



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2012

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

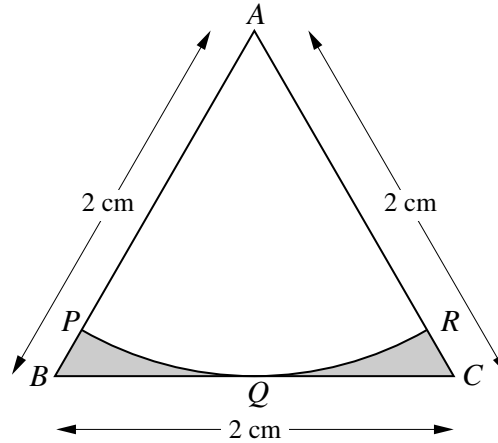
This document consists of **4** printed pages.



1 Solve the equation $\sin 2x = 2 \cos 2x$, for $0^\circ \leq x \leq 180^\circ$. [4]

2 Find the coefficient of x^6 in the expansion of $\left(2x^3 - \frac{1}{x^2}\right)^7$. [4]

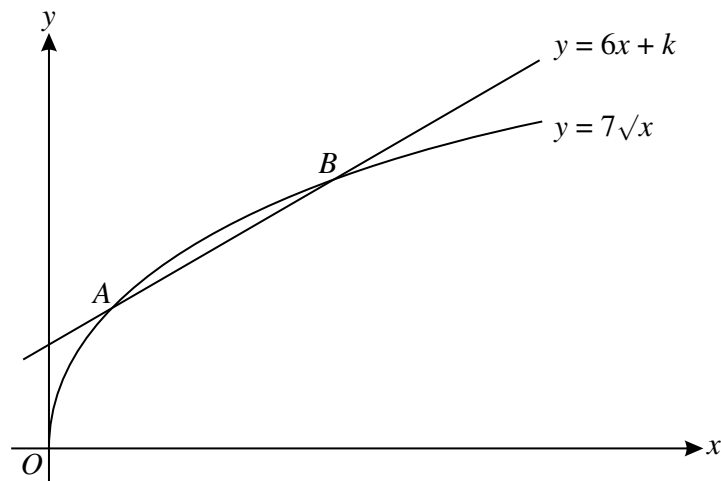
3



In the diagram, ABC is an equilateral triangle of side 2 cm. The mid-point of BC is Q . An arc of a circle with centre A touches BC at Q , and meets AB at P and AC at R . Find the total area of the shaded regions, giving your answer in terms of π and $\sqrt{3}$. [5]

4 A watermelon is assumed to be spherical in shape while it is growing. Its mass, M kg, and radius, r cm, are related by the formula $M = kr^3$, where k is a constant. It is also assumed that the radius is increasing at a constant rate of 0.1 centimetres per day. On a particular day the radius is 10 cm and the mass is 3.2 kg. Find the value of k and the rate at which the mass is increasing on this day. [5]

5



The diagram shows the curve $y = 7\sqrt{x}$ and the line $y = 6x + k$, where k is a constant. The curve and the line intersect at the points A and B .

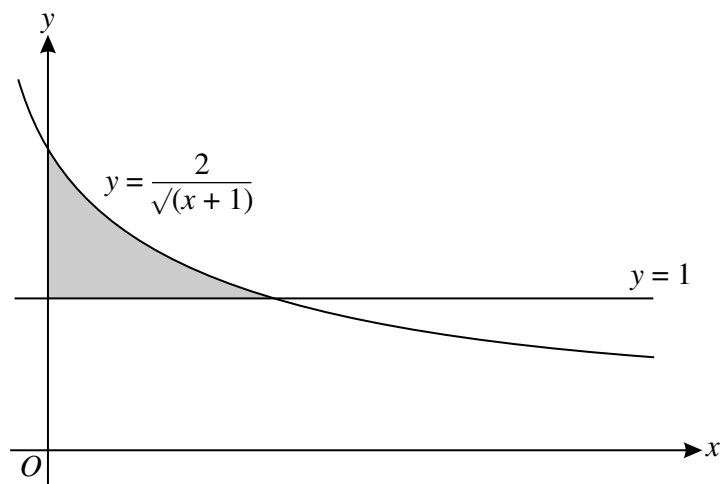
(i) For the case where $k = 2$, find the x -coordinates of A and B . [4]

(ii) Find the value of k for which $y = 6x + k$ is a tangent to the curve $y = 7\sqrt{x}$. [2]

