

MARK SCHEME for the May/June 2013 series

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

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 - A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
 - B Mark for a correct result or statement independent of method marks.
- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
 - The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
 - Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

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1	$f'(x) = (2x - 5)^2 \times 2 + 1$ or $24\left(x - \frac{5}{2}\right)^2 + 1$ > 0 (allow $\geq$)	B1B1 B1 $\frac{1}{2}$ [3]	B1 for $3(2x - 5)^2$, B1 for $(\times 2 + 1)$ SC B1 for $24x^2 - 120x + 151$ Dep on $k(2x - 5)^2 + c$ ($k > 0$), ($c \geq 0$) Subst of particular values is B0
2	(i) $1 - 6px + 15p^2x^2$ (ii) $15p^2 \times 1 - 6p \times -1$ $3p(5p + 2) = 0$ $p = -\frac{2}{5}$ oe	B1B1 [2] M1 DM1 A1 [3]	Simplificn of nCr can be scored in (ii) Obtain & attempt to solve quadratic Allow $p = 0$ in addition
3	(i) $(OAB) = \frac{1}{2} \times 8^2 \alpha$, $(OAC) = \frac{1}{2} \times \pi \times 4^2$ $\alpha = \frac{\pi}{8}$ (ii) $8 + 8 \times \text{their } \alpha + \frac{1}{2} \times 8 \times \pi$ $8 + 5\pi$	B1B1 B1 [3] B1 $\frac{1}{2}$ B1 [2]	Accept 25.1 (for OAC) 23.7 gets B1B0 SC B1 for e.g. 5π (omitted OB)
4	(i) $ar^2 = -108$, $ar^5 = 32$ $r^3 = \frac{32}{-108} = \left(-\frac{8}{27}\right)$ $r = \left(-\frac{2}{3}\right)$ or -0.666 or -0.667 (ii) $a = -243$ (iii) $S_\infty = \frac{-243}{1 + \frac{2}{3}} = -\frac{729}{5}$ or -145.8	B1 M1 A1 [3] B1 $\frac{1}{2}$ [1] M1A1 [2]	Eliminating a $-\frac{2}{3}$ from little or no working $\rightarrow \frac{3}{3}$ www ft on <i>their</i> $r \left(-\frac{108}{r^2} \text{ or } \frac{32}{r^5}\right)$ Accept -146 . For M1 $ r $ must be < 1
5	(i) $\frac{\sin \theta (\sin \theta - \cos \theta) + \cos \theta (\sin \theta + \cos \theta)}{(\sin \theta + \cos \theta)(\sin \theta - \cos \theta)}$ $\frac{\sin^2 \theta - \sin \theta \cos \theta + \cos \theta \sin \theta + \cos^2 \theta}{\sin^2 \theta - \cos^2 \theta}$ $\frac{1}{\sin^2 \theta - \cos^2 \theta}$ AG	M1 A1 A1 [3]	www

