv \frac{\cos \theta (1 - \cos \theta)}{(1 - \cos \theta) \sin \theta}$ $\equiv \frac{1}{\tan \theta}$ (ii) $\tan \theta = (\pm) \frac{1}{2}$ 26.6°, 153.4°	B1 M1 M1 A1 [4] M1 A1A1 $\sqrt{h}$ [3]	Put over common denominator Use $s^2 = 1 - c^2$ oe Correct factorisation from line 2 AG Ft for $180 - 1^{st}$ answer

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10 (i) $-5 \leq f(x) \leq 4$ For $f(x)$ allow x or y; allow $<$, $[-5, 4]$, $(-5, 4)$ (ii) $f^{-1}(x)$ approximately correct (independent of f) Closed region between $(1, 1)$ and $(4, 4)$; line reaches x-axis (iii) LINE: $f^{-1}(x) = \frac{1}{3}(x+2)$ for $-5 \leq x \leq 1$ CURVE: $5 - y = \frac{4}{x}$ OR $x = 5 - \frac{4}{y}$ $f^{-1}(x) = 5 - \frac{4}{x}$ oe for $1 < x \leq 4$	B1 [1] B1 DB1 [2] B1 B1B1 M1 A1 B1 [6]	Allow less explicit answers (eg $-5 \rightarrow 4$) Ignore line $y = x$ Allow $y = \dots$ but must be a function of x cao but allow $<$ cao cao but allow $<$ or $<$
11 (i) $x^2 + 4x + c - 8 (= 0)$ $16 - 4(c - 8) = 0$ $c = 12$ OR $-2 - 2x = 2 \rightarrow x = (-2)$ $-4 + c = 8 + 4 - 4$ $c = 12$ (ii) $x^2 + 4x + 3 \rightarrow (x + 1)(x + 3) (= 0) \rightarrow$ $x = -1$ or -3 $\int (8 - 2x - x^2) - [\int (2x + 11) \text{ or area of trapezium}]$ $\left[8x - x^2 - \frac{x^3}{3} \right] - \left[x^2 + 11x \right] \text{ or } \left[8x - x^2 - \frac{x^3}{3} \right] - \frac{1}{2}(5+9) \times 2$ Apply <i>their</i> limits to at least integral for curve $1\frac{1}{3}$ oe	M1 M1 A1 M1 M1 A1 [3] B1 M1M1 A1B1 M1 A1 [7]	Attempt to simplify to 3-term quadratic Apply $b^2 - 4ac = 0$. '$= 0$' soi Equate derivs of curve and line. Expect $x = -2$ Sub <i>their</i> $x = -2$ into line and curve, and equate Attempt to integrate. At some stage subtract A1 for curve, B1 for line OR $\left[-3x - 2x^2 - \frac{x^3}{3} \right] \text{ A2, 1, 0}$ For M marks allow reversed limits and/or subtraction of areas but then final A0

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12 (i) $y = \frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}} + (c)$ oe $\frac{2}{3} = \frac{16}{3} - 4 + c$ $c = -\frac{2}{3}$ (ii) $\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}x^{-\frac{3}{2}}$ oe (iii) $x^{\frac{1}{2}} - x^{-\frac{1}{2}} = 0 \rightarrow \frac{x-1}{\sqrt{x}} = 0$ $x = 1$ When $x = 1$, $y = \frac{2}{3} - 2 - \frac{2}{3} = -2$ When $x = 1$, $\frac{d^2y}{dx^2}(=1) > 0$ Hence minimum	B1B1 M1 A1 [4] B1B1 [2] M1 A1 M1A1 B1 [5]	Attempt to integrate Sub $\left(4, \frac{2}{3}\right)$. Dependent on c present Equate to zero and attempt to solve Sub. <i>their</i> '1' into <i>their</i> 'y' Everything correct on final line. Also dep on correct (ii). Accept other valid methods
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1 (2, 7) to (10, 3) Mid-point (6, 5) Gradient = $-\frac{1}{2}$ Perp gradient = 2 Eqn $y - 5 = 2(x - 6)$ Sets y to 0, $\rightarrow (3\frac{1}{2}, 0)$	B1 B1 B1✓ M1 A1 [5]	co co co Must be correct form of Perp co $x = 3\frac{1}{2}$ only is ok.