- 6 Two vectors $\mathbf{u}$ and $\mathbf{v}$ are such that $\mathbf{u} = \begin{pmatrix} p^2 \\ -2 \\ 6 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 2 \\ p-1 \\ 2p+1 \end{pmatrix}$, where p is a constant.
- (i) Find the values of p for which $\mathbf{u}$ is perpendicular to $\mathbf{v}$. [3]
- (ii) For the case where $p = 1$, find the angle between the directions of $\mathbf{u}$ and $\mathbf{v}$. [4]
- 7 (a) The first two terms of an arithmetic progression are 1 and $\cos^2 x$ respectively. Show that the sum of the first ten terms can be expressed in the form $a - b \sin^2 x$, where a and b are constants to be found. [3]
- (b) The first two terms of a geometric progression are 1 and $\frac{1}{3} \tan^2 \theta$ respectively, where $0 < \theta < \frac{1}{2}\pi$.
- (i) Find the set of values of θ for which the progression is convergent. [2]
- (ii) Find the exact value of the sum to infinity when $\theta = \frac{1}{6}\pi$. [2]
- 8 The function $f : x \mapsto x^2 - 4x + k$ is defined for the domain $x \geq p$, where k and p are constants.
- (i) Express $f(x)$ in the form $(x + a)^2 + b + k$, where a and b are constants. [2]
- (ii) State the range of f in terms of k . [1]
- (iii) State the smallest value of p for which f is one-one. [1]
- (iv) For the value of p found in part (iii), find an expression for $f^{-1}(x)$ and state the domain of f^{-1} , giving your answers in terms of k . [4]
- 9 The coordinates of A are $(-3, 2)$ and the coordinates of C are $(5, 6)$. The mid-point of AC is M and the perpendicular bisector of AC cuts the x -axis at B .
- (i) Find the equation of MB and the coordinates of B . [5]
- (ii) Show that AB is perpendicular to BC . [2]
- (iii) Given that $ABCD$ is a square, find the coordinates of D and the length of AD . [2]
- 10 It is given that a curve has equation $y = f(x)$, where $f(x) = x^3 - 2x^2 + x$.
- (i) Find the set of values of x for which the gradient of the curve is less than 5. [4]
- (ii) Find the values of $f(x)$ at the two stationary points on the curve and determine the nature of each stationary point. [5]

[Question 11 is printed on the next page.]

11



The diagram shows the line $y = 1$ and part of the curve $y = \frac{2}{\sqrt{x+1}}$.

- (i) Show that the equation $y = \frac{2}{\sqrt{x+1}}$ can be written in the form $x = \frac{4}{y^2} - 1$. [1]
- (ii) Find $\int \left(\frac{4}{y^2} - 1 \right) dy$. Hence find the area of the shaded region. [5]
- (iii) The shaded region is rotated through 360° about the **y-axis**. Find the exact value of the volume of revolution obtained. [5]



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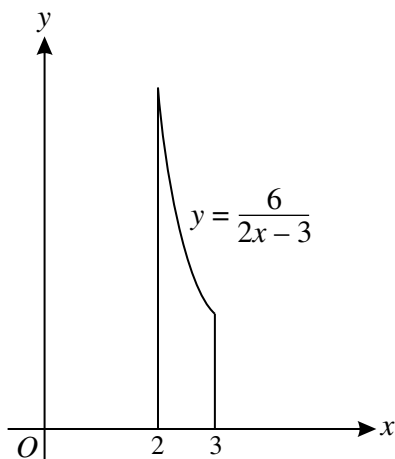
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1



The diagram shows the region enclosed by the curve $y = \frac{6}{2x-3}$, the x -axis and the lines $x = 2$ and $x = 3$. Find, in terms of π , the volume obtained when this region is rotated through 360° about the x -axis. [4]

2 The equation of a curve is $y = 4\sqrt{x} + \frac{2}{\sqrt{x}}$.

(i) Obtain an expression for $\frac{dy}{dx}$. [3]

(ii) A point is moving along the curve in such a way that the x -coordinate is increasing at a constant rate of 0.12 units per second. Find the rate of change of the y -coordinate when $x = 4$. [2]

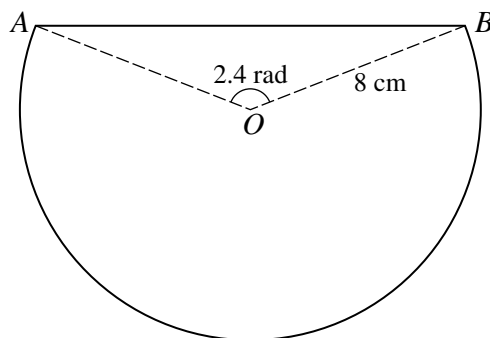
3 The coefficient of x^3 in the expansion of $(a+x)^5 + (2-x)^6$ is 90. Find the value of the positive constant a . [5]