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(ii) $s^2 - (1 - s^2) = \frac{1}{3}$ or $1 - c^2 - c^2 = \frac{1}{3}$ or $3(s^2 - c^2) = c^2 + s^2$ $\sin \theta = (\pm)\sqrt{\frac{2}{3}}$ or $\cos \theta = (\pm)\sqrt{\frac{1}{3}}$ or $\tan \theta = (\pm)\sqrt{2}$ $\theta = 54.7^\circ, 125.3^\circ, 234.7^\circ, 305.3^\circ$	M1 A1 A1A1 [4]	Applying $c^2 + s^2 = 1$ Or $s = (\pm) 0.816, c = (\pm) 0.577,$ $t = (\pm) 1.414$ <u>any</u> 2 solutions for 1st A1 >4 solutions in range max A1A0
6 (i) OA.OC $= -4p^2 - q^2 + 4p^2 + q^2$ $= 0$ (ii) CA = OA – OC $= (\pm)(1 + 4p^2 + q^2)$ (i) $\mathbf{CA} = 1 + 4p^2 + q^2$ (iii) BA = OA – OB $= \mathbf{i} + 6\mathbf{j} + 2\mathbf{k} - (2\mathbf{j} - 6\mathbf{k})$ $= (\pm)(\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$ $\frac{x\mathbf{i} + y\mathbf{j} + z\mathbf{k}}{\sqrt{x^2 + y^2 + z^2}} \rightarrow \frac{1}{9}(\mathbf{i} + 4\mathbf{j} + 8\mathbf{k})$	M1 A1 [2] M1 A1 [2] M1 M1A1 [3]	Attempt scalar product. Allow M1 even for e.g. OA.OB $= 2pq - 2pq$ etc. Ignore CA = OC – OA Not $\sqrt{(1 + 4p^2 + q^2)^2}$ Allow subtn reversed for both M marks M1 independent of 1st M1
7 (i) $x^2 - 4x + 4 = x \Rightarrow x^2 - 5x + 4 = 0$ $(x - 1)(x - 4) = 0$ or other valid method (1, 1), (4, 4) Mid-point $= (2\frac{1}{2}, 2\frac{1}{2})$ (ii) $x^2 - (4 + m)x + 4 = 0 \rightarrow (4 + m)^2 - 4(4) = 0$ $4 + m = \pm 4$ or $m(8 + m) = 0$ $m = -8$ $x^2 + 4x + 4 = 0$ $x = -2, y = 16$ Alt (ii) $2x - 4 = m$ $x^2 - 4x + 4 = (2x - 4)x$ $x = -2$ (ignore +2) $m = -8$ (ignore 0) $y = 16$	M1 M1 A1 A1 ✓ [4] M1 DM1 A1 M1 A1 [5] M1 DM1 A1 A1 A1	Eliminate y to reach 3-term quadratic Attempt solution ft dependent on 1st M1 Applying $b^2 - 4ac = 0$ Attempt solution Ignore $m = 0$ in addition Sub non-zero m and attempt to solve Ignore (2, 0) solution from $m = 0$ OR $2x - 4 = m$ Sub $x = \frac{m + 4}{2}, y = \frac{m(m + 4)}{2}$ into quad $m = -8$ from resulting quad $m(m + 8) = 0$ $x = -2$ $y = 16$
8 (i) $2(x - 3)^2 - 5$ or $a = 2, b = -3, c = -5$ (ii) 3	B1B1B1 [3] B1 ✓ [1]	ft on – <i>their</i> b. Allow $k \geq 3$ or $x \geq 3$

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(iii) $(y) \geq 27$ (iv) $2(x-3)^2 = (y+5)$ $x-3 = (\pm)\sqrt{\frac{1}{2}(y+5)}$ $x = 3 + / \pm \sqrt{\frac{1}{2}(y+5)}$ $(f^{-1}(x)) = 3 + \sqrt{\frac{1}{2}(x+5)}$ for $x \geq 27$	B1 [1] M1 M1 A1 ✓ A1B1 ✓ [5]	Allow $>$. Allow $27 \leq y \leq \infty$ etc. OR (x/y interchange as 1st operation) $x = 2(y-3)^2 - 5$ $(y-3)^2 = \frac{1}{2}(x+5)$ $y-3 = (\pm)\sqrt{\frac{1}{2}(x+5)}$ ft on <i>their</i> 27 from (iii)
9 (i) $3u + \frac{3}{u} - 10 = 0$ $3u^2 - 10u + 3 = 0 \Rightarrow (3u-1)(u-3) = 0$ $\sqrt{x} = \frac{1}{3}$ or 3 $\sqrt{x} = \frac{1}{9}$ or 9 (ii) $f''(x) = \frac{3}{2}x^{-\frac{1}{2}} - \frac{3}{2}x^{-\frac{3}{2}}$ At $x = \frac{1}{9}$ $f''(x) = \frac{3}{2}(3) - \frac{3}{2}(27) (= -36) < 0 \rightarrow \text{Max}$ At $x = 9$ $f''(x) = \frac{3}{2} \times \frac{1}{3} - \frac{3}{2} \times \frac{1}{27} (= \frac{4}{9}) > 0 \rightarrow \text{Min}$ (iii) $f(x) = 2x^{\frac{3}{2}} + 6x^{\frac{1}{2}} - 10x (+c)$ $-7 = 16 + 12 - 40 + c$ $c = 5$	B1 M1 A1 A1 [4] B1 M1 A1 [3] B2 M1 A1 [4]	Or $3x - 10\sqrt{x} + 3 = 0$ Or $(3\sqrt{x} - 1)(\sqrt{x} - 3)$ or apply formula etc. Allow anywhere Valid method. Allow innac subs, even $3, \frac{1}{3}$ Fully correct. No working, no marks. B1 for 2/3 terms correct. Allow in (i) Sub (4, -7). c must be present.
10 (i) $\frac{dy}{dx} = 4(x-2)^3$ Grad of tangent = -4 Eq. of tangent is $y - 1 = -4(x - 1)$ $\rightarrow B(\frac{5}{4}, 0)$ Grad of normal = $\frac{1}{4}$ Eq. of normal is $y - 1 = \frac{1}{4}(x - 1) \rightarrow C(0, \frac{3}{4})$	B1 M1 M1 A1 M1 A1 [6]	Or $4x^3 - 24x^2 + 48x - 32$ Sub $x = 1$ into <i>their</i> derivative Line thru (1, 1) and with m from deriv Use of $m_1 m_2 = -1$

