



UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

MATHEMATICS

9709/11

Paper 1 Pure Mathematics 1 (P1)

May/June 2010

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
 Graph Paper
 List of Formulae (MF9)



READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.

Write your Centre number, candidate number and name on all the work you hand in.

Write in dark blue or black pen.

You may use a soft pencil for any diagrams or graphs.

Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

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The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **4** printed pages.



- 1 The acute angle x radians is such that $\tan x = k$, where k is a positive constant. Express, in terms of k ,
- (i) $\tan(\pi - x)$, [1]
 - (ii) $\tan(\frac{1}{2}\pi - x)$, [1]
 - (iii) $\sin x$. [2]

- 2 (i) Find the first 3 terms in the expansion of $\left(2x - \frac{3}{x}\right)^5$ in descending powers of x . [3]

- (ii) Hence find the coefficient of x in the expansion of $\left(1 + \frac{2}{x^2}\right)\left(2x - \frac{3}{x}\right)^5$. [2]

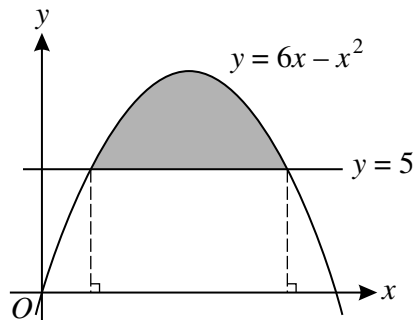
- 3 The ninth term of an arithmetic progression is 22 and the sum of the first 4 terms is 49.

- (i) Find the first term of the progression and the common difference. [4]

The n th term of the progression is 46.

- (ii) Find the value of n . [2]

4



The diagram shows the curve $y = 6x - x^2$ and the line $y = 5$. Find the area of the shaded region. [6]

- 5 The function f is such that $f(x) = 2\sin^2 x - 3\cos^2 x$ for $0 \leq x \leq \pi$.

- (i) Express $f(x)$ in the form $a + b\cos^2 x$, stating the values of a and b . [2]

- (ii) State the greatest and least values of $f(x)$. [2]

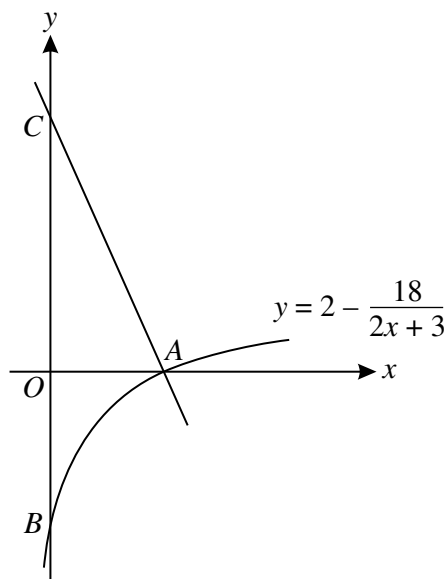
- (iii) Solve the equation $f(x) + 1 = 0$. [3]

- 6 A curve is such that $\frac{dy}{dx} = 3x^{\frac{1}{2}} - 6$ and the point $(9, 2)$ lies on the curve.

- (i) Find the equation of the curve. [4]

- (ii) Find the x -coordinate of the stationary point on the curve and determine the nature of the stationary point. [3]

7

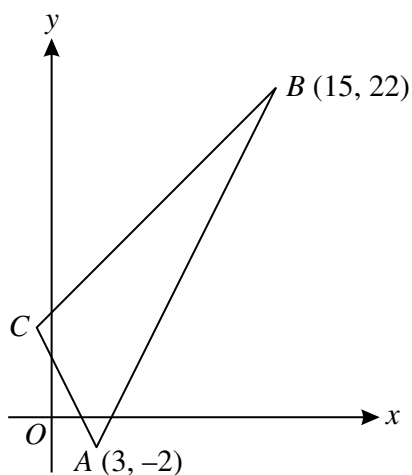


The diagram shows part of the curve $y = 2 - \frac{18}{2x + 3}$, which crosses the x -axis at A and the y -axis at B . The normal to the curve at A crosses the y -axis at C .

(i) Show that the equation of the line AC is $9x + 4y = 27$. [6]

(ii) Find the length of BC . [2]

8



The diagram shows a triangle ABC in which A is $(3, -2)$ and B is $(15, 22)$. The gradients of AB , AC and BC are $2m$, $-2m$ and m respectively, where m is a positive constant.

(i) Find the gradient of AB and deduce the value of m . [2]

(ii) Find the coordinates of C . [4]

The perpendicular bisector of AB meets BC at D .

