

MARK SCHEME for the May/June 2012 question paper
for the guidance of teachers

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

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Page 2	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	11

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- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

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Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	11

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Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	11

1	$\tan 2x = 2$ $2x = 63.4$ or 243.4 $x = 31.7$ or 121.7 (allow 122)	M1 A1 A1A1 [4]	1 solution sufficient For 2 nd A1 allow 90 + 1 st soln prov. only 2 solns in range. Alt methods possible
2	$[7C3] \times [(2x^3)^4] \times [(-1/x^2)^3]$ seen soi $35 \times 2^4 \times (-1)^3$ leading to their answer soi $-560(x^6)$ as answer	B1B1 B1 B1 [4]	2 elements correct, 3 rd element correct 2 elements correct. Identifying reqd term SC B3 for $[560(x)^6]$ as answer
3	AQ (or r) = $\sqrt{3}$ Area $\Delta = \sqrt{3}$ (or area $\Delta AQC = \frac{\sqrt{3}}{2}$) Area sector $APR = \frac{1}{2}(\sqrt{3})^2 \times \frac{\pi}{3} = \frac{\pi}{2}$ Shaded region = $\sqrt{3} - \frac{\pi}{2}$ oe cao	B1 B1 M1A1 A1 [5]	soi Allow 1.73 soi ft <i>their</i> $\sqrt{3}$ Allow 1.73 ft <i>their</i> $\sqrt{3}$. Allow 1.57. SCA1 for $\pi/4$ from $\frac{1}{2}(\sqrt{3})^2 \times \frac{\pi}{6}$ provided $\Delta = \frac{\sqrt{3}}{2}$
4	$1000k = 3.2 \Rightarrow k = \frac{3.2}{1000}$ or $\frac{2}{625}$ or 0.0032 oe $\left(\frac{dM}{dr}\right) = 3kr^2$ $\frac{dM}{dt} = \frac{dM}{dr} \times \frac{dr}{dt}$ used e.g. $3 \times k \times 10^2 \times 0.1$ 0.096	M1A1 B1 M1 A1 [5]	Must eventually make dM/dt subject cao. Non-calculus methods (e.g. $\rightarrow$ 0.09696) can score only 1 st 2 marks
5	(i) $6x + 2 = 7\sqrt{x} \Rightarrow 6(\sqrt{x})^2 - 7\sqrt{x} + 2 = 0$ $(3\sqrt{x} - 2)(2\sqrt{x} - 1) = 0$ $\sqrt{x} = \frac{2}{3}$ or $\frac{1}{2}$ $x = \frac{4}{9}$ or $\frac{1}{4}$ (or 0.444, 0.25) OR $(6x + 2)^2 = 49x \rightarrow 36x^2 - 25x + 4 = 0$ $(9x - 4)(4x - 1) = 0$ $x = \frac{4}{9}$ or $\frac{1}{4}$ (or 0.444, 0.25) oe (ii) $7^2 - 4 \times 6 \times k (= 0)$ $k = \frac{49}{24}$ or 2.04 OR $\frac{d}{dx}(7x^{\frac{1}{2}}) = \frac{d}{dx}(6x + k) \rightarrow \frac{7}{2}x^{-\frac{1}{2}} = 6$ $x = \frac{49}{144}$, $y = \frac{49}{12} \rightarrow k = \frac{49}{24}$ or 2.04	M1 M1 A1 A1 M1A1 M1 A1 M1 A1 M1 A1 [4] M1 A1 M1 A1 [2]	Expressing as a clear quadratic soi oe e.g. $(3t - 2)(2t - 1) = 0$ 1 solution sufficient. Accept e.g. $t = 2/3$ Both solutions required cao Attempt to square both sides Attempt to solve (or formula etc.) Apply $b^2 - 4ac (= 0)$ Attempt to equate derivatives

Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	11

6 (i) $2p^2 - 2p + 2 + 12p + 6 \rightarrow 2p^2 + 10p + 8$ $\mathbf{u \cdot v} = 0$ $(p+1)(p+4) = 0 \rightarrow p = -1$ or $p = -4$ (ii) $\mathbf{u \cdot v} = 2 + 0 + 18 = 20$ $\mathbf{u} = \sqrt{41}$ or $\mathbf{v} = \sqrt{13}$ $20 = \sqrt{41} \times \sqrt{13} \times \cos \theta$ oe $\theta = 30.0^\circ$ or 0.523 rads	M1 B1 A1 [3] M1 M1 M1 A1 [4]	Correct method for scalar product Scalar product = 0 cao Both solutions required Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct method for moduli All connected correctly cao
7 (a) $S_{10} = \frac{10}{2[2 + 9(\cos^2 x - 1)]}$ $S_{10} = 5[2 - 9\sin^2 x]$ $S_{10} = 10 - 45\sin^2 x$ (b) (i) $(0 <) \frac{1}{3} \tan^2 \theta < 1$ oe $(0 <) \theta < \frac{\pi}{3}$ (ii) $S_\infty = \frac{1}{1 - \frac{1}{3} \tan^2 \frac{\pi}{6}}$ $S_\infty = \frac{9}{8}$ or 1.125	M1 M1 A1 [3] M1 A1 [2] M1 A1 [2]	Correct formula with $d = \pm(\cos^2 x - 1)$ Use of $c^2 + s^2 = 1$ in a correct S_{10} Or $a = 10, b = 45$ Allow < cao Allow < cao
8 (i) $(x-2)^2 - 4 + k$ (ii) $f(x) > k - 4$ or $[k - 4, \infty]$ or $(k - 4, \infty)$ oe (iii) smallest value of $p = 2$ (iv) $x - 2 = (\pm)\sqrt{y + 4 - k}$ $x = 2 + \sqrt{y + 4 - k}$ $f^{-1}(x) = 2 + \sqrt{x + 4 - k}$ Domain is $x > k - 4$ or $[k - 4, \infty]$ or $(k - 4, \infty)$ oe	B1B1 [2] B1✓ [1] B1✓ [1] M1 A1✓ A1 B1✓ [4]	$a = -2, b = -4$ ft <i>their</i> $k - 4$. Accept > ft <i>their</i> 2 ft from <i>their</i> part (i) cao ft from <i>their</i> part (ii). Accept >

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9 (i) $M = (1, 4)$ gradient $= \frac{1}{2}$ soi grad of $MB = -2$ soi Equation $MB: y - 4 = -2(x - 1)$ When $y = 0, x = 3$ or $B = (3, 0)$ (ii) grad of $AB = -\frac{2}{6}$; grad of $BC = \frac{6}{2}$ oe $m_1 m_2 = -1 (\Rightarrow AB \perp AC)$ (iii) $D = (-1, 8)$ $AD = \sqrt{40}$ or 6.32	B1B1 M1 A1[✓] A1[✓] [5] M1[✓] A1 [2] B1 B1 [2]	Use of $m_1 m_2 = -1$ Or $y = -2x + 6$ ft on <i>their</i> $\frac{1}{2}$ or M ft result of putting $y = 0$ into <i>their</i> eqn At least one correct [✓] AG Allow omitted conclusion
10 (i) $3x^2 - 4x + 1 (<) 5$ $(3x + 2)(x - 2) < 0$ $-\frac{2}{3} < x < 2$ or $\left[-\frac{2}{3}, 2\right]$ or $\left(-\frac{2}{3}, 2\right)$. Allow < (ii) $3x^2 - 4x + 1 = 0 \Rightarrow (3x - 1)(x - 1) = 0$ $x = \frac{1}{3}$ or 1 $y = \frac{4}{27}$ or 0 $f''(x) = 6x - 4 \rightarrow f''\left(\frac{1}{3}\right) = -2 (< 0);$ $f''(1) = 2 (> 0)$ max at $\left(\frac{1}{3}, \frac{4}{27}\right)$; min at $(1, 0)$ cao	M1 M1 A2 [4] M1 A1 A1 M1 A1 [5]	Attempt differentiate & put 5 on RHS Attempt to factorise or solve SC Allow A1 for $-\frac{2}{3}$ and 2 seen Derivative = 0 & any attempt to solve Both Both Or other valid method Allow just x values or just y values given for identification

