

MARK SCHEME for the May/June 2011 question paper
for the guidance of teachers

9709 MATHEMATICS

9709/11

Paper 1, maximum raw mark 75

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A Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).

B Mark for a correct result or statement independent of method marks.

- When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly when there are several B marks allocated. The notation DM or DB (or dep*) is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- The symbol $\checkmark$ implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only. A and B marks are not given for fortuitously “correct” answers or results obtained from incorrect working.
- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

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AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\sqrt{}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
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1 ${}^7C_2 x^5 \left(\frac{2}{x^2}\right)^2$ SOI and leading to final answer 84 or $84x$ as final answer	B2 B1 [3]	B1 for 2/3 parts correct leading to ans. If no answer; $84x$ seen scores B2, else ${}^7C_2 x^5 \left(\frac{2}{x^2}\right)^2$ scores SCB1 only
2 $\left(\frac{dv}{dr} = \right) 4\pi r^2$ $= 4\pi \times 10^2$ $\frac{dr}{dt} = \frac{dv}{dt} \frac{dv}{dr}$ OE used $\frac{50}{4\pi \times 10^2} = \frac{1}{8\pi}$ or 0.0398	M1 A1 M1 A1 [4]	SOI at any point Correct link between differentials with $\frac{dr}{dt}$ finally as subject Allow $\frac{50}{400\pi}$. Non-calculus methods $\frac{0}{4}$
3 (i) Correct shape – touching positive x-axis (ii) $(\pi) \int (x-2)^4 dx$ $(\pi) \left[\frac{(x-2)^5}{5} \right]$ $(\pi) [0 - (-32)/5]$ $\frac{32\pi}{5}$ or 6.4π	B1 [1] M1 A1 M1 A1 [4]	Ignore intersections with axes Use $(\pi) \int y^2 dx$ & attempt integrate but expansion before integ needs 5 terms Use of limits 0, 2 on <i>their</i> $(\pi) \int y^2 dx$ cao Rotation about y-axis max 1/5
4 (i) $\overrightarrow{CP} = -6\mathbf{i} + 6\mathbf{j} - 2\mathbf{k}$ $\overrightarrow{CQ} = -6\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}$ (ii) Scalar product = $36 + 36 - 6$ $66 = \overrightarrow{CP} \overrightarrow{CQ} \cos \theta$ $\overrightarrow{CP} = \sqrt{76}$, $\overrightarrow{CQ} = \sqrt{81}$ Angle $PCQ = 32.7^\circ$ (or 0.571 rad)	B1 B1 [2] M1 M1 M1 A1 [4]	Use of $x_1x_2 + y_1y_2 + z_1z_2$ Linking everything correctly Correct magnitude for either cao 147.3° converted to 32.7° gets A0