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(ii) $AC^2 = 1^2 + \left(\frac{1}{4}\right)^2$ $\frac{\sqrt{17}}{4}$ (iii) $\int (x-2)^4 dx = \frac{(x-2)^5}{5}$ $\left[0 - \left(-\frac{1}{5}\right)\right] = \frac{1}{5}$ $\Delta = \frac{1}{2} \times 1 \times \left(\text{their } \frac{5}{4} - 1\right) = \frac{1}{8}$ $\frac{1}{5} - \frac{1}{8} = \frac{3}{40}$ or 0.075	M1 A1 [2] B1 M1 M1 A1 [4]	Allow $\sqrt{\frac{17}{16}}$ Or $\frac{x^5}{5} - 2x^4 + 8x^3 - 16x^2 + 16x$ Apply limits $1 \rightarrow 2$ for curve Or $\int_1^{\frac{5}{4}} (-4x + 5) dx = \frac{1}{8}$
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1 $\frac{dy}{dx} = \frac{6}{x^2}$ $y = -6x^{-1} + c$ Uses (2, 9) $\rightarrow c = 12$ $y = -6x^{-1} + 12$	B1 M1 A1 [3]	Integration only – unsimplified Uses (2, 9) in an integral
2 $\left(2x - \frac{1}{2x}\right)^6$ (i) Coeff of $x^2 = 15 \times 16 \times (-\frac{1}{2})^2 = 60$ (ii) Constant term is $20 \times 8x^3 \times (-1 \div 8x^3)$ $\times (1 + x^2)$ needs to consider 2 terms $\rightarrow 60 - 20 = 40$	B1 B1 [2] B1 M1 A1 [3]	B1 for 2/3 parts. B1 B1 unsimplified Needs to consider the constant term
3 $mx + 14 = \frac{12}{x} + 2 \rightarrow mx^2 + 12x - 12 = 0$ Uses $b^2 = 4ac \rightarrow m = -3$ $-3x^2 + 12x - 12 = 0 \rightarrow P(2, 8)$ [Or $m = -12x^{-2}$ M1 Sub M1 $x = 2$ A1] $\rightarrow m = -3$ and $y = 8$ M1 A1]	M1 M1 A1 DM1 A1 [5]	Eliminates x (or y) Any use of discriminant Any valid method.
4 (i) $BOC = 2\tan^{-1}\frac{1}{2} = 0.9273$ (ii) $OB = \sqrt{(10^2 + 5^2)}$ or $11.2 = r$ Arc $BXC = \sqrt{125} \times 0.9273$ $\rightarrow$ Perimeter = 20.4 cm (iii) Area = $\frac{1}{2}r^2\theta$ $= \frac{1}{2} \cdot 10 \cdot 10 \rightarrow 7.96 \text{ cm}^2$.	M1 A1 [2] B1 M1 A1 [3] M1 A1 [2]	Correct trigonometry. (ans given) Use of trig (or Pyth) for the $OB = \sqrt{125}$. Use of $s = r\theta$ with θ in rads, $r \neq 10$ Correct formula used with rads, $r \neq 10$. Allow 7.95 or 7.96
5 $a = \sin \theta - 3 \cos \theta$, $b = 3 \sin \theta + \cos \theta$ (i) $a^2 + b^2 =$ $(s^2 + 9c^2 - 6sc) + (9s^2 + c^2 + 6sc)$ $10c^2 + 10s^2 = 10$ (ii) $2s - 6c = 3s + c \rightarrow s = -7c$ $\rightarrow \tan \theta = -7$ $\rightarrow 98.1^\circ$ and 278.1°	B1 M1 A1 [3] M1 A1 A1 A1 [4]	Correct squaring Use of $s^2 + c^2 = 1$ to get constant. (can get 2/3 for missing $6sc$) Collecting and $t = s \div c$ For 180° + first answer, providing no extra answers in the range.