4 The point A has coordinates $(-1, -5)$ and the point B has coordinates $(7, 1)$. The perpendicular bisector of AB meets the x -axis at C and the y -axis at D . Calculate the length of CD . [6]

5 (i) Prove the identity $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$. [2]

(ii) Solve the equation $\frac{2}{\sin x \cos x} = 1 + 3 \tan x$, for $0^\circ \leq x \leq 180^\circ$. [4]

6



The diagram shows a metal plate made by removing a segment from a circle with centre O and radius 8 cm. The line AB is a chord of the circle and angle $AOB = 2.4$ radians. Find

- (i) the length of AB , [2]
- (ii) the perimeter of the plate, [3]
- (iii) the area of the plate. [3]

- 7 (a) In an arithmetic progression, the sum of the first n terms, denoted by S_n , is given by

$$S_n = n^2 + 8n.$$

Find the first term and the common difference. [3]

- (b) In a geometric progression, the second term is 9 less than the first term. The sum of the second and third terms is 30. Given that all the terms of the progression are positive, find the first term. [5]

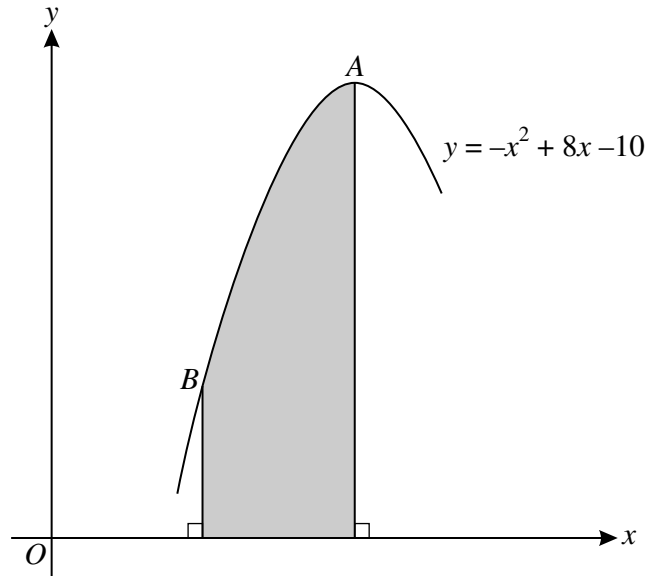
- 8 (i) Find the angle between the vectors $3\mathbf{i} - 4\mathbf{k}$ and $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. [4]

The vector $\overrightarrow{OA}$ has a magnitude of 15 units and is in the same direction as the vector $3\mathbf{i} - 4\mathbf{k}$. The vector $\overrightarrow{OB}$ has a magnitude of 14 units and is in the same direction as the vector $2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$.

- (ii) Express $\overrightarrow{OA}$ and $\overrightarrow{OB}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]
- (iii) Find the unit vector in the direction of $\overrightarrow{AB}$. [3]

[Questions 9 and 10 are printed on the next page.]

9



The diagram shows part of the curve $y = -x^2 + 8x - 10$ which passes through the points A and B . The curve has a maximum point at A and the gradient of the line BA is 2.

(i) Find the coordinates of A and B . [7]

(ii) Find $\int y \, dx$ and hence evaluate the area of the shaded region. [4]

10 Functions f and g are defined by

$$f : x \mapsto 2x + 5 \quad \text{for } x \in \mathbb{R},$$

$$g : x \mapsto \frac{8}{x-3} \quad \text{for } x \in \mathbb{R}, x \neq 3.$$

(i) Obtain expressions, in terms of x , for $f^{-1}(x)$ and $g^{-1}(x)$, stating the value of x for which $g^{-1}(x)$ is not defined. [4]

(ii) Sketch the graphs of $y = f(x)$ and $y = f^{-1}(x)$ on the same diagram, making clear the relationship between the two graphs. [3]

(iii) Given that the equation $fg(x) = 5 - kx$, where k is a constant, has no solutions, find the set of possible values of k . [5]

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- 1 (i) Prove the identity $\tan^2 \theta - \sin^2 \theta \equiv \tan^2 \theta \sin^2 \theta$. [3]

- (ii) Use this result to explain why $\tan \theta > \sin \theta$ for $0^\circ < \theta < 90^\circ$. [1]

- 2 Relative to an origin O , the position vectors of the points A , B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$$

Find

- (i) the unit vector in the direction of $\overrightarrow{AB}$, [3]