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5 (i) $\frac{2\sin^2 \theta \sin^2 \theta}{1 - \sin^2 \theta} = 1$ $2\sin^4 \theta + \sin^2 \theta - 1 = 0$ AG (ii) $(2\sin^2 \theta - 1)(\sin^2 \theta + 1) = 0$ $\sin \theta = \frac{(\pm)1}{\sqrt{2}}$ $\theta = 45^\circ, 135^\circ$ $\theta = 225^\circ, 315^\circ$	M1 A1 [2] M1 A1 A1 A1 [4]	Equation as function of $\sin \theta$ Or use formula on quadratic in $\sin^2 \theta$ Provided no excess solutions in range
6 (i) $z = 3x + 2\left(\frac{600}{x}\right)$ or $x\frac{(z-3x)}{2} = 600$ OE → AG (ii) $\frac{dz}{dx} = 3 - \frac{1200}{x^2}$ or $\frac{dz}{dy} = 2 - \frac{1800}{y^2}$ $= 0 \rightarrow x = 20$ or $= 0 \rightarrow y = 30$ $z = 60 + \frac{120}{20} = 120$ $\frac{d^2z}{dx^2} = \frac{2400}{x^3}$ $> 0 \Rightarrow$ minimum	B1 [1] B1 M1A1 A1✓ B1✓ B1 [6]	Set to 0 & attempt to solve. Allow ± 20 Ft from <i>their</i> x provided positive Or other valid method Dep. on $\frac{d^2z}{dx^2} = \frac{k}{x^3}$ ($k > 0$) or other valid method.
7 (i) $\frac{3(1+2x)^{-1}}{-1} + (c)$ $y = \frac{3(1+2x)^{-1}}{-2} + (c)$ Sub (1, (1/2)) $\frac{1}{2} = \frac{3}{-6} + c \Rightarrow c = 1$ (ii) $(1+2x)^2(>)9$ or $4x^2 + 4x - 8(>)0$ OE 1, -2 $x > 1, x < -2$ ISW	B1 B1(indep) M1 A1 [4] M1 A1 A1 [3]	Division by 2 $y =$ necessary Dependent on c present Use of $y = mx + c$ etc. gets 0/4
8 (i) 1000, 2000, 3000... or 50, 100, 150... $\frac{40}{2(1000 + 40000)}$ or $\frac{40}{2(2000 + 39000)}$ × 5% of attempt at valid sum 41000 (ii) 1000, 1000×1.1, $1000 \times 1.1^2 + \dots$ or with $a = 50$ $\frac{1000(1.1^{40} - 1)}{1.1 - 1}$ 22100	M1 M1 M1 A1 [4] M1 M1 A1 [3]	Recognise series, correct a/d (or 3 terms) Correct use of formula Can be awarded in either (i) or (ii) cao Recognise series, correct a/r (or 3 terms) Correct use of formula. Allow e.g. $r = 0.1$ Or answers rounding to this

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9 (i) $AS = r \tan \theta$ Area $OAB = r^2 \tan \theta$ or $(OAS) = \frac{1}{2} r^2 \tan \theta$ Area of sector $= \frac{1}{2} r^2 \times 2\theta (= r^2 \theta)$ Shaded area $= r^2 (\tan \theta - \theta)$ OE (ii) $\cos \frac{\pi}{3} = \frac{6}{OA} \Rightarrow OA = 12$ $AP = 6$ $AS = 6 \tan \frac{\pi}{3} (\Rightarrow AB = 12\sqrt{3})$ Arc $(PST) = 12 \frac{\pi}{3}$ Perimeter $= 12 + 12\sqrt{3} + 4\pi$	M1 A1 B1 A1 [4] M1 A1 B1 B1 A1 [5]	Or $(AB) = 2r \tan \theta$ or $(AO) = \frac{r}{\cos \theta}$ Or $OAB = \frac{1}{2} \frac{r^2}{\cos 2\theta} \sin 2\theta$ Or area sector $(OPS) = \frac{1}{2} r^2 \theta$ Allow e.g. $r^2 \tan \theta - \frac{1}{2} r^2 2\theta$ Or arc $(PS) = 6 \frac{\pi}{3}$ or arc $(ST) = 6 \frac{\pi}{3}$ Allow unsimplified 4π
10 (i) $2(x-1)^2 - 1$ OR $a = 2, b = -1, c = -1$ $A = (1, -1)$ (ii) $2x^2 - 5x - 3 = 0 \Rightarrow (2x+1)(x-3) = 0$ OE in y $x = -\frac{1}{2}, y = 3\frac{1}{2}$ (iii) Mid-point of $AP = (2, 3)$ Gradient of line $= \frac{\frac{1}{2}}{\frac{-5}{2}} = \frac{-1}{5}$ Equation is $y - 3 = \frac{-1}{5}(x - 2)$ OE	B1, B1, B1 B1✓ [4] M1, M1 A1 [3] B1✓ B1 B1 [3]	Allow alt. method for final mark Complete elim & simplify, attempt soln. Additional (3, 7) not penalised Follow through on <i>their A</i> Or $y - 3\frac{1}{2} = -\frac{1}{5(x + \frac{1}{2})}$
11 (i) $fg(x) = 2x^2 - 3, gf(x) = 4x^2 + 4x - 1$ (ii) $2a^2 - 3 = 4a^2 + 4a - 1 \Rightarrow 2a^2 + 4a + 2 = 0$ $(a+1)^2 = 0$ $a = -1$ (iii) $b^2 - b - 2 = 0 \rightarrow (b+1)(b-2) = 0$ $b = 2$ Allow $b = -1$ in addition (iv) $f^{-1}(x) = \frac{1}{2}(x-1)$ $f^{-1}g(x) = \frac{1}{2}(x^2 - 3)$ (v) $x = (\pm)\sqrt{y+2}$ $h^{-1}(x) = -\sqrt{x+2}$	B1, B1 [2] M1 M1 A1 [3] M1 A1 [2] B1 B1✓ [2] M1 A1 [2]	fg & gf clearly transposed gets B0B0 Dep. quadratic. Allow x for all 3 marks Allow marks in (ii) if transposed in (i) Allow in terms of x for M1 only Correct answer without working B2 Must be simplified. Ft from <i>their</i> f^{-1}