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	11

11 (i) $x = \frac{4}{y^2} - 1$ (ii) $\int \left(\frac{4}{y^2} - 1 \right) dy = \left[-\frac{4}{y} - y \right]$ Upper limit = 2 $\left[\left(-\frac{4}{2} - 2 \right) - \left(-4 - 1 \right) \right]$ 1 (iii) $(\pi) \int x^2 dy = (\pi) \int \left(\frac{16}{y^4} - \frac{8}{y^2} + 1 \right) dy$ $(\pi) \left[\frac{-16}{3y^3} + \frac{8}{y} + y \right]$ $(\pi) \left[\left(\frac{-16}{24} + 4 + 2 \right) - \left(\frac{-16}{3} + 8 + 1 \right) \right]$ $\frac{5\pi}{3}$	B1 [1] B1B1 B1 M1 A1 [5] B1B1 B1 M1 A1 [5]	AG At least 1 step of working needed For $-\frac{4}{y}, -y$ Apply limits 1 and <i>their</i> 2 ‘correctly’ SC B2 for $\int 2(x+1)^{-\frac{1}{2}} dx - 3 \rightarrow 1$ Apply limits 1 and <i>their</i> 2 ‘correctly’
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	GCE AS/A LEVEL – May/June 2012	9709	12

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	GCE AS/A LEVEL – May/June 2012	9709	12

1 $y = \frac{6}{2x-3}$ Integral of $y^2 = \frac{-36}{(2x-3)} \div 2$ Use of limits 2, 3 $\rightarrow 12\pi$ or 37.7	B1 B1 M1 A1 [4]	co allow 2nd B1 independent of 1st. Used as 2 to 3 or 3 to 2 in integral of y^2. Co (uses area 0/4. No π Max 3/4)
2 $y = 4\sqrt{x} + \frac{2}{\sqrt{x}}$ (i) $\frac{dy}{dx} = 4 \cdot \frac{1}{2} x^{-1/2} + 2 \cdot (-1/2) x^{-1.5}$ or $y = \frac{2}{\sqrt{x}} - \frac{1}{x^{1.5}}$ (ii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ used $\rightarrow = \frac{7}{8} \times 0.12 = 0.105$	M1 A1 A1 [3] M1 A1 [2]	Reducing “their” power by 1 once. Allow unsimplified Must be used correctly co – fraction or decimal.
3 Coeff of x^3 in $(a+x)^5 = 10 \times a^2$ Coeff of x^3 in $(2-x)^6 = -160$ $\rightarrow 10a^2 - 160 = 90$ $\rightarrow a = 5$	B1 B1 B1 M1 A1 [5]	co co co forms an equation from 2 terms co
4 $A(-1, -5), B(7, 1).$ $M(3, -2)$ Gradient = $\frac{3}{4}$ Perpendicular gradient = $-\frac{4}{3}$ Eqn $y + 2 = -\frac{4}{3}(x - 3)$ Sets x and y to 0 $C(\frac{3}{2}, 0)$ $D(0, 2)$ $\rightarrow$ Pythagoras $\rightarrow CD = 2.5$	B1 B1 M1 M1 M1A1 [6]	co co needs to be perp. through M. Setting one of x or y to 0. Correct method. co.