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6 $\vec{OA} = \mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$, $\vec{OB} = 3\mathbf{i} + p\mathbf{j} + q\mathbf{k}$ (i) $p = -6, q = 6$ (ii) dot product $= 0 \rightarrow 3 - 2p + 4p = 0$ $\rightarrow p = -1.5$ (iii) $\vec{AB} = \mathbf{b} - \mathbf{a} = 2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ Unit vector $= (2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}) \div 7$	B1 B1 [2] M1 A1 [2] B1 M1 A1[✓] [3]	Use of $x_1x_2 + y_1y_2 + z_1z_2 = 0$ not for $\mathbf{b} - \mathbf{a}$. M1 for division by modulus. [✓] on B1.
7 $3y + 2x = 33$. Gradient of line $= -\frac{2}{3}$ Gradient of perpendicular $= 3/2$ Eqn of perp $y - 3 = \frac{3}{2}(x + 1)$ Sim Eqns $\rightarrow (3, 9)$ $(-1, 3) \rightarrow (3, 9) \rightarrow (7, 15)$	B1 M1 M1 M1 A1 M1 A1 [7]	Use of $m_1m_2 = -1$ with gradient of line Correct form of perpendicular eqn. Sim eqns. Vectors or other method.
8 (i) $\pi r^2 h = 250\pi \rightarrow h = \frac{250}{r^2}$ $\rightarrow S = 2\pi r h + 2\pi r^2$ $\rightarrow S = 2\pi r^2 + \frac{500\pi}{r}$ (ii) $\frac{dS}{dr} = 4\pi r - \frac{500\pi}{r^2}$ $= 0$ when $r^3 = 125 \rightarrow r = 5$ $\rightarrow S = 150\pi$ (iii) $\frac{d^2S}{dr^2} = 4\pi + \frac{1000\pi}{r^3}$ This is positive $\rightarrow$ Minimum	M1 M1 [2] B1 B1 M1 A1 [4] M1 A1 [2]	Makes h the subject. $\pi r^2 h$ must be right Ans given – check all formulae.. B1 for each term Sets differential to 0 + attempt at soln Any valid method. 2nd differential must be correct – no need for numerical answer or correct r.
9 $f(x) = \frac{5}{1-3x}, x \geq 1$ (i) $f'(x) = \frac{-5}{(1-3x)^2} \times -3$ (ii) $15 > 0$ and $(1-3x)^2 > 0, f'(x) > 0$ $\rightarrow$ increasing (iii) $y = \frac{5}{1-3x} \rightarrow 3x = 1 - \frac{5}{y}$ $\rightarrow f^{-1}(x) = \frac{x-5}{3x}$ or $\frac{1}{3} - \frac{5}{3x}$ Range is ≥ 1 Domain is $-2.5 \leq x < 0$	B1 B1 [2] B1[✓] [1] M1 A1 B1 B1 B1 [5]	B1 without $\times -3$. B1 for $\times -3$, even if first B mark is incorrect [✓] providing $()^2$ in denominator. Attempt to make x the subject. Must be in terms of x. must be $\geq$ condone $<$

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10 (a) $57 = 2(24 + 3d) \rightarrow d = 1.5$ $48 = 12 + (n - 1)1.5 \rightarrow n = 25$ (b) $ar^2 = 4a \quad r = \pm 2$ $\frac{a(r^6 - 1)}{r - 1} = ka$ $\rightarrow k = 63 \text{ or } k = -21$	M1 A1 M1 A1 [4] B1 B1 B1 B1 [4]	Use of correct S_n formula. Use of correct T_n formula. (allow for $r = 2$)
11 $y = \sqrt{1 + 4x}$ (i) $\frac{dy}{dx} = \frac{1}{2}(1 + 4x)^{-\frac{1}{2}} \times 4$ $= 2 \text{ at } B(0, 1)$ Gradient of normal $= -\frac{1}{2}$ Equation $y - 1 = -\frac{1}{2}x$ (ii) At A $x = -\frac{1}{4}$ $\int \sqrt{1 + 4x} dx = \frac{(1 + 4x)^{\frac{3}{2}}}{\frac{3}{2}} \div 4$ Limits $-\frac{1}{4} \text{ to } 0 \rightarrow \frac{1}{6}$ Area $BOC = \frac{1}{2} \times 2 \times 1 = 1$ $\rightarrow \text{Shaded area} = \frac{7}{6}$	B1 B1 M1 M1 A1 [5] B1 B1 B1 B1 B1 [5]	B1 Without “$\times 4$”. B1 for “$\times 4$” even if first B mark lost. Use of $m_1 m_2 = -1$ Correct method for eqn. B1 Without the “$\div 4$”. For “$\div 4$” even if first B mark lost. For 1 + his “$1/6$”.