(iii) Find the coordinates of D . [4]

9 The function f is defined by $f : x \mapsto 2x^2 - 12x + 7$ for $x \in \mathbb{R}$.

(i) Express $f(x)$ in the form $a(x - b)^2 - c$. [3]

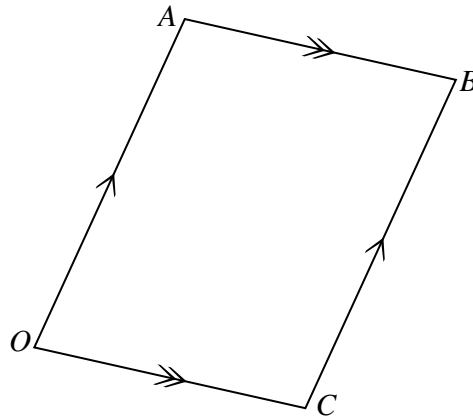
(ii) State the range of f . [1]

(iii) Find the set of values of x for which $f(x) < 21$. [3]

The function g is defined by $g : x \mapsto 2x + k$ for $x \in \mathbb{R}$.

(iv) Find the value of the constant k for which the equation $gf(x) = 0$ has two equal roots. [4]

10



The diagram shows the parallelogram $OABC$. Given that $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j} + 3\mathbf{k}$ and $\overrightarrow{OC} = 3\mathbf{i} - \mathbf{j} + \mathbf{k}$, find

(i) the unit vector in the direction of $\overrightarrow{OB}$, [3]

(ii) the acute angle between the diagonals of the parallelogram, [5]

(iii) the perimeter of the parallelogram, correct to 1 decimal place. [3]



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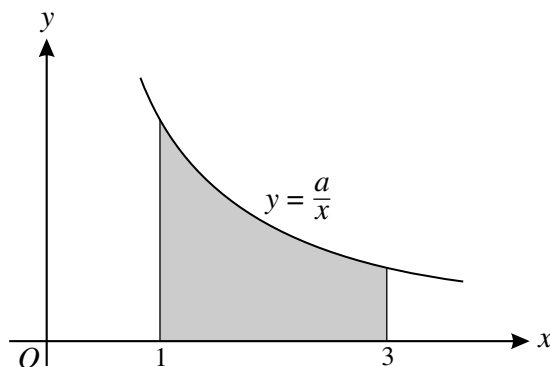
- 1 (i) Show that the equation

$$3(2 \sin x - \cos x) = 2(\sin x - 3 \cos x)$$

can be written in the form $\tan x = -\frac{3}{4}$. [2]

- (ii) Solve the equation $3(2 \sin x - \cos x) = 2(\sin x - 3 \cos x)$, for $0^\circ \leq x \leq 360^\circ$. [2]

2



The diagram shows part of the curve $y = \frac{a}{x}$, where a is a positive constant. Given that the volume obtained when the shaded region is rotated through 360° about the x -axis is 24π , find the value of a . [4]

- 3 The functions f and g are defined for $x \in \mathbb{R}$ by

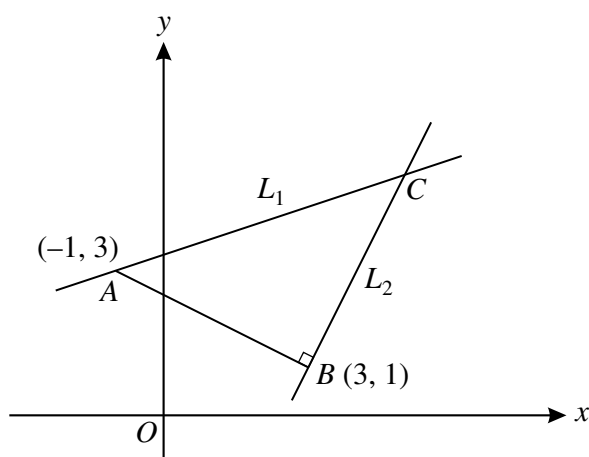
$$f : x \mapsto 4x - 2x^2,$$

$$g : x \mapsto 5x + 3.$$

- (i) Find the range of f . [2]

- (ii) Find the value of the constant k for which the equation $gf(x) = k$ has equal roots. [3]

4



In the diagram, A is the point $(-1, 3)$ and B is the point $(3, 1)$. The line L_1 passes through A and is parallel to OB . The line L_2 passes through B and is perpendicular to AB . The lines L_1 and L_2 meet at C . Find the coordinates of C . [6]

- 5 Relative to an origin O , the position vectors of the points A and B are given by

$$\overrightarrow{OA} = \begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ 1 \\ p \end{pmatrix}.$$

