



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2011

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of 4 printed pages.



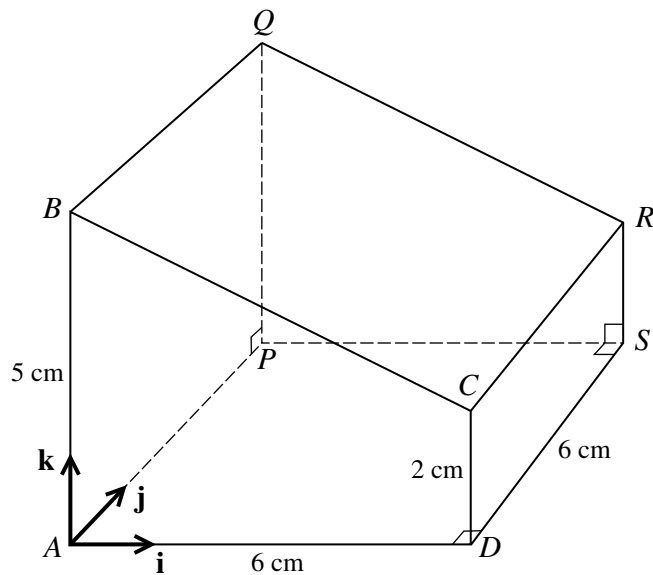
1 Find the coefficient of x in the expansion of $\left(x + \frac{2}{x^2}\right)^7$. [3]

2 The volume of a spherical balloon is increasing at a constant rate of 50 cm^3 per second. Find the rate of increase of the radius when the radius is 10 cm. [Volume of a sphere = $\frac{4}{3}\pi r^3$.] [4]

3 (i) Sketch the curve $y = (x - 2)^2$. [1]

(ii) The region enclosed by the curve, the x -axis and the y -axis is rotated through 360° about the x -axis. Find the volume obtained, giving your answer in terms of π . [4]

4



The diagram shows a prism $ABCDPQRS$ with a horizontal square base $APSD$ with sides of length 6 cm. The cross-section $ABCD$ is a trapezium and is such that the vertical edges AB and DC are of lengths 5 cm and 2 cm respectively. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to AD , AP and AB respectively.

(i) Express each of the vectors $\overrightarrow{CP}$ and $\overrightarrow{CQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [2]

(ii) Use a scalar product to calculate angle PCQ . [4]

5 (i) Show that the equation $2 \tan^2 \theta \sin^2 \theta = 1$ can be written in the form

$$2 \sin^4 \theta + \sin^2 \theta - 1 = 0. \quad [2]$$

(ii) Hence solve the equation $2 \tan^2 \theta \sin^2 \theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 6 The variables x , y and z can take only positive values and are such that

$$z = 3x + 2y \quad \text{and} \quad xy = 600.$$

(i) Show that $z = 3x + \frac{1200}{x}$. [1]

(ii) Find the stationary value of z and determine its nature. [6]

- 7 A curve is such that $\frac{dy}{dx} = \frac{3}{(1+2x)^2}$ and the point $(1, \frac{1}{2})$ lies on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the set of values of x for which the gradient of the curve is less than $\frac{1}{3}$. [3]

- 8 A television quiz show takes place every day. On day 1 the prize money is \$1000. If this is not won the prize money is increased for day 2. The prize money is increased in a similar way every day until it is won. The television company considered the following two different models for increasing the prize money.

Model 1: Increase the prize money by \$1000 each day.