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1	$\int \left(x^3 + \frac{1}{x^3} \right) dx = \frac{x^4}{4} + \frac{x^{-2}}{-2} + c$	3 × B1 [3]	Allow unsimplified, 1 mark for each term, including “c”
2	$\left(1 - \frac{3}{2}x \right)^6$ (i) Term in x^2 ${}^6C_2 \times \left(\frac{\pm 3x}{2} \right)^2 = \frac{135x^2}{4}$ Term in x^3 ${}^6C_3 \times \left(\frac{\pm 3x}{2} \right)^3 = \frac{-540x^3}{8}$ (ii) Term in $x^3 = \frac{270x^3}{4} - \frac{135kx^3}{2}$ $\rightarrow k = 1.$	M1 A1 A1 [3] M1 A1 [2]	For either unsimplified term co co (omission or error with “–” can still gain 2 out of 3) considers exactly 2 terms in x^3 co
3	(i) $x^2 + px + q = (x+3)(x-5)$ $\rightarrow p = -2, q = -15.$ (any other method ok) (ii) $x^2 + px + q + r = 0$ Use of “ $b^2 - 4ac$ ” Uses a, b and c correctly $r = 16$ or $= (x+k)^2 \rightarrow 2k = p$ (M1) $k^2 = q + r$ (M1) $\rightarrow k = -1 \rightarrow r = 16$ (A1)	M1 A1 [2] M1 DM1 A1 [3]	Must be $(x+3)$ and $(x-5)$. co Any use of “ $b^2 - 4ac$ ” c must include both q and r . co
4	$y = \frac{4}{3x-4}$ (i) $\frac{dy}{dx} = -4(3x-4)^{-2} \times 3$ If $x = 2, m = -3$ Eqn of tangent $y - 2 = -3(x - 2)$ (ii) $\tan \theta = \pm(-3)$ $\rightarrow \theta = \pm 108.4^\circ$ (or $\pm 71.6^\circ$) or scalar product, $\tan \theta = y\text{-step} \div x\text{-step}$ or use of $\tan(A - B)$ M1A1 for each	B1 B1 M1 A1 [4] M1 A1√ [2]	Correct without $\times 3$. For $\times 3$. Correct line eqn. co (for normal M0A0) Correct link with ($\pm$ his gradient) co (accept acute or obtuse) or -71.6° or radians