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- When a part of a question has two or more "method" steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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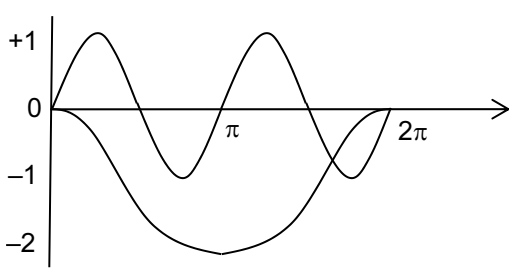
The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through $\sqrt{}$ " marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

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1	$\frac{dy}{dx} = \sqrt{2x+5}$ $\frac{(2x+5)^{\frac{3}{2}}}{\frac{3}{2}} \div 2 \quad (+c)$ Uses (2, 5) $\rightarrow c = -4$	B1 B1 M1 A1	[4]	B1 Everything without “÷2”. B1 “÷2” Uses point in an integral.
2	(i) $\frac{1}{2} \cdot 3^2\pi = \frac{1}{2}9^2\theta - \frac{1}{2}3^2\theta$ $\rightarrow \theta = \frac{1}{4}\pi$ (ii) $P = 6+6+3 \times \frac{1}{4}\pi + 9 \times \frac{1}{4}\pi = 21.4 \text{ cm.}$ or $12 + 3\pi$	M1 A1 A1 M1 A1	[3] [2]	M1 needs $\frac{1}{2}r^2\theta$ once. A1 all correct. Answer given M1 is for use of $s=r\theta$ once.
3	(i) $2\cos^2\theta = \tan^2\theta$ $\rightarrow 2\cos^2\theta = \frac{\sin^2\theta}{\cos^2\theta}$ $\rightarrow$ Uses $c^2 + s^2 = 1 \rightarrow 2c^4 = 1 - c^2$ (ii) $(2c^2 - 1)(c^2 + 1) = 0 \rightarrow c = \pm \frac{1}{\sqrt{2}}$ $\rightarrow \theta = \frac{1}{4}\pi \text{ or } \frac{3}{4}\pi.$	M1 A1 M1 A1 A1√	[2] [3]	Use of $t^2 = s^2 \div c^2$ or alternative. Correct eqn. Method of solving for 3-term quadratic. (in terms of π). √ for $\pi - 1^{\text{st}}$ ans. Cannot gain A1√ if other answers given in the range.
4	(i) $(2+ax)^5 = 32 + 80ax + 80a^2x^2$ (ii) $\times (1+2x)$ $240 = 80a^2 + 160a$ $\rightarrow a = 1 \text{ or } a = -3.$	$3 \times \text{B1}$ M1 DM1A1	[3] [3]	B1 for each term. Realises need to consider 2 terms. Solution of 3-term quadratic.
5	(i)  (ii) (a) $\sin 2x = -\frac{1}{2} \rightarrow 4 \text{ solutions}$ (b) $\sin 2x + \cos x + 1 = 0 \rightarrow 3 \text{ solutions.}$	B1 DB1 B1 DB1 B1√ B1√	[4] [1] [1]	$y = \sin 2x$ has 2 cycles, starts and finishes on the x-axis, max comes first. From +1 to -1. Smooth curves. $y = \cos x - 1$ has one cycle, starts and finishes on x-axis, with a minimum pt. From 0 to -2, smooth curve, flattens. √ for their curve. √ for intersections of their curves.