2 $(1 + x^2)\left(\frac{x}{2} - \frac{4}{x}\right)^6$. Term in $x^2 = 15 \times \frac{1}{16} \times (-4)^2 = 15$ Constant term = $20 \times \frac{1}{8} \times (-4)^3 = -160$ Coefficient of $x^2 = -145$	B1 B1 B1 B1 B1✓ [5]	B1 unsimplified. B1 15. B1 unsimplified. B1 -160 Uses 2 terms. ✓ on previous answers
3 reflex angle θ is such that $\cos\theta = k$, (i) (a) $\sin\theta = -\sqrt{1 - k^2}$ (b) Uses $t=s/c \rightarrow \frac{-\sqrt{1 - k^2}}{k}$ (ii) θ is in 4th quadrant. 2θ lies between 540° and 720° $\sin 2\theta$ is negative in both these quadrants.	B1 B1 [2] B1✓ [1] B1 B1 [2]	(-) B1 rest B1 ✓ for (i) $\div k$. co co
4 (i) $\frac{1}{2}r^2\theta = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$ $\rightarrow 2\sin\theta = \theta \rightarrow p = 2$. (ii) Chord length = $8\sin 1.2 \times 2$ (14.9) (or from cosine rule) Arc length = 2.4×8 (19.2) Perimeter = sum of these = 34.1	B1 B1 [2] M1 B1 A1 [3]	Correct equation. All ok – answer given. Needs $\times 2$. Any method ok. co
5 (i) $\frac{1}{\cos\theta} - \frac{\cos\theta}{1 + \sin\theta} \equiv \tan\theta$. LHS = $\frac{1 + s - c^2}{c(1 + s)} = \frac{s^2 + s}{c(1 + s)} = \frac{s}{c}$ = $\tan\theta$ (ii) $\rightarrow \tan\theta + 2 = 0$ ie $\tan\theta = -2$ $\rightarrow \theta = 116.6^\circ$ or 296.6°	M1 M1M1 A1 [4] M1 A1 A1✓ [3]	Correct addition of fractions Use of $s^2 + c^2 = 1$. $(1 + s)$ cancelled. $\rightarrow$ answer given. Uses part (i) . Allow $\tan\theta = \pm 2$ Co. ✓ for $180^\circ +$ and no other solutions in the range.

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6 (i) GP $8 \quad 8r \quad 8r^2$ AP $8 \quad 8 + 8d \quad 8 + 20d$ $8r = 8 + 8d$ and $8r^2 = 8 + 20d$ Eliminates $d \rightarrow 2r^2 - 5r + 3 = 0$ $\rightarrow r = 1.5$ (or 1) (ii) 4th term of GP $= ar^3 = 8 \times 27/8 = 27$ If $r = 1.5$, $d = 0.5$ 4th term of AP $= a + 3d = 9\frac{1}{2}$	B1 B1 M1 A1 [4] B1✓ M1A1 [3]	B1 for each equation. Correct elimination. co (no penalty for including $r = 1$) co needs $a + 3d$ and correct method for d
7 (i) $(\mathbf{b} - \mathbf{a}) \cdot (\mathbf{b} - \mathbf{c}) = \begin{pmatrix} -2 \\ -1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 2 \\ 4 \end{pmatrix}$ $\rightarrow -6 - 2 + 8 = 0 \rightarrow 90^\circ$ (ii) Unit vector $= \frac{1}{3} \begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ $\mathbf{CD} = 12 \times \text{unit vector} = \pm \begin{pmatrix} 8 \\ 4 \\ -8 \end{pmatrix}$ $\mathbf{OD} = \mathbf{OC} + \mathbf{CD} = \begin{pmatrix} 12 \\ 9 \\ -2 \end{pmatrix}$	M1 M1 A1 [3] M1 M1 M1 A1 [4]	$\mathbf{AB} = \mathbf{b} - \mathbf{a}$ once ($\mathbf{a} - \mathbf{b}$ is ok) Use of $x_1x_2\dots$ with $\mathbf{AB}$ and $\mathbf{CB}$ All correct Method for unit vector. Knows to multiply by 12 or $\pm 4\mathbf{BA}$ Correct method. co