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5 (i) $\frac{\cos \theta}{\tan \theta(1 - \sin \theta)} \equiv \frac{\cos^2 \theta}{\sin \theta(1 - \sin \theta)}$ $= \frac{1 - \sin^2 \theta}{\sin \theta(1 - \sin \theta)}$ $= \frac{1 + \sin \theta}{\sin \theta} = \frac{1}{\sin \theta} + 1$ (ii) $\frac{\cos \theta}{\tan \theta(1 - \sin \theta)} = 4 \rightarrow \frac{1}{\sin \theta} + 1 = 4$ $\rightarrow \sin \theta = \frac{1}{3} \rightarrow \theta = 19.5^\circ, 160.5^\circ$	M1 M1 A1 [3] M1 A1 A1√ [3]	Use of $t = s \div c$ Replaces $\cos^2 \theta$ with $1 - \sin^2 \theta$ to form $f(\sin \theta)$. AG. Ensure all ok. Must show difference of 2 squares. Linking up to obtain $\sin \theta = k$. co. $\sqrt{180^\circ - 1^\text{st}}$ answer providing there are no other solutions in the range 0° to 360°.
6 (i) $f(x) = \frac{x+3}{2x-1}$ $ff(x) = \frac{\frac{x+3}{2x-1} + 3}{\frac{2(x+3)}{2x-1} - 1} = \frac{7x}{7} = x$ (ii) $y = \frac{x+3}{2x-1}$ $\rightarrow 2xy - y = x + 3$ $\rightarrow x(2y - 1) = y + 3$ $\rightarrow f^{-1}(x) = \frac{x+3}{2x-1}$ or since $ff(x) = x$, $f^{-1}(x) = f(x) = \frac{x+3}{2x-1}$ (M1, A1)	B1 M1 A1 [3] M1 A1 [2]	Replacing “x” twice - must be correct Correct algebra – clearing $(2x - 1)$ AG – all correct. Attempt to make x the subject and complete method co
7 (i) (2, 5) to (10, 9) gradient = $\frac{1}{2}$ Equation of L_2 $y = \frac{1}{2}x$. Gradient of perpendicular = -2 Eqn of Perp $y - 5 = -2(x - 2)$ Sim Eqns $\rightarrow C(3.6, 1.8)$ (ii) $d^2 = 1.6^2 + 3.2^2 \rightarrow d = 3.58$	B1 B1√ M1 M1 A1 [5] M1 A1 [2]	co √ on gradient of L_1 Use of $m_1 m_2 = -1$ Correct form of line eqn co Correct method for AC co (accept with $\sqrt{5}$ in answer)

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8 (i) $\overrightarrow{BA} \cdot \overrightarrow{BC}$ or $\overrightarrow{AB} \cdot \overrightarrow{CB}$ $\overrightarrow{BA} = \begin{pmatrix} -2 \\ 1 \\ 2 \end{pmatrix}, \quad \overrightarrow{BC} = \begin{pmatrix} 6 \\ -2 \\ 3 \end{pmatrix}$ $\overrightarrow{BA} \cdot \overrightarrow{BC} = -8$ $= 3 \times 7 \times \cos \theta$ $\rightarrow \theta = 112.4^\circ \text{ or } 1.96 \text{ radians}$ (ii) $\overrightarrow{OD} = \overrightarrow{OA} + \overrightarrow{AD} = \overrightarrow{OA} + \overrightarrow{BC}$ $= \begin{pmatrix} 8 \\ 1 \\ 8 \end{pmatrix}$	B1 M1 M1 M1 M1 A1 [6] M1 A1√ [2]	Correct two vectors for angle ABC. Correct method for one of the sides. Correct use for any pair of vectors. Correct method for moduli. All linked correctly. co (67.6° usually gets 4/6) Correct method. (allow for $\mathbf{d} = \mathbf{a} + \mathbf{b} - \mathbf{c}$ or for $\mathbf{d} = \mathbf{a} + \mathbf{c} - \mathbf{b}$ or for $\mathbf{d} = \mathbf{b} + \mathbf{c} - \mathbf{a}$) A1√ for his $\overrightarrow{BC}$.
9 (i) (a) $f(x) = 3 - 4\cos^2 x$. One limit is -1 Other limit is 3 (b) $3 - 4\cos^2 x = 1 \rightarrow \cos^2 x = \frac{1}{2}$ $\rightarrow \cos x = \pm \frac{1}{\sqrt{2}}$ $\rightarrow x = \frac{1}{4}\pi \text{ or } \frac{3}{4}\pi$ (ii) (a) (b) f has an inverse since it is 1:1 or increasing or no turning points.	B1 B1 [2] M1 A1 A1√ [3] B1 B1 [2] B1 [1]	co irrespective of inequalities co irrespective of inequalities Makes $\cos x$ the subject. co (radians). √ for “$\pi - (1^{\text{st}} \text{ answer})$” (“exact” means that decimal answers only earn A0 A1√) Joins $(0, -1)$ to $(\pi, 7)$, providing increasing function Not a line, flattens at extremities-needs inflexion. co independent of part (i)