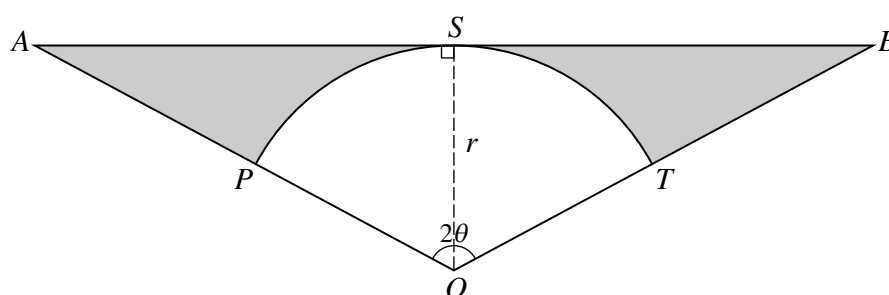
Model 2: Increase the prize money by 10% each day.

On each day that the prize money is not won the television company makes a donation to charity. The amount donated is 5% of the value of the prize on that day. After 40 days the prize money has still not been won. Calculate the total amount donated to charity

(i) if Model 1 is used, [4]

(ii) if Model 2 is used. [3]

9



In the diagram, OAB is an isosceles triangle with $OA = OB$ and angle $AOB = 2\theta$ radians. Arc PST has centre O and radius r , and the line ASB is a tangent to the arc PST at S .

(i) Find the total area of the shaded regions in terms of r and θ . [4]

(ii) In the case where $\theta = \frac{1}{3}\pi$ and $r = 6$, find the total perimeter of the shaded regions, leaving your answer in terms of $\sqrt{3}$ and π . [5]

[Questions 10 and 11 are printed on the next page.]

- 10** (i) Express $2x^2 - 4x + 1$ in the form $a(x + b)^2 + c$ and hence state the coordinates of the minimum point, A , on the curve $y = 2x^2 - 4x + 1$. [4]

The line $x - y + 4 = 0$ intersects the curve $y = 2x^2 - 4x + 1$ at points P and Q . It is given that the coordinates of P are $(3, 7)$.

- (ii) Find the coordinates of Q . [3]

- (iii) Find the equation of the line joining Q to the mid-point of AP . [3]

- 11** Functions f and g are defined for $x \in \mathbb{R}$ by

$$f : x \mapsto 2x + 1,$$

$$g : x \mapsto x^2 - 2.$$

- (i) Find and simplify expressions for $fg(x)$ and $gf(x)$. [2]

- (ii) Hence find the value of a for which $fg(a) = gf(a)$. [3]

- (iii) Find the value of b ($b \neq a$) for which $g(b) = b$. [2]

- (iv) Find and simplify an expression for $f^{-1}g(x)$. [2]

The function h is defined by

$$h : x \mapsto x^2 - 2, \quad \text{for } x \leq 0.$$

- (v) Find an expression for $h^{-1}(x)$. [2]



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- 1 Find $\int \left(x^3 + \frac{1}{x^3}\right) dx$. [3]
- 2 (i) Find the terms in x^2 and x^3 in the expansion of $\left(1 - \frac{3}{2}x\right)^6$. [3]
- (ii) Given that there is no term in x^3 in the expansion of $(k + 2x)\left(1 - \frac{3}{2}x\right)^6$, find the value of the constant k . [2]
- 3 The equation $x^2 + px + q = 0$, where p and q are constants, has roots -3 and 5 .
- (i) Find the values of p and q . [2]
- (ii) Using these values of p and q , find the value of the constant r for which the equation $x^2 + px + q + r = 0$ has equal roots. [3]
- 4 A curve has equation $y = \frac{4}{3x-4}$ and $P(2, 2)$ is a point on the curve.
- (i) Find the equation of the tangent to the curve at P . [4]
- (ii) Find the angle that this tangent makes with the x -axis. [2]
- 5 (i) Prove the identity $\frac{\cos \theta}{\tan \theta(1 - \sin \theta)} \equiv 1 + \frac{1}{\sin \theta}$. [3]
- (ii) Hence solve the equation $\frac{\cos \theta}{\tan \theta(1 - \sin \theta)} = 4$, for $0^\circ \leq \theta \leq 360^\circ$. [3]
- 6 The function f is defined by $f : x \mapsto \frac{x+3}{2x-1}$, $x \in \mathbb{R}$, $x \neq \frac{1}{2}$.
- (i) Show that $ff(x) = x$. [3]
- (ii) Hence, or otherwise, obtain an expression for $f^{-1}(x)$. [2]
- 7 The line L_1 passes through the points $A(2, 5)$ and $B(10, 9)$. The line L_2 is parallel to L_1 and passes through the origin. The point C lies on L_2 such that AC is perpendicular to L_2 . Find
- (i) the coordinates of C , [5]
- (ii) the distance AC . [2]