- (i) Find the value of p for which $\overrightarrow{OA}$ is perpendicular to $\overrightarrow{OB}$. [2]
- (ii) Find the values of p for which the magnitude of $\overrightarrow{AB}$ is 7. [4]
- 6 (i) Find the first 3 terms in the expansion of $(1 + ax)^5$ in ascending powers of x . [2]
- (ii) Given that there is no term in x in the expansion of $(1 - 2x)(1 + ax)^5$, find the value of the constant a . [2]
- (iii) For this value of a , find the coefficient of x^2 in the expansion of $(1 - 2x)(1 + ax)^5$. [3]
- 7 (a) Find the sum of all the multiples of 5 between 100 and 300 inclusive. [3]
- (b) A geometric progression has a common ratio of $-\frac{2}{3}$ and the sum of the first 3 terms is 35. Find
- (i) the first term of the progression, [3]
- (ii) the sum to infinity. [2]
- 8 A solid rectangular block has a square base of side x cm. The height of the block is h cm and the total surface area of the block is 96 cm^2 .
- (i) Express h in terms of x and show that the volume, $V \text{ cm}^3$, of the block is given by

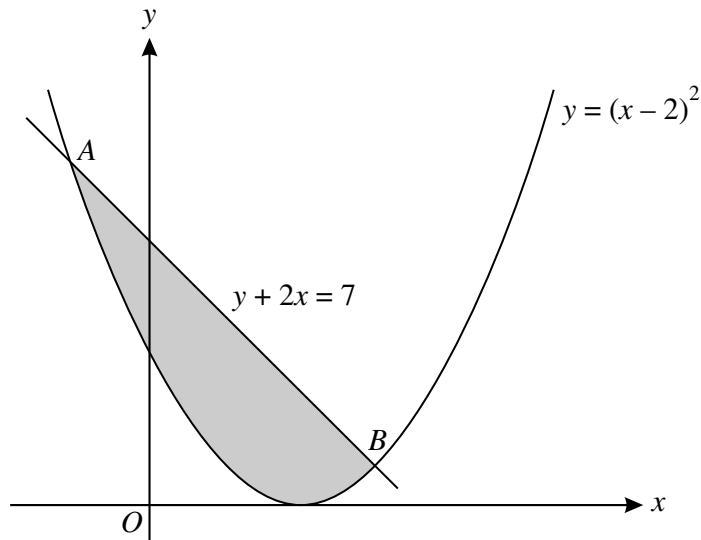
$$V = 24x - \frac{1}{2}x^3. \quad [3]$$

Given that x can vary,

- (ii) find the stationary value of V , [3]
- (iii) determine whether this stationary value is a maximum or a minimum. [2]

[Questions 9, 10 and 11 are printed on the next page.]

9



The diagram shows the curve $y = (x - 2)^2$ and the line $y + 2x = 7$, which intersect at points A and B . Find the area of the shaded region. [8]

10 The equation of a curve is $y = \frac{1}{6}(2x - 3)^3 - 4x$.

(i) Find $\frac{dy}{dx}$. [3]

(ii) Find the equation of the tangent to the curve at the point where the curve intersects the y -axis. [3]

(iii) Find the set of values of x for which $\frac{1}{6}(2x - 3)^3 - 4x$ is an increasing function of x . [3]

11 The function $f : x \mapsto 4 - 3 \sin x$ is defined for the domain $0 \leq x \leq 2\pi$.

(i) Solve the equation $f(x) = 2$. [3]

(ii) Sketch the graph of $y = f(x)$. [2]

(iii) Find the set of values of k for which the equation $f(x) = k$ has no solution. [2]

The function $g : x \mapsto 4 - 3 \sin x$ is defined for the domain $\frac{1}{2}\pi \leq x \leq A$.

(iv) State the largest value of A for which g has an inverse. [1]

(v) For this value of A , find the value of $g^{-1}(3)$. [2]