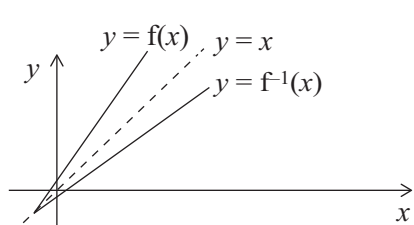
Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	12

5 $\tan x + \frac{1}{\tan x} \equiv \frac{1}{\sin x \cos x}$ (i) $\text{LHS} = \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}$ $= \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} = \frac{1}{\sin x \cos x}$ (ii) $\frac{2}{\sin x \cos x} = 3 \tan x + 1$ Uses (i) $2(\tan x + \frac{1}{\tan x}) = 3 \tan x + 1$ $\rightarrow \tan^2 x + \tan x - 2 = 0$ $\rightarrow \tan x = 1 \text{ or } -2$ $\rightarrow x = 45^\circ \text{ or } 116.6^\circ$	M1 M1 [2] M1 DM1 B1 A1 [4]	Use of $\tan = \sin/\cos$ twice Use of $s^2 + c^2 = 1$ appropriately – everything correct. Uses part (i) to obtain eqn in $\tan x$ only Correct soln of quadratic eqn co. Must have correct quadratic co
6 (i) cosine rule or $2 \times r \times \sin \frac{1}{2}(2.4)$ $\rightarrow 14.9 \text{ cm}$ (ii) Perimeter = (i) + $r\theta$ $\theta = 2\pi - 2.4,$ $\rightarrow 46.0 \text{ cm}$ (iii) Area = Sector + triangle $\frac{1}{2} \times 8^2 (2\pi - 2.4) + \frac{1}{2} \times 8^2 \sin 2.4$ $124.3 + 21.6 \rightarrow 146 \text{ cm}^2.$	M1 A1 [2] M1 B1 A1 $\sqrt{h}$ [3] M1 M1 A1 [3]	Any complete valid method. co Uses $s = r\theta$ with 2.4, or $\pi - 2.4$, or $2\pi - 2.4$ Anywhere in parts (ii) or (iii). Adds 31.1 to (i) for $\sqrt{h}$. Uses $\frac{1}{2}r^2\theta$. Uses any valid method. co
7 (a) $S_n = n^2 + 8n.$ $S_1 = 9 \rightarrow a = 9$ $S_2 = 20 \rightarrow a + d = 11 \rightarrow d = 2$ (or equating $n^2 + 8n$ with S_n and comparing coefficients) (b) $a - ar = 9$ $ar + ar^2 = 30$ Eliminates $a \rightarrow 3r^2 + 13r - 10 = 0$ or $\rightarrow 2a^2 - 57a + 81 = 0$ $\rightarrow r = \frac{2}{3}$ $\rightarrow a = 27$	B1 M1 A1 [3] B1 B1 M1 A1 A1 [5]	co Realises that S_2 is $a + (a + d)$. co co co Complete elimination of r or a Correct quadratic. co (condone 27 or 1.5)