- 8 Relative to the origin O , the position vectors of the points A , B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 3 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 10 \\ 0 \\ 6 \end{pmatrix}.$$

- (i) Find angle ABC . [6]

The point D is such that $ABCD$ is a parallelogram.

- (ii) Find the position vector of D . [2]

- 9 The function f is such that $f(x) = 3 - 4 \cos^k x$, for $0 \leq x \leq \pi$, where k is a constant.

- (i) In the case where $k = 2$,

- (a) find the range of f , [2]

- (b) find the exact solutions of the equation $f(x) = 1$. [3]

- (ii) In the case where $k = 1$,

- (a) sketch the graph of $y = f(x)$, [2]

- (b) state, with a reason, whether f has an inverse. [1]

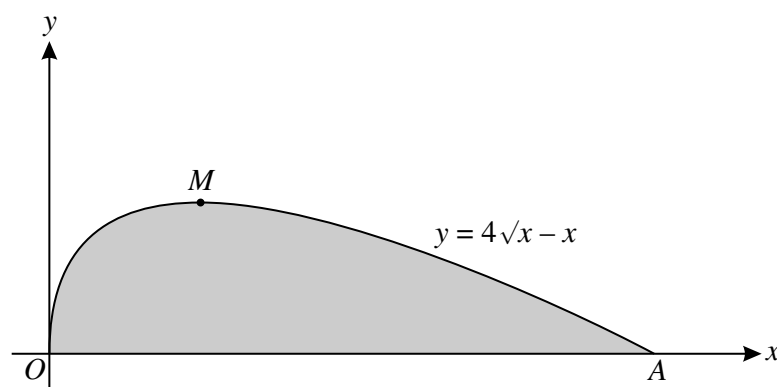
- 10 (a) A circle is divided into 6 sectors in such a way that the angles of the sectors are in arithmetic progression. The angle of the largest sector is 4 times the angle of the smallest sector. Given that the radius of the circle is 5 cm, find the perimeter of the smallest sector. [6]

- (b) The first, second and third terms of a geometric progression are $2k + 3$, $k + 6$ and k , respectively. Given that all the terms of the geometric progression are positive, calculate

- (i) the value of the constant k , [3]

- (ii) the sum to infinity of the progression. [2]

11



The diagram shows part of the curve $y = 4\sqrt{x} - x$. The curve has a maximum point at M and meets the x -axis at O and A .

- (i) Find the coordinates of A and M . [5]

- (ii) Find the volume obtained when the shaded region is rotated through 360° about the x -axis, giving your answer in terms of π . [6]