8 $\frac{d^2y}{dx^2} = 2x - 1$ $\rightarrow \int \frac{dy}{dx} = x^2 - x + c$ $= 0$ when $x = 3 \rightarrow c = -6$ $x^2 - x - 6 = 0$ when $x = -2$ (or 3) $\rightarrow \int y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 6x (+k)$ $= -10$ when $x = 3$ $\rightarrow k = 3\frac{1}{2}$ $\rightarrow y = 10\frac{5}{6}$	B1 M1 A1 A1 B1✓B1✓ M1 A1 [8]	Correct integration (ignore $+c$) Uses a constant of integration. co Puts dy/dx to 0 ✓ first 2 terms, ✓ for cx. Correct method for k Co $\rightarrow 10.8$

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9 $y = 8 - \sqrt{4 - x}$ (i) $\frac{dy}{dx} = -\frac{1}{2}(4 - x)^{-\frac{1}{2}} \times -1$ $\int y \, dx = 8x - \frac{(4 - x)^{\frac{3}{2}}}{\frac{3}{2}} \div -1$ (ii) Eqn $y - 7 = \frac{1}{2}(x - 3)$ $\rightarrow y = \frac{1}{2}x + 5\frac{1}{2}$ (iii) Area under curve = $\int$ from 0 to 3 (58/3) Area under line = $\frac{1}{2}(5\frac{1}{2} + 7) \times 3$ Or $\left[\frac{1}{4}x^2 + \frac{11x}{2} \right]$ from 0 to 3 $\rightarrow \frac{58}{3} - \frac{75}{4} = \frac{7}{12}$	B1 B1 3 $\times$ B1 [5] M1A1 [2] M1 M1 M1 A1 [4]	Without (-1). For $(\times -1)$. B1 for "8x" and "+c". B1 for all except $\div(-1)$. B1 for $\div(-1)$. (n.b. these 5 marks can be gained in(ii) or (iii)) M1 unsimplified. A1 as $y=mx+c$ Use of limits – needs use of "0" Correct method M1 Subtraction. A1 co
10 $f : x \mapsto 2x - 3, x \in \mathbb{R},$ $g : x \mapsto x^2 + 4x, x \in \mathbb{R}.$ (i) $ff = 2(2x - 3) - 3$ Solves $= 11 \rightarrow x = 5$ (or $2x - 3 = 11, x = 7. 2x - 3 = 7 \rightarrow x = 5$) (ii) min at $x = -2$ $\rightarrow$ Range ≥ -4 (iii) $x^2 + 4x - 12 (> 0)$ $\rightarrow x = 2$ or -6 $\rightarrow x < -6, x > 2.$ (iv) $gf(x) = (2x - 3)^2 + 4(2x - 3) = p$ $\rightarrow 4x^2 - 4x - 3 - p = 0$ Uses "$b^2 - 4ac$" $16 = 16(-3 - p)$ $\rightarrow p = -4$ (v) -2 (vi) $y = (x + 2)^2 - 4$ $\sqrt{y + 4} = x + 2$ $h^{-1}(x) = \sqrt{x + 4} - 2$	M1 A1 [2] M1 A1 [2] M1 A1 A1 [3] B1 M1 A1 [3] B1 [1] B2,1 M1 A1 [4]	Either forms ff correctly, or solves 2 equations co Any valid method – could be guesswork. Makes quadratic $= 0 + 2$ solutions Correct limits – even if $>, <, \geq, \leq, =$ co co unsimplified Use of discriminant co co -1 for each error Correct order of operations co with x, not y. $\pm$ left A0.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only – often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR –2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 $\left(x^2 - \frac{2}{x}\right)^5$ Term in x is $10 \times (x^2)^2 \times \left(\frac{-2}{x}\right)^3$ Coefficient = $-80(x)$	B1 B1 B1 [3]	B1 10 or ${}^5C_2$ or ${}^5C_3$, B1 $\left(\frac{-2}{x}\right)^3$ co Must be identified
