



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2013

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

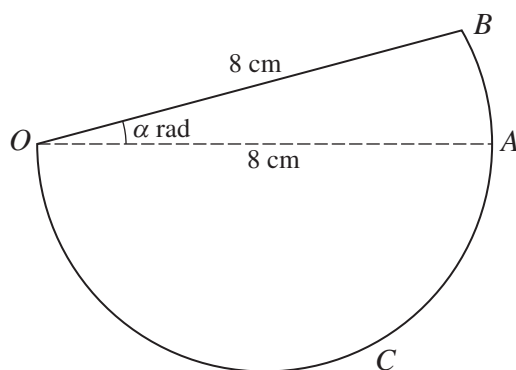
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



- 1 It is given that $f(x) = (2x - 5)^3 + x$, for $x \in \mathbb{R}$. Show that f is an increasing function. [3]
- 2 (i) In the expression $(1 - px)^6$, p is a non-zero constant. Find the first three terms when $(1 - px)^6$ is expanded in ascending powers of x . [2]
- (ii) It is given that the coefficient of x^2 in the expansion of $(1 - x)(1 - px)^6$ is zero. Find the value of p . [3]

3



In the diagram, OAB is a sector of a circle with centre O and radius 8 cm . Angle BOA is α radians. OAC is a semicircle with diameter OA . The area of the semicircle OAC is twice the area of the sector OAB .

- (i) Find α in terms of π . [3]
- (ii) Find the perimeter of the complete figure in terms of π . [2]
- 4 The third term of a geometric progression is -108 and the sixth term is 32 . Find
- (i) the common ratio, [3]
- (ii) the first term, [1]
- (iii) the sum to infinity. [2]
- 5 (i) Show that $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} \equiv \frac{1}{\sin^2 \theta - \cos^2 \theta}$. [3]
- (ii) Hence solve the equation $\frac{\sin \theta}{\sin \theta + \cos \theta} + \frac{\cos \theta}{\sin \theta - \cos \theta} = 3$, for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 6 Relative to an origin O , the position vectors of three points, A , B and C , are given by

$$\overrightarrow{OA} = \mathbf{i} + 2p\mathbf{j} + q\mathbf{k}, \quad \overrightarrow{OB} = q\mathbf{j} - 2p\mathbf{k} \quad \text{and} \quad \overrightarrow{OC} = -(4p^2 + q^2)\mathbf{i} + 2p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

- (i) Show that $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OC}$ for all non-zero values of p and q . [2]

- (ii) Find the magnitude of $\overrightarrow{CA}$ in terms of p and q . [2]

- (iii) For the case where $p = 3$ and $q = 2$, find the unit vector parallel to $\overrightarrow{BA}$. [3]

- 7 A curve has equation $y = x^2 - 4x + 4$ and a line has equation $y = mx$, where m is a constant.

- (i) For the case where $m = 1$, the curve and the line intersect at the points A and B . Find the coordinates of the mid-point of AB . [4]

- (ii) Find the non-zero value of m for which the line is a tangent to the curve, and find the coordinates of the point where the tangent touches the curve. [5]

- 8 (i) Express $2x^2 - 12x + 13$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

- (ii) The function f is defined by $f(x) = 2x^2 - 12x + 13$ for $x \geq k$, where k is a constant. It is given that f is a one-one function. State the smallest possible value of k . [1]

The value of k is now given to be 7.

- (iii) Find the range of f . [1]

- (iv) Find an expression for $f^{-1}(x)$ and state the domain of f^{-1} . [5]

- 9 A curve has equation $y = f(x)$ and is such that $f'(x) = 3x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} - 10$.