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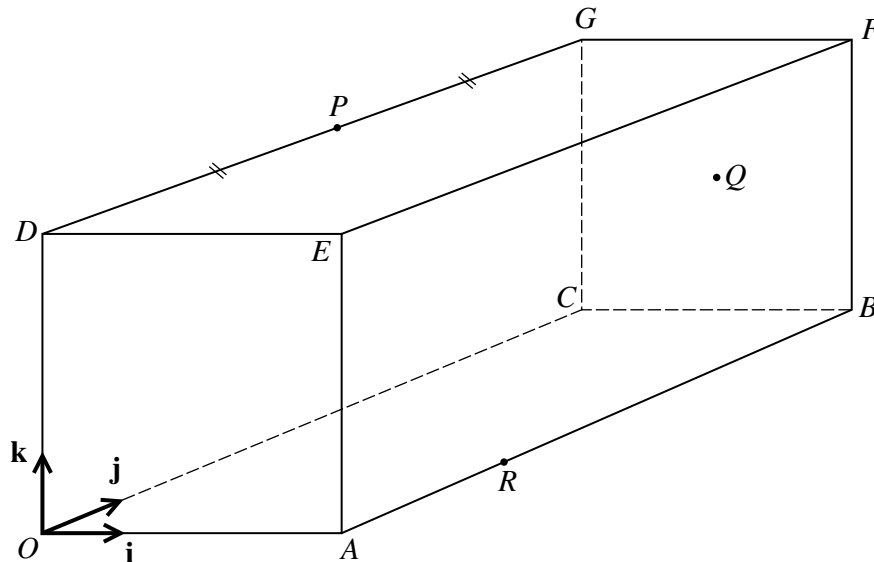
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- 1 The coefficient of x^3 in the expansion of $(a+x)^5 + (1-2x)^6$, where a is positive, is 90. Find the value of a . [5]
- 2 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at two distinct points. [5]
- 3 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, meets the x -axis at P and the y -axis at Q . Given that $PQ = \sqrt{45}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b . [5]
- 4 (a) Differentiate $\frac{2x^3 + 5}{x}$ with respect to x . [3]
- (b) Find $\int (3x - 2)^5 dx$ and hence find the value of $\int_0^1 (3x - 2)^5 dx$. [4]

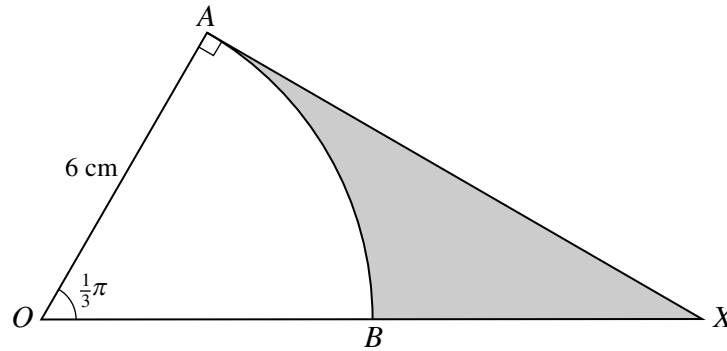
5



In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG , Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.

- (i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$. [3]
- (ii) Use a scalar product to find angle RQP . [4]
- 6 (a) A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]
- (b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]

7



In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A , and OBX is a straight line.

- (i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

- (ii) the area of the shaded region, [3]

- (iii) the perimeter of the shaded region. [4]

- 8 (i) Prove the identity $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta}\right)^2 \equiv \frac{1 - \cos \theta}{1 + \cos \theta}$. [3]

- (ii) Hence solve the equation $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta}\right)^2 = \frac{2}{5}$, for $0^\circ \leq \theta \leq 360^\circ$. [4]

- 9 A curve is such that $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ and $P(9, 5)$ is a point on the curve.

- (i) Find the equation of the curve. [4]

- (ii) Find the coordinates of the stationary point on the curve. [3]

- (iii) Find an expression for $\frac{d^2y}{dx^2}$ and determine the nature of the stationary point. [2]

- (iv) The normal to the curve at P makes an angle of $\tan^{-1}k$ with the positive x -axis. Find the value of k . [2]

- 10 Functions f and g are defined by

$$f : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$

$$g : x \mapsto 2(x - 1)^3 + 8, \quad x > 1.$$

- (i) Evaluate $fg(2)$. [2]

- (ii) Sketch in a single diagram the graphs of $y = f(x)$ and $y = f^{-1}(x)$, making clear the relationship between the graphs. [3]

- (iii) Obtain an expression for $g'(x)$ and use your answer to explain why g has an inverse. [3]

- (iv) Express each of $f^{-1}(x)$ and $g^{-1}(x)$ in terms of x . [4]

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