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	12

8 (i) $3\mathbf{i} - 4\mathbf{k}, 2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}$. Dot product = $6 + 24 = 30$ $= \sqrt{25} \times \sqrt{49} \cos \theta$ $\rightarrow$ angle = 31° or $0.54(1)$ radians. (ii) $\mathbf{OA} = (3\mathbf{i} - 4\mathbf{k}) \times (15 \div 5)$ $\rightarrow 9\mathbf{i} - 12\mathbf{k}$ $\mathbf{OB} = (2\mathbf{i} + 3\mathbf{j} - 6\mathbf{k}) \times (14 \div 7)$ $\rightarrow 4\mathbf{i} + 6\mathbf{j} - 12\mathbf{k}$. (iii) $\mathbf{AB} = \mathbf{b} - \mathbf{a} = -5\mathbf{i} + 6\mathbf{j}$ $\rightarrow$ Magnitude of $\sqrt{61}$ or 7.81 $\rightarrow$ Unit vector of $(-5\mathbf{i} + 6\mathbf{j}) \div \sqrt{61}$	M1 M1 M1 A1 [4] M1 A1 A1 [3] M1 M1 A1 [3]	Uses $x_1x_2 + y_1y_2 + z_1z_2$ Method for modulus Links everything correctly. co M mark for $\times(15 \div 5)$ or $\times(14 \div 7)$ A1 for $\mathbf{OA}$ A1 for $\mathbf{OB}$ Correct use for either $\mathbf{AB}$ or $\mathbf{BA}$ Complete method for unit vector. co
9 $y = -x^2 + 8x - 10$ (i) $\frac{dy}{dx} = -2x + 8$ $= 0$ when $x = 4$, A is $(4, 6)$ Equation of AB is $y - 6 = 2(x - 4)$ Sim eqns with eqn of curve $\rightarrow x^2 - 6x + 8 = 0$ or $y^2 - 8y + 12 = 0$ $\rightarrow B(2, 2)$ (ii) $\int -x^2 + 8x - 10 \, dx = -\frac{x^3}{3} + 4x^2 - 10x$ Uses his x limits 2 to 4 $\rightarrow 9\frac{1}{3}$	B1 M1A1 M1 M1 A1 A1 [7] B2,1 M1 A1 [4]	co Sets to 0 and attempt to solve for x. co. Correct form of equation. Eliminates x or y completely Method for quadratic eqn = 0. co (Must not be guessed from diagram) 3 terms, loses 1 for each error Uses x limits correctly – allow $\pm$ co – allow $\pm$ (2 must have been correctly found, not guessed)

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10 $f: x \mapsto 2x+5$ $g: x \mapsto \frac{8}{x-3}$ (i) $f^{-1} = \frac{1}{2}(x-5)$ $g^{-1} = \frac{8}{x} + 3, x \neq 0$ (ii)  (iii) $fg(x) = \frac{16}{x-3} + 5$ Forms eqn $\frac{16}{x-3} + 5 = 5 - kx$ $kx^2 - 3kx + 16 = 0$ Uses $b^2 = 4ac \rightarrow k = \frac{64}{9}$ or 0 Set of values $0 < k < \frac{64}{9}$	B1 M1 A1 B1 $\frac{1}{2}$ [4] B1 B1 B1 [3] B1 M1 M1 A1 A1 [5]	co Attempt at x the subject. co but $(f(x))$ Allow if a linear denominator. +ve gradient, +ve y intercept +ve gradient, +ve y intercept States, or shows the line $y = x$ as a line of symmetry. co Must lead to a quadratic Use of $b^2 - 4ac$ even if $<0, >0$. co Condone $<$ co
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- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously "correct" answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

Page 3	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	13

The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no "follow through" from a previous error is allowed)
CWO	Correct Working Only - often written by a 'fortuitous' answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR -1	A penalty of MR -1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become "follow through ✓" marks. MR is not applied when the candidate misreads his own figures - this is regarded as an error in accuracy. An MR-2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA -1	This is deducted from A or B marks in the case of premature approximation. The PA -1 penalty is usually discussed at the meeting.