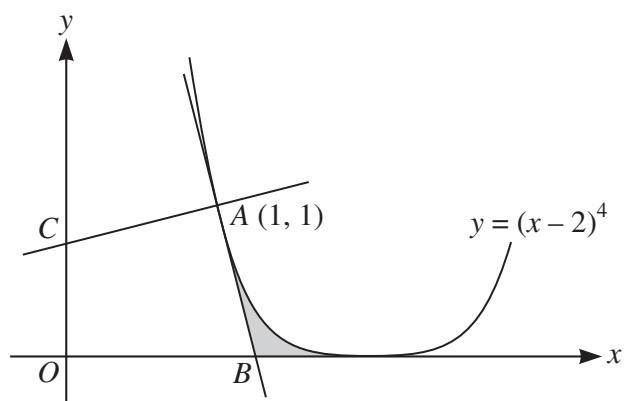
- (i) By using the substitution $u = x^{\frac{1}{2}}$, or otherwise, find the values of x for which the curve $y = f(x)$ has stationary points. [4]

- (ii) Find $f''(x)$ and hence, or otherwise, determine the nature of each stationary point. [3]

- (iii) It is given that the curve $y = f(x)$ passes through the point $(4, -7)$. Find $f(x)$. [4]

[Question 10 is printed on the next page.]

10



The diagram shows part of the curve $y = (x - 2)^4$ and the point $A(1, 1)$ on the curve. The tangent at A cuts the x -axis at B and the normal at A cuts the y -axis at C .

- (i) Find the coordinates of B and C . [6]
- (ii) Find the distance AC , giving your answer in the form $\frac{\sqrt{a}}{b}$, where a and b are integers. [2]
- (iii) Find the area of the shaded region. [4]



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1 A curve is such that $\frac{dy}{dx} = \frac{6}{x^2}$ and $(2, 9)$ is a point on the curve. Find the equation of the curve. [3]

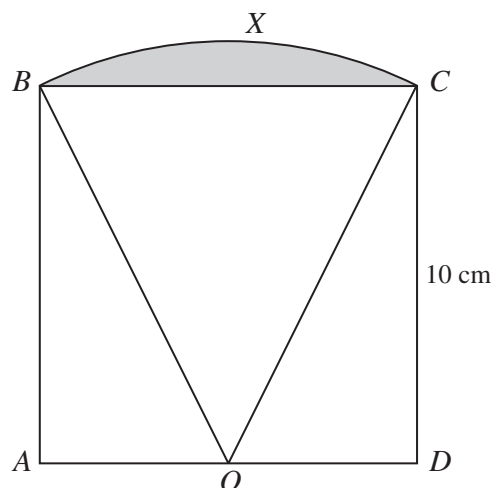
2 Find the coefficient of x^2 in the expansion of

(i) $\left(2x - \frac{1}{2x}\right)^6$, [2]

(ii) $(1 + x^2)\left(2x - \frac{1}{2x}\right)^6$. [3]

3 The straight line $y = mx + 14$ is a tangent to the curve $y = \frac{12}{x} + 2$ at the point P . Find the value of the constant m and the coordinates of P . [5]

4



The diagram shows a square $ABCD$ of side 10 cm. The mid-point of AD is O and BXC is an arc of a circle with centre O .

(i) Show that angle BOC is 0.9273 radians, correct to 4 decimal places. [2]

(ii) Find the perimeter of the shaded region. [3]

(iii) Find the area of the shaded region. [2]

5 It is given that $a = \sin \theta - 3 \cos \theta$ and $b = 3 \sin \theta + \cos \theta$, where $0^\circ \leq \theta \leq 360^\circ$.

(i) Show that $a^2 + b^2$ has a constant value for all values of θ . [3]

(ii) Find the values of θ for which $2a = b$. [4]

- 6 Relative to an origin O , the position vectors of points A and B are given by

$$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k} \quad \text{and} \quad \overrightarrow{OB} = 3\mathbf{i} + p\mathbf{j} + q\mathbf{k},$$

where p and q are constants.

- (i) State the values of p and q for which $\overrightarrow{OA}$ is parallel to $\overrightarrow{OB}$. [2]

- (ii) In the case where $q = 2p$, find the value of p for which angle BOA is 90° . [2]

- (iii) In the case where $p = 1$ and $q = 8$, find the unit vector in the direction of $\overrightarrow{AB}$. [3]

- 7 The point R is the reflection of the point $(-1, 3)$ in the line $3y + 2x = 33$. Find by calculation the coordinates of R . [7]

- 8 The volume of a solid circular cylinder of radius r cm is 250π cm³.