2 36, 32, ... (i) $r = \frac{8}{9}$ $S_\infty = (\text{their } a) \div (1 - \text{their } r)$ $S_\infty = 36 \div \frac{1}{9} = 324$ (ii) $d = -4$ $0 = \frac{n}{2} (72 + (n-1)(-4))$ $\rightarrow n = 19$	M1 A1 [2] B1 M1 A1 [3]	Method for r and S_∞ ok. ($r < 1$) co co S_n formula ok and a value for d ($\neq \frac{8}{9}$) Condone $n = 0$ but no other soln
3 (i) $s = r\theta$ Angle of major arc = $2\pi - 2.2 = (4.083)$ Perimeter = $12 + 24.5 = 36.5$ or $12\pi - 1.2$ (or full circle – minor arc B1) (ii) Area of major sector = $\frac{1}{2}r^2\theta = (73.49)$ Area of triangle = $\frac{1}{2} \cdot 6^2 \sin 2.2 = (14.55)$ Ratio = $5.05 : 1$ (Allow $5.03 \rightarrow 5.06$)	M1 B1 A1 [3] M1 M1 A1 [3]	Used with major or minor arc Could be gained in (ii). co Used with major/minor sector. Correct formula or method. ($2\pi - 2.2$)/$\sin 2.2$ gets M1M1 co
4 $\frac{\tan x + 1}{\sin x \tan x + \cos x} \equiv \sin x + \cos x$ (i) LHS $\frac{\left(\frac{s}{c}\right) + 1}{\left(\frac{s^2}{c} + c\right)} = \frac{s+c}{s^2+c^2}$ = RHS (ii) $s + c = 3s - 2c$ $\rightarrow \tan x = \frac{3}{2}$ Allow $\cos^2 = \frac{4}{13}$, $\sin^2 = \frac{9}{13}$ $\rightarrow x = 0.983$ and 4.12 or 4.13	M1 M1 A1 [3] M1 A1 A1✓ [3]	Use of $t = s/c$ twice Correct algebra and use of $s^2 + c^2 = 1$ AG all ok Uses (i) and $t = \frac{s}{c}$ $t = \frac{2}{3}$ or 0 is M0 co. ✓ 1st + π, providing no excess solns in range. Allow 0.313π, 1.31π

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5 $f(x) = \frac{15}{2x+3}$ (i) $f'(x) = \frac{-15}{(2x+3)^2} \times 2$ $()^2$ always +ve $\rightarrow f'(x) < 0$ (No turning points) – therefore an inverse (ii) $y = \frac{15}{2x+3} \rightarrow 2x+3 = \frac{15}{y}$ $\rightarrow x = \frac{\frac{15}{y}-3}{2} \rightarrow \frac{15-3y}{2y}$ (Range) $0 \leq f^{-1}(x) \leq 6$. Allow $0 \leq y \leq 6, [0,6]$ (Domain) $1 \leq x \leq 5$. Allow $[1, 5]$	B1 B1 B1 ↗ [3] M1 A1 B1 B1 [4]	Without the “$\times 2$”. For “$\times 2$” (indep of 1st B1). ↗ providing $()^2$ in $f'(x)$. 1–1 insuff. Order of ops – allow sign error co as function of x. Allow $y = \dots$ For range/domain ignore letters unless range/domain not identified
6 $\frac{dy}{dx} = \frac{12}{\sqrt{4x+a}}$ P (2, 14) Normal $3y + x = 44$ (i) m of normal $= -\frac{1}{3}$ $\frac{dy}{dx} = 3 = \frac{12}{\sqrt{4x+a}} \rightarrow a = 8$ (ii) $\int y = 12(4x+a)^{\frac{1}{2}} \div \frac{1}{2} \div 4 (+c)$ Uses (2, 14) $c = -10$	B1 M1 A1 [3] B1 B1 M1 A1 [4]	co Use of $m_1 m_2 = -1$. AG. Correct without “$\div 4$”. for “$\div 4$”. Uses in an integral only. Dep ‘c’. co All 4 marks can be given in (i)