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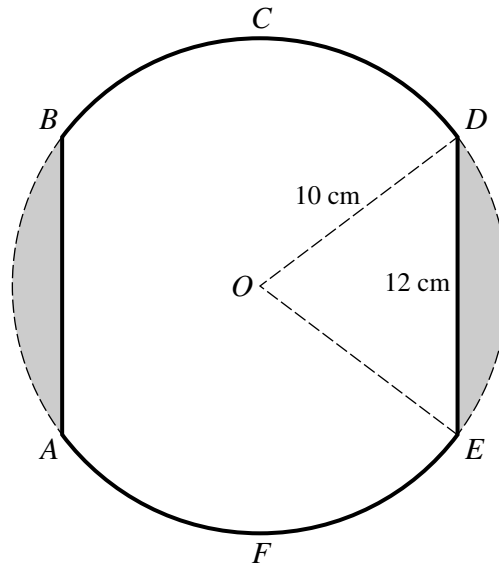
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- 1** The first term of a geometric progression is 12 and the second term is -6 . Find
- (i) the tenth term of the progression, [3]
 - (ii) the sum to infinity. [2]
- 2** (i) Find the first three terms, in descending powers of x , in the expansion of $\left(x - \frac{2}{x}\right)^6$. [3]
- (ii) Find the coefficient of x^4 in the expansion of $(1 + x^2)\left(x - \frac{2}{x}\right)^6$. [2]
- 3** The function $f : x \mapsto a + b \cos x$ is defined for $0 \leq x \leq 2\pi$. Given that $f(0) = 10$ and that $f\left(\frac{2}{3}\pi\right) = 1$, find
- (i) the values of a and b , [2]
 - (ii) the range of f , [1]
 - (iii) the exact value of $f\left(\frac{5}{6}\pi\right)$. [2]
- 4** (i) Show that the equation $2 \sin x \tan x + 3 = 0$ can be expressed as $2 \cos^2 x - 3 \cos x - 2 = 0$. [2]
- (ii) Solve the equation $2 \sin x \tan x + 3 = 0$ for $0^\circ \leq x \leq 360^\circ$. [3]
- 5** The equation of a curve is such that $\frac{dy}{dx} = \frac{6}{\sqrt{3x-2}}$. Given that the curve passes through the point $P(2, 11)$, find
- (i) the equation of the normal to the curve at P , [3]
 - (ii) the equation of the curve. [4]
- 6** Relative to an origin O , the position vectors of the points A , B and C are given by
- $$\overrightarrow{OA} = \mathbf{i} - 2\mathbf{j} + 4\mathbf{k}, \quad \overrightarrow{OB} = 3\mathbf{i} + 2\mathbf{j} + 8\mathbf{k}, \quad \overrightarrow{OC} = -\mathbf{i} - 2\mathbf{j} + 10\mathbf{k}.$$
- (i) Use a scalar product to find angle ABC . [6]
 - (ii) Find the perimeter of triangle ABC , giving your answer correct to 2 decimal places. [2]

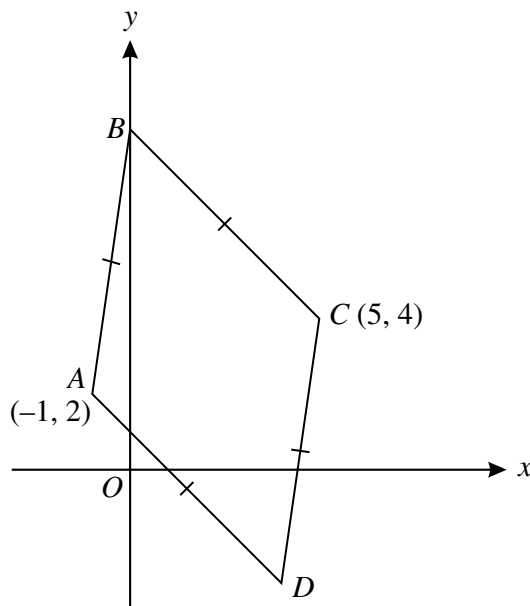
7



The diagram shows a metal plate $ABCDEF$ which has been made by removing the two shaded regions from a circle of radius 10 cm and centre O . The parallel edges AB and ED are both of length 12 cm .

- (i) Show that angle DOE is 1.287 radians, correct to 4 significant figures. [2]
- (ii) Find the perimeter of the metal plate. [3]
- (iii) Find the area of the metal plate. [3]

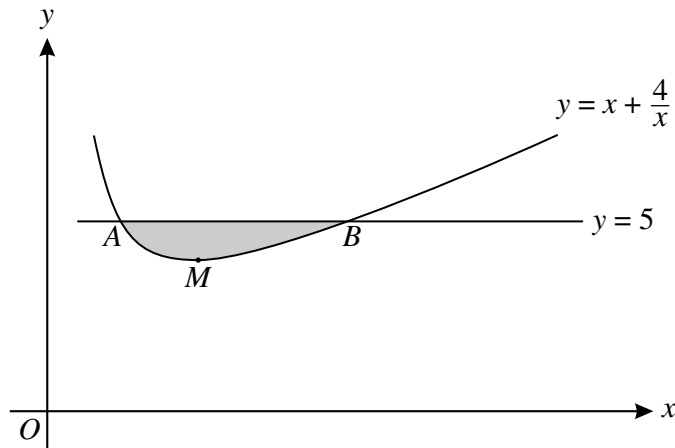
8



The diagram shows a rhombus $ABCD$ in which the point A is $(-1, 2)$, the point C is $(5, 4)$ and the point B lies on the y -axis. Find

- (i) the equation of the perpendicular bisector of AC , [3]
- (ii) the coordinates of B and D , [3]
- (iii) the area of the rhombus. [3]

9



The diagram shows part of the curve $y = x + \frac{4}{x}$ which has a minimum point at M . The line $y = 5$ intersects the curve at the points A and B .

(i) Find the coordinates