- (i) Show that the total surface area, S cm², of the cylinder is given by

$$S = 2\pi r^2 + \frac{500\pi}{r}. \quad [2]$$

- (ii) Given that r can vary, find the stationary value of S . [4]

- (iii) Determine the nature of this stationary value. [2]

- 9 A function f is defined by $f(x) = \frac{5}{1-3x}$, for $x \geq 1$.

- (i) Find an expression for $f'(x)$. [2]

- (ii) Determine, with a reason, whether f is an increasing function, a decreasing function or neither. [1]

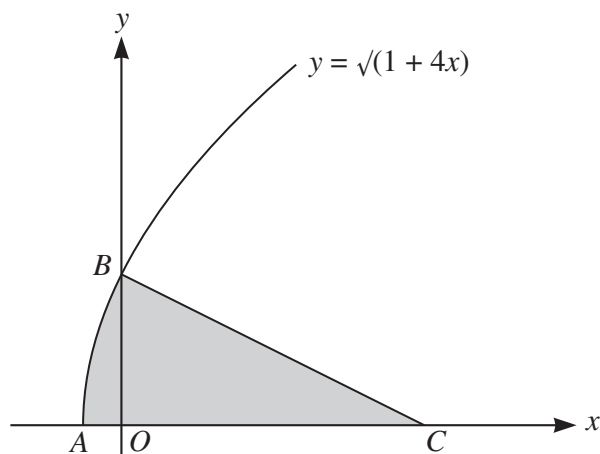
- (iii) Find an expression for $f^{-1}(x)$, and state the domain and range of f^{-1} . [5]

- 10 (a) The first and last terms of an arithmetic progression are 12 and 48 respectively. The sum of the first four terms is 57. Find the number of terms in the progression. [4]

- (b) The third term of a geometric progression is four times the first term. The sum of the first six terms is k times the first term. Find the possible values of k . [4]

[Question 11 is printed on the next page.]

11



The diagram shows the curve $y = \sqrt{1 + 4x}$, which intersects the x -axis at A and the y -axis at B . The normal to the curve at B meets the x -axis at C . Find

- (i) the equation of BC , [5]
- (ii) the area of the shaded region. [5]



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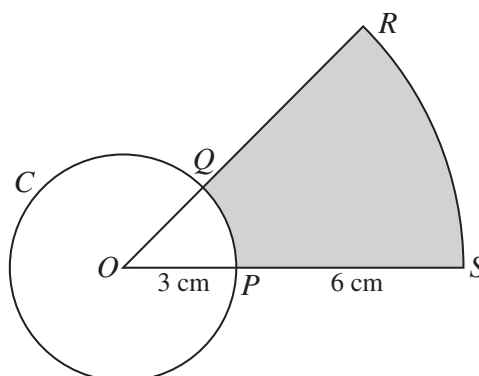
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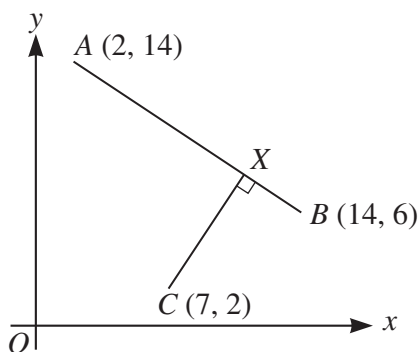
2



The diagram shows a circle C with centre O and radius 3 cm. The radii OP and OQ are extended to S and R respectively so that ORS is a sector of a circle with centre O . Given that $PS = 6$ cm and that the area of the shaded region is equal to the area of circle C ,