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10 (a) $a + 5d = 4a$ or $\frac{(a+4a)}{2} \times 6$ $\frac{6}{2}(2a+5d)$ or $\frac{(a+4a)}{2} \times 6 = 360$ Sim Eqns $a = 24^\circ$ or $\frac{2\pi}{15}$ rads Arc length = 5θ Perimeter = 12.1. (b) (i) $\frac{k+6}{2k+3} = \frac{k}{k+6}$ $\rightarrow k^2 - 9k - 36 = 0 \rightarrow k = 12$ (NB stating a, ar, ar^2 as $f(k)$ gets M1) (ii) $r = \frac{2}{3}, a = 27$ $\rightarrow S_\infty = 27 \div \frac{1}{3} = 81.$	B1 M1 A1 A1 M1 A1 [6] M1 A1 A1 [3] M1 A1 [2]	co Correct left-hand side. All correct. Either answer. Correct use of arc length with θ in rads. co Correct eqn for k. Co condone inclusion of $k = -3$. Correct formula for S_∞ must have $-1 \leq r \leq 1$. co.
11 $y = 4\sqrt{x} - x.$ (i) At A, $4\sqrt{x} - x = 0 \rightarrow A(16, 0)$ $\frac{dy}{dx} = 2x^{-\frac{1}{2}} - 1$ = 0 when $x = 4 \rightarrow (4, 4)$ (ii) Vol = $\pi \int y^2 dx =$ $\pi \int (16x + x^2 - 8x^{\frac{3}{2}}) dx$ $\pi [8x^2 + \frac{x^3}{3} - 8 \frac{x^{\frac{5}{2}}}{\frac{5}{2}}]$ Limits 0 to 16 $\rightarrow 136.5\pi$. (or 137π)	B1 B1 B1 M1 A1 [5] M1 A3,2,1 DM1 A1 [6]	co – independent of working. B1 for each part. Sets to 0 and solves his eqn. co Use of correct formula + attempt at integration One mark for each term – unsimplified Correct use of his limits. co – (429 ok)

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- Note: B2 or A2 means that the candidate can earn 2 or 0.
B2/1/0 means that the candidate can earn anything from 0 to 2.

The marks indicated in the scheme may not be subdivided. If there is genuine doubt whether a candidate has earned a mark, allow the candidate the benefit of the doubt. Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored.

- Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.
- For a numerical answer, allow the A or B mark if a value is obtained which is correct to 3 s.f., or which would be correct to 3 s.f. if rounded (1 d.p. in the case of an angle). As stated above, an A or B mark is not given if a correct numerical answer arises fortuitously from incorrect working. For Mechanics questions, allow A or B marks for correct answers which arise from taking g equal to 9.8 or 9.81 instead of 10.

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The following abbreviations may be used in a mark scheme or used on the scripts:

AEF	Any Equivalent Form (of answer is equally acceptable)
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
BOD	Benefit of Doubt (allowed when the validity of a solution may not be absolutely clear)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only often written by a ‘fortuitous’ answer
ISW	Ignore Subsequent Working
MR	Misread
PA	Premature Approximation (resulting in basically correct work that is insufficiently accurate)
SOS	See Other Solution (the candidate makes a better attempt at the same question)
SR	Special Ruling (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)

Penalties

MR –1	A penalty of MR –1 is deducted from A or B marks when the data of a question or part question are genuinely misread and the object and difficulty of the question remain unaltered. In this case all A and B marks then become “follow through $\sqrt{}$ ” marks. MR is not applied when the candidate misreads his own figures – this is regarded as an error in accuracy. An MR–2 penalty may be applied in particular cases if agreed at the coordination meeting.
PA –1	This is deducted from A or B marks in the case of premature approximation. The PA –1 penalty is usually discussed at the meeting.