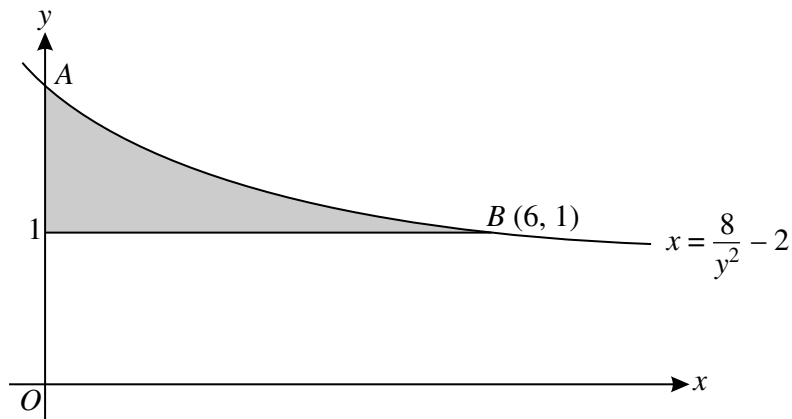
- (ii) the value of the constant p for which angle $BOC = 90^\circ$. [2]

- 3 The first three terms in the expansion of $(1 - 2x)^2(1 + ax)^6$, in ascending powers of x , are $1 - x + bx^2$. Find the values of the constants a and b . [6]

- 4 (i) Solve the equation $\sin 2x + 3 \cos 2x = 0$ for $0^\circ \leq x \leq 360^\circ$. [5]

- (ii) How many solutions has the equation $\sin 2x + 3 \cos 2x = 0$ for $0^\circ \leq x \leq 1080^\circ$? [1]

5



The diagram shows part of the curve $x = \frac{8}{y^2} - 2$, crossing the y -axis at the point A . The point $B(6, 1)$ lies on the curve. The shaded region is bounded by the curve, the y -axis and the line $y = 1$. Find the exact volume obtained when this shaded region is rotated through 360° about the y -axis. [6]

- 6 The first term of an arithmetic progression is 12 and the sum of the first 9 terms is 135.

- (i) Find the common difference of the progression. [2]

The first term, the ninth term and the n th term of this arithmetic progression are the first term, the second term and the third term respectively of a geometric progression.

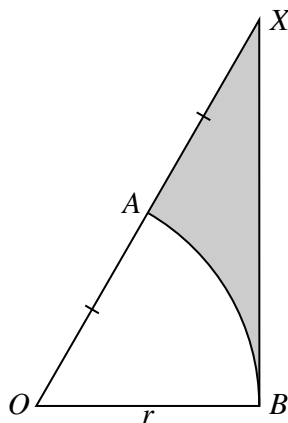
- (ii) Find the common ratio of the geometric progression and the value of n . [5]

- 7 The curve $y = \frac{10}{2x+1} - 2$ intersects the x -axis at A . The tangent to the curve at A intersects the y -axis at C .

(i) Show that the equation of AC is $5y + 4x = 8$. [5]

(ii) Find the distance AC . [2]

8



In the diagram, AB is an arc of a circle with centre O and radius r . The line XB is a tangent to the circle at B and A is the mid-point of OX .

(i) Show that angle $AOB = \frac{1}{3}\pi$ radians. [2]

Express each of the following in terms of r , π and $\sqrt{3}$:

(ii) the perimeter of the shaded region, [3]

(iii) the area of the shaded region. [2]

- 9 A curve is such that $\frac{d^2y}{dx^2} = -4x$. The curve has a maximum point at $(2, 12)$.

(i) Find the equation of the curve. [6]

A point P moves along the curve in such a way that the x -coordinate is increasing at 0.05 units per second.

(ii) Find the rate at which the y -coordinate is changing when $x = 3$, stating whether the y -coordinate is increasing or decreasing. [2]

- 10 The equation of a line is $2y + x = k$, where k is a constant, and the equation of a curve is $xy = 6$.

(i) In the case where $k = 8$, the line intersects the curve at the points A and B . Find the equation of the perpendicular bisector of the line AB . [6]

(ii) Find the set of values of k for which the line $2y + x = k$ intersects the curve $xy = 6$ at two distinct points. [3]

11 The function f is such that $f(x) = 8 - (x - 2)^2$, for $x \in \mathbb{R}$.

- (i) Find the coordinates and the nature of the stationary point on the curve $y = f(x)$. [3]

The function g is such that $g(x) = 8 - (x - 2)^2$, for $k \leq x \leq 4$, where k is a constant.

- (ii) State the smallest value of k for which g has an inverse. [1]

For this value of k ,

- (iii) find an expression for $g^{-1}(x)$, [3]

- (iv) sketch, on the same diagram, the graphs of $y = g(x)$ and $y = g^{-1}(x)$. [3]