Page 4	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	13

1 $\tan^2 \theta - \sin^2 \theta = \tan^2 \theta \sin^2 \theta$ (i) $\frac{s^2}{c^2} - s^2$ $\rightarrow \frac{s^2 - s^2 c^2}{c^2} = \frac{s^2(1 - c^2)}{c^2}$ $\rightarrow t^2 s^2$ (ii) $\text{RHS} > 0 \rightarrow \tan^2 \theta > \sin^2 \theta$ QED $\tan \theta > \sin \theta$ if θ acute.	M1 M1 A1 [3] B1 [1]	Use of $s \div c = t$ Use of $s^2 + c^2 = 1$ All ok Realises $\text{RHS} > 0$
2 $\vec{OA} = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}, \vec{OB} = \begin{pmatrix} 4 \\ 2 \\ -2 \end{pmatrix}, \vec{OC} = \begin{pmatrix} 1 \\ 3 \\ p \end{pmatrix}.$ (i) $\vec{AB} = \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ Modulus = $\sqrt{(4+9+36)}$ Unit Vector = $\frac{1}{7} \begin{pmatrix} 2 \\ 3 \\ -6 \end{pmatrix}$ (ii) $\vec{OB} \cdot \vec{OC} = 4 + 6 - 2p$ $= 0 \rightarrow p = 5$	B1 M1 A1✓ [3] M1A1 [2]	co. Correct method for modulus co for his vector AB. Dot product = 0. co
3 $(1 - 2x)^2(1 + ax)^6$ Coeff of x in $(1 + ax)^6 = 6ax$ Coeff of x^2 in $(1 + ax)^6 = 15a^2x^2$ Multiplies by $(1 - 4x + 4x^2)$ 2 terms in x $6a - 4 = -1$ $\rightarrow a = \frac{1}{2}$ 3 terms in x^2 $15a^2 - 24a + 4 = b$ $\rightarrow b = -4\frac{1}{4}$	B1 B1 M1 A1 M1 A1 [6]	6C1 needs removing (here or later) 6C2 needs removing (here or later) Needs to consider 2 terms in equation Co Needs to consider 3 terms in equation
4 $\sin 2x + 3 \cos 2x = 0$ (i) $\rightarrow \tan 2x = -3$ $2x = 180 - 71.6$ or $360 - 71.6$ $x = 54.2^\circ$ or 144.2° Also 234.2° and 324.2° (ii) 12 answers.	M1 M1 A1A1✓ A1✓ [5] B1✓ [1]	Uses $\tan 2x = k$ and works with “2x”. Finds “2x” before $\div 2$ co. co✓ (both of these need 2nd M) for 180° + his answer(s) for 3 times the number of solns to (i).

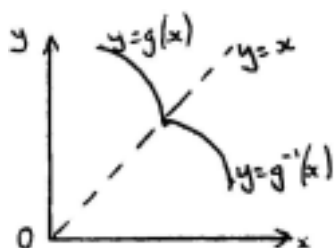
Page 5	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	13

5 $x = \frac{8}{y^2} - 2$; at $x = 0, y = 2$ $\rightarrow x^2 = \frac{64}{y^4} - \frac{32}{y^2} + 4$ Integral of $x^2 = \frac{64y^{-3}}{-3} - \frac{32y^{-1}}{-1} + 4y$ Uses limits 1 to 2 $\rightarrow 6\frac{2}{3}\pi$	B1 B1B1B1 M1 A1 [6]	co All co. Uses 1 to 2 or 2 to 1. co.
6 (i) Uses S_n $\frac{9}{2}(24 + 8d) = 135 \rightarrow d = \frac{3}{4}$ (ii) 9th term of AP = $12 + 8 \times \frac{3}{4} = 18$ GP 1st term 12, 2nd term 18 Common ratio = $r = 18 \div 12 = 1\frac{1}{2}$ 3rd term of GP = $ar^2 = 27$ nth term of AP is $12 + (n - 1)\frac{3}{4}$ $12 + (n - 1)\frac{3}{4} = 27 \rightarrow n = 21$	M1 A1 [2] B1✓ M1 M1 M1A1 [5]	Uses correct formula co ✓ on “d” Uses “ar” Uses ar^2 or “ar” $\times r$ Links AP with GP. co
7 $y = \frac{10}{2x+1} - 2$ (i) $\frac{dy}{dx} = \frac{-10}{(2x+1)^2} \times 2$ At A, $y = 0, \rightarrow x = 2$ m at $x = 2$, is $-\frac{4}{5}$ Eqn of tangent is $y = -\frac{4}{5}(x - 2)$ $\rightarrow 5y + 4x = 8$ (ii) C (0, 1.6) $d = \sqrt{(1.6^2 + 2^2)} = 2.56$	B1 B1 B1 M1 A1 [5] M1 A1 [2]	Without the “$\times 2$”. For the “$\times 2$”. For $x = 2$ Must be using differential as m co – answer given. Correct method – needs ✓. co
8 (i) $OBX = 90^\circ, \cos \theta = \frac{r}{2r}$ $\rightarrow \theta = \frac{1}{3}\pi$. (ii) Arc length $AB = \frac{1}{3}r\pi$ $BX = r \tan(\frac{1}{3}\pi) = r\sqrt{3}$ $P = r + (\frac{1}{3}r\pi + r\sqrt{3})$ (iii) Area = $\frac{1}{2}r^2\sqrt{3} - \frac{1}{6}r^2\pi$	M1 A1 [2] B1 B1 B1 [3] B1✓ B1 [2]	Needs $90^\circ + \cos$ (or Pyth + sin or tan) co ag r + sum of other two ✓ on $\tan(\frac{1}{3}\pi)$. co