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1 $(a+x)^5 + (1-2x)^6$ Coeff of x^3 in $1^{\text{st}} = 10 \times a^2$ Coeff of x^3 in $2^{\text{nd}} = 20 \times (-2)^3$ $\rightarrow 10a^2 - 160 = 90$ $\rightarrow a = 5$	B1 B1 + B1 M1 A1 [5]	co co Forming an equation for a + solution co (condone $\pm$)
2 $y = mx + 4$ $y = 3x^2 - 4x + 7$ Equate $\rightarrow 3x^2 - (4+m)x + 3 = 0$ Uses $b^2 - 4ac \rightarrow (4+m)^2 - 36$ Solution of quadratic $m = 2$ or -10 Set of values $m > 2$ or $m < -10$	M1 M1 DM1 A1 A1 [5]	Eliminates y (or x) completely Any use of $b^2 - 4ac$ Method shown. Correct end-values co
3 $\frac{x}{a} + \frac{y}{b} = 1$ $P(a, 0)$ and $Q(0, b)$ Distance $\rightarrow \sqrt{(a^2 + b^2)} = \sqrt{45}$ Gradients $\rightarrow \frac{-a}{b} = \frac{-1}{2}$ Solution of sim eqns $\rightarrow a = 6, b = 3$	M1 A1 M1 A1 A1 [5]	M1 even if sign(s) incorrect. Correct values a and b (both)
4 (a) $y = \frac{2x^3 + 5}{x} = 2x^2 + \frac{5}{x}$ $d/dx = 4x - \frac{5}{x^2}$ or $4x - 5x^{-2}$ (b) $\int (3x-2)^5 dx = \frac{(3x-2)^6}{6} \div 3 (+c)$ $\int_0^1 (3x-2)^5 dx = \left[\frac{(3x-2)^6}{18} \right]$ Limits used correctly $\rightarrow -3\frac{1}{2}$	M1 A1 + A1 [3] B1 B1 M1 A1 [4]	Knows to divide numerator by x co B1 without “ $\div 3$ ”. B1 for “ $\div 3$ ”. (ignore $(+c)$) Uses limits after integration. co
5 (i) $\overrightarrow{PQ} = 3\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}$ $\overrightarrow{RQ} = -3\mathbf{i} + 8\mathbf{j} + 3\mathbf{k}$ (ii) $\overrightarrow{PQ} \cdot \overrightarrow{RQ} = -9 + 48 - 9 = 30$ $= \sqrt{54} \sqrt{82} \cos RQP$ $\rightarrow RQP = 63.2^\circ$	B2,1 B1 [3] M1 M1 M1 A1 [4]	Allow B2,1 for either one, B1 for the other. Use of $x_1x_2 + y_1y_2 + z_1z_2$ Correct use of modulus All linked correctly co