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6	$u = x^2y \quad y + 3x = 9$ $u = x^2(9 - 3x) \text{ or } \left(\frac{9-y}{3}\right)^2 y$ $\frac{du}{dx} = 18x - 9x^2 \text{ or } \frac{du}{dy} = 27 - 12y + y^2$ $= 0 \text{ when } x = 2 \text{ or } y = 3 \rightarrow u = 12$ $\frac{d^2u}{dx^2} = 18 - 18x \text{ -ve}$	M1 DM1A1 DM1A1 DM1A1	 	Expressing u in terms of 1 variable Knowing to differentiate. Setting differential to 0. Any valid method
7	$A(2, 14), B(14, 6) \text{ and } C(7, 2).$ (i) $m \text{ of } AB = -\frac{2}{3}$ $m \text{ of perpendicular} = \frac{3}{2}$ eqn of $AB \quad y - 14 = -\frac{2}{3}(x - 2)$ eqn of $CX \quad y - 2 = \frac{3}{2}(x - 7)$ Sim Eqns $\rightarrow X(11, 8)$ (ii) $AX : XB = 14 - 8 : 8 - 6 = 3 : 1$ Or $\sqrt{(9^2 + 6^2)} : \sqrt{(3^2 + 2^2)} = 3 : 1$	B1 M1 M1 M1 M1 A1 M1 A1	 	For use of $m_1 m_2 = -1$ Allow M1 for unsimplified eqn Allow M1 for unsimplified eqn For solution of sim eqns. Vector steps or Pythagoras.
8	$\vec{OA} = \begin{pmatrix} 3 \\ 3 \\ -4 \end{pmatrix} \text{ and } \vec{OB} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}.$ (i) $\mathbf{OC} = \mathbf{AB} = \mathbf{b} - \mathbf{a} = \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$ Uses OC and OB $\mathbf{OC} \cdot \mathbf{OB} = 22 = 7 \times \sqrt{29} \cos BOC$ $\rightarrow \text{Angle } BOC = 54.3^\circ \text{ (or } 0.948 \text{ rad)}$	M1 B1 M1 M1 M1 A1	 	Knowing how to find OC Using OC.OB or CO.BO M1 Use of $x_1 x_2 + \dots$ M1 for modulus M1 everything linked. (nb uses BO.OC loses B1 A1) (nb uses other vectors – max M1M1)

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	(ii)	Modulus of OC = 7 Vector = $35 \div 7 \times \mathbf{OC}$ $\rightarrow \pm 5 \begin{pmatrix} 2 \\ -3 \\ 6 \end{pmatrix}$	M1 A1✓		Knows to scale by factor of $35 \div \text{Mod}$ For their OC .
				[2]	
9	(a)	$S_n = 2n^2 + 8n$ $S_1 = 10 = a$ $S_2 = 24 = a + (a + d) \Rightarrow d = 4$	B1 M1 A1	[3]	correct use of S_n formula.
	(b)	GP $a = 64$ $ar = 48 \rightarrow r = \frac{3}{4}$ $\rightarrow$ 3rd term is $ar^2 = 36$ AP $a = 64$, $a + 8d = 48 \rightarrow d = -2$ $36 = 64 + (n - 1)(-2)$ $\rightarrow n = 15$.	B1 M1 B1 M1 A1	[5]	ar^2 numerical – for their r correct use of $a + (n - 1)d$
10		$f: x \mapsto 2x + k$, $g: x \mapsto x^2 - 6x + 8$,			
	(i)	$2(2x + 3) + 3 = 25$ $\rightarrow x = 4$ or $\{f(11) = 25, f(4) = 11\}$	M1 A1	[2]	$ff(x)$ needs to be correctly formed
	(ii)	$x^2 - 6x + 8 = 2x + k$ $x^2 - 8x + 8 - k = 0$ Uses $b^2 - 4ac < 0$ $\rightarrow k < -8$	M1 M1 A1	[3]	Eliminates y to form eqn in x . Uses the discriminant – even if $=0, >0$
	(iii)	$x^2 - 6x + 8 = (x - 3)^2 - 1$ $y = (x - 3)^2 - 1$ Makes x the subject $\rightarrow \pm\sqrt{(x + 1) + 3}$ Needs specifically to lose the “–”.	B1 B1 M1 A1✓	[4]	For “–3” and “–1” Makes x the subject, in terms of x and without – or $\pm$.

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11	$y = \frac{8}{\sqrt{x}} - x$			
(i)	$\frac{dy}{dx} = -4x^{-\frac{3}{2}} - 1$ $= -\frac{3}{2} \text{ when } x = 4.$ Eqn of BC $y - 0 = -\frac{3}{2}(x - 4)$ $\rightarrow C(1, 4\frac{1}{2})$	B1 M1 M1 A1		needs both Subs $x = 4$ into dy/dx Must be using differential + correct form of line at $B(4,0)$.
(ii)	area under curve = $\int (\frac{8}{\sqrt{x}} - x)$ $= \frac{8x^{\frac{1}{2}}}{\frac{1}{2}} - \frac{1}{2}x^2$ Limits 1 to 4 $\rightarrow 8\frac{1}{2}$ Area under tangent = $\frac{1}{2} \times 4\frac{1}{2} \times 3 = 6\frac{3}{4}$ Shaded area = $1\frac{3}{4}$	B1 B1 M1 M1 A1	[4] (both unsimplified) Using correct limits. Or could use calculus) [5]	