Page 6	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	13

9 $\frac{d^2y}{dx^2} = -4x$ (i) $\frac{dy}{dx} = -2x^2 + c$ $\frac{dy}{dx} = 0$ when $x = 2$, $\rightarrow c = 8$ $y = -\frac{2x^3}{3} + 8x (+C)$ Subs (2, 12) $\rightarrow C = \frac{4}{3}$ (iii) $\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$ $= -10 \times 0.05$ $\rightarrow$ decreasing at 0.5 units per second	B1 B1 B1 B1 $\frac{1}{3}$ M1 A1 [6] M1 A1 [2]	For $-2x^2$ $c = 8$ For each term – $\frac{1}{3}$ on “c” – ignore (+C) Uses (2, 12) to find C. Must use. Enough to see product of gradient and rate. bod over notation.
10 $2y + x = k \quad xy = 6$ (i) $2y + x = 8 \rightarrow y(8 - 2y) = 6$ $2y^2 - 8y + 6 = 0$ or $x^2 - 8x + 12 = 0$ $\rightarrow (6, 1)$ and $(2, 3)$ Midpoint $M(4, 2)$ $m = -\frac{1}{2}$ Perpendicular $m = 2$ $\rightarrow y - 2 = 2(x - 4)$ (ii) $(k - 2y)y = 6$ $\rightarrow 2y^2 - ky + 6 = 0$ or $x^2 - kx + 12 = 0$ Uses $b^2 - 4ac$ (0) $\rightarrow k^2 > 48$ $\rightarrow k < -\sqrt{48}$ and $k > \sqrt{48}$	M1 DM1A1 M1 M1 A1 [6] M1 A1 A1 [3]	Complete elimination of x (or y) DM1 soln of quadratic. co for their 2 points Uses $m_1m_2 = -1$ to find perp. gradient co unsimplified Any use of $b^2 - 4ac$ on a quadratic = 0 For $\sqrt{48}$ on its own All correct.

Page 7	Mark Scheme: Teachers' version	Syllabus	Paper
	GCE AS/A LEVEL – May/June 2012	9709	13

11 $f(x) = 8 - (x - 2)^2$, (i) Stationary point at $x = 2$ y – coordinate = 8 Nature Maximum (or $y = -x^2 + 4x + 4$ $-2x + 4 = 0 \rightarrow (2, 8)$ Max) (ii) $k = 2$ (iii) $y = 8 - (x - 2)^2$ $\rightarrow (x - 2)^2 + y = 8$ $\rightarrow (x - 2) = \pm \sqrt{8 - y}$ $\rightarrow g^{-1} = 2 + \sqrt{8 - x}$ (iv) 	B1 B1 B1 [3] B1✓^h [1] M1 M1 A1 [3] B1 B1 B1 [3]	co co co independent of first two marks ✓^h on “x-value” Attempt to make x the subject Order of operations correct Must be $f(x)$. B1 arc 1st quad (no tp, no axes) B1 Evidence of symmetry about $y = x$ B1 all correct as shown left
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