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7 (i) Angle BAC needs sides AB, AC or BA, CA $\mathbf{AB} \cdot \mathbf{AC} = (\mathbf{b} - \mathbf{a}) \cdot (\mathbf{c} - \mathbf{a})$ $= \begin{pmatrix} 4 \\ -2 \\ 4 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 3 \\ 4 \end{pmatrix} = 10$ $= \sqrt{36} \times \sqrt{25} \cos BAC$ $\rightarrow BAC = \cos^{-1} \frac{1}{3} \quad \text{AG}$ (ii) $\sin BAC = \sqrt{1 - \frac{1}{9}}$ $\text{Area} = \frac{1}{2} \times 6 \times 5 \times \sqrt{\frac{8}{9}} = 5\sqrt{8} \text{ oe}$	B1 M1 M1M1 A1 [5] B1 M1 A1 [3]	Ignore <i>their</i> labels: One of AB, BA, AC, CA correct Use of $x_1x_2 + y_1y_2$, etc. M1 prod of moduli. M1 all linked If e.g. BA.OC max B1M1M1. If both vectors wrong 0/5. If e.g. BA.AC used $\rightarrow \cos^{-1}\left(-\frac{1}{3}\right)$ final mark A0 Use of $s^2 + c^2 = 1$ – not decimals Correct formula for area. Decimals seen A0
8 $2x^2 - 10x + 8 \rightarrow a(x + b)^2 + c$ (i) $a = 2, b = -2\frac{1}{2}, c = -4\frac{1}{2}$ $\rightarrow$ min value is $-4\frac{1}{2}$ Allow $(2\frac{1}{2}, -4\frac{1}{2})$ (ii) $2x^2 - 10x + 8 - kx = 0$ Use of “$b^2 - 4ac$” $(-10 - k)^2 - 64 < 0$ or $k^2 + 20k + 36 < 0$ $\rightarrow k = -18$ or -2 $-18 < k < -2$	$3 \times \text{B1}$ B1✓ [4] M1 M1 A1 A1 [4]	Or $2\left(x - 2\frac{1}{2}\right)^2 - 4\frac{1}{2}$ Can score by sub $x = 2\frac{1}{2}$ into original but not by differentiation Sets equation to 0 and uses discriminant correctly Realises discriminant < 0. Allow $\leq$ co Dep on 1st M1 only co
9 (i) $3x^2y = 288$ y is the height $A = 2(3x^2 + xy + 3xy)$ Sub for $y \rightarrow A = 6x^2 + \frac{768}{x}$ (ii) $\frac{dA}{dx} = 12x - \frac{768}{x^2}$ $= 0$ when $x = 4 \rightarrow A = 288$. Allow $(4, 288)$ $\frac{d^2A}{dx^2} = 12 + \frac{1536}{x^3}$ $(= 36) > 0$ Minimum	B1 M1 A1 [3] B1 M1 A1 M1 A1 [5]	co Considers at least 5 faces ($y \neq x$) co answer given co Sets differential to 0 + solution. co Any valid method co www dep on correct f'' and $x = 4$

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10 pts of intersection $2x + 1 = -x^2 + 12x - 20$ $\rightarrow x = 3, 7$ Area of trapezium $= \frac{1}{2}(4)(7 + 15) = 44$ (or $\int (2x+1) dx$ from 3 to 7 = 44) Area under curve $= -\frac{1}{3}x^3 + 6x^2 - 20x$ Uses 3 to 7 $\rightarrow (54\frac{2}{3})$ Shaded area $= 10\frac{2}{3}$ OR $\int_3^7 (-x^2 + 10x - 21) = -\frac{x^3}{3} + 5x^2 - 21x$ M1 subtraction, A1A1A1 for integrated terms, DM1 correct use of limits, A1	M1A1 M1A1 B2,1 DM1 A1 [8]	Attempt at soln of sim eqns. co Either method ok. co -1 each term incorrect Correct use of limits (Dep 1st M1) co Functions subtracted before integration Subtraction reversed allow A3A0. Limits reversed allow DM1A0
11 Sim eqns $\rightarrow A(1, 3)$ Vectors or mid-point $\rightarrow C(12, 14)$ Eqn of BC $4y = x + 44$ or CD $y = 3x - 22$ Sim eqns $\rightarrow B(4, 12)$ or $D(9, 5)$ Vectors or mid-point $\rightarrow B(4, 12)$ or $D(9, 5)$	M1 A1 M1 A1^h M1 DM1A1 DM1A1 [9]	co Allow answer only B2 Allow answer only B2^h equation ok – unsimplified Sim eqns. co Valid method (or sim eqns) co