- (i) show that angle $POQ = \frac{1}{4}\pi$ radians, [3]
 - (ii) find the perimeter of the shaded region. [2]
- 3 (i) Express the equation $2 \cos^2 \theta = \tan^2 \theta$ as a quadratic equation in $\cos^2 \theta$. [2]
- (ii) Solve the equation $2 \cos^2 \theta = \tan^2 \theta$ for $0 \leq \theta \leq \pi$, giving solutions in terms of π . [3]
- 4 (i) Find the first three terms in the expansion of $(2 + ax)^5$ in ascending powers of x . [3]
- (ii) Given that the coefficient of x^2 in the expansion of $(1 + 2x)(2 + ax)^5$ is 240, find the possible values of a . [3]
- 5 (i) Sketch, on the same diagram, the curves $y = \sin 2x$ and $y = \cos x - 1$ for $0 \leq x \leq 2\pi$. [4]
- (ii) Hence state the number of solutions, in the interval $0 \leq x \leq 2\pi$, of the equations
- (a) $2 \sin 2x + 1 = 0$, [1]
 - (b) $\sin 2x - \cos x + 1 = 0$. [1]
- 6 The non-zero variables x , y and u are such that $u = x^2y$. Given that $y + 3x = 9$, find the stationary value of u and determine whether this is a maximum or a minimum value. [7]

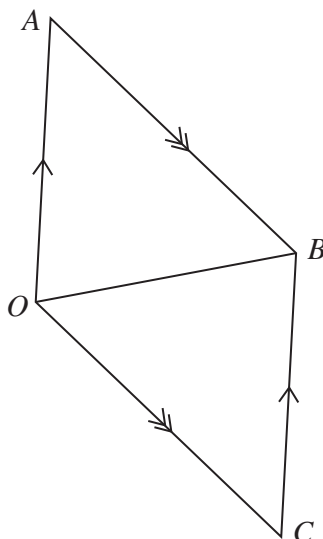
7



The diagram shows three points $A(2, 14)$, $B(14, 6)$ and $C(7, 2)$. The point X lies on AB , and CX is perpendicular to AB . Find, by calculation,

- (i) the coordinates of X , [6]
- (ii) the ratio $AX : XB$. [2]

8



The diagram shows a parallelogram $OACB$ in which

$$\overrightarrow{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$$

- (i) Use a scalar product to find angle BOC . [6]
- (ii) Find a vector which has magnitude 35 and is parallel to the vector $\overrightarrow{OC}$. [2]

- 9
- (a) In an arithmetic progression, the sum, S_n , of the first n terms is given by $S_n = 2n^2 + 8n$. Find the first term and the common difference of the progression. [3]
 - (b) The first 2 terms of a geometric progression are 64 and 48 respectively. The first 3 terms of the geometric progression are also the 1st term, the 9th term and the n th term respectively of an arithmetic progression. Find the value of n . [5]

10 The function f is defined by $f : x \mapsto 2x + k$, $x \in \mathbb{R}$, where k is a constant.

- (i) In the case where $k = 3$, solve the equation $ff(x) = 25$. [2]

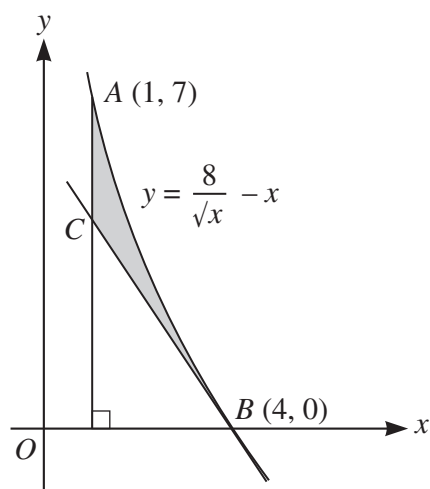
The function g is defined by $g : x \mapsto x^2 - 6x + 8$, $x \in \mathbb{R}$.

- (ii) Find the set of values of k for which the equation $f(x) = g(x)$ has no real solutions. [3]

The function h is defined by $h : x \mapsto x^2 - 6x + 8$, $x > 3$.

- (iii) Find an expression for $h^{-1}(x)$. [4]

11



The diagram shows part of the curve $y = \frac{8}{\sqrt{x}} - x$ and points $A(1, 7)$ and $B(4, 0)$ which lie on the curve. The tangent to the curve at B intersects the line $x = 1$ at the point C .

- (i) Find the coordinates of C . [4]
- (ii) Find the area of the shaded region. [5]