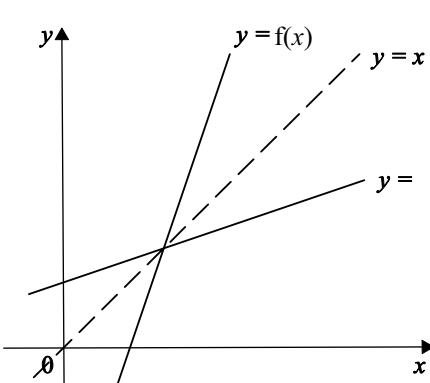
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6 (a) $ar^2 = 20$ $\frac{a}{1-r} = 3a$ Soln of equations $\rightarrow (r = \frac{2}{3}) a = 45$ (b) $a + 7d = 3(a + 2d)$ $\rightarrow 2a = d$ $S_8 = 4(2a + 7d) = 32d$ or $64a$ $S_4 = 2(2a + 3d) = 8d$ or $16a$	B1 B1 M1 A1 [4] M1 A1 M1 A1 [4]	co co Complete method to find a. co Use of $a + (n - 1)d$ co correct use of S_n formula once. ag
7 (i) $AX = 6 \tan \frac{\pi}{3} = 6\sqrt{3}$ (ii) Area of triangle $= \frac{1}{2} \times 6 \times 6\sqrt{3}$ Area of sector $= \frac{1}{2} 6^2 \times \frac{\pi}{3}$ Area shaded $= 18\sqrt{3} - 6\pi$ (iii) Arc $AB = 6 \times \frac{\pi}{3} = 2\pi$ $OX = 6 \div \cos \frac{\pi}{3} = 12$, $BX = 6$ Perimeter $= 6\sqrt{3} + 2\pi + 6$	B1 [1] M1 M1 A1 [3] M1 B1 M1 A1 [4]	ag Use of $\frac{1}{2}bh$ Use of $\frac{1}{2}r^2\theta$ co Use of $r\theta$ Use of trig to find (OX and then) BX.
8 (i) $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta} \right)^2 = \frac{1 - \cos \theta}{1 + \cos \theta}$ $\left(\frac{1}{\sin \theta} - \frac{\cos \theta}{\sin \theta} \right)^2 = \frac{(1 - \cos \theta)^2}{\sin^2 \theta}$ $= \frac{(1 - \cos \theta)(1 - \cos \theta)}{1 - \cos^2 \theta} = \frac{1 - \cos \theta}{1 + \cos \theta}$ (ii) $\left(\frac{1}{\sin \theta} - \frac{1}{\tan \theta} \right)^2 = \frac{2}{5}$ $\frac{1 - \cos \theta}{1 + \cos \theta} = \frac{2}{5}$ $\cos \theta = \frac{3}{7}$ $\theta = 64.6^\circ$ or 295.4°	M1 M1 A1 [3] M1 A1 A1 A1 $\sqrt{\quad}$ [4]	Use of $\tan = \sin/\cos$ Use of $\sin^2 + \cos^2 = 1$. All correct. (NB ag. – ensure cancelling has been done) Uses part (i) to obtain an eqn in $\cos \theta$ co co. $\sqrt{\quad}$ for 360 – “1st answer”.

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9 $\frac{dy}{dx} = \frac{2}{\sqrt{x}} - 1$ $P(9, 5)$ (i) $y = 4\sqrt{x} - x (+c)$ Uses (9, 5) in an integrated expression $\rightarrow c = 2$ (ii) $\frac{dy}{dx} = 0 \rightarrow x = 4, y = 6$ (iii) $\frac{d^2y}{dx^2} = -x^{-\frac{3}{2}} \rightarrow -ve \rightarrow \text{Max}$ (iv) $\frac{dy}{dx} = -\frac{1}{3}$ Perpendicular $m = 3$ $\tan\theta = 3$ Angle is $\tan^{-1}3$ $k = 3$	B1 B1 M1 A1 [4] M1 A1 A1 [3] B1 B1√ [2] M1 A1 [2]	Ignore + c. Substitution of point after integration. co. Attempt to solve $dy/dx = 0$. x correct. y correct. co. √ for correct deduction. Use of $m_1m_2 = -1$ Needs $k = 3$
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10 $f : x \mapsto 3x - 4$ $g : x \mapsto 2(x - 1)^3 + 8$ (i) $fg(2) = f(10) = 26$ $f^{-1}(x)$ (ii)  (iii) $g'(x) = 6(x - 1)^2$ $g'(x) > 0 \rightarrow$ no turning points $\rightarrow g$ is 1 : 1, g has an inverse. (iv) $f^{-1}(x) = \frac{x + 4}{3}$ Attempt at making x Order correct. $-8, \div 2, \sqrt[3]{}, +1$ $g^{-1}(x) = \sqrt[3]{\frac{x - 8}{2}} + 1$	M1 A1 [2] B1 B1 B1 [3] B1 B1√ B1√ [3] B1 M1 M1 A1 [4]	Must use g first, then f. co $y = f(x)$ correct in 1st, 4th quadrants. $y = f^{-1}(x)$ correct in 1st, 2nd quadrants. $y = x$ marked, or quoted. co allow only for incorrect “6” following from incorrect “6” co May change x and y first. Must all be correct, but allow for $+8, -1$ co as function of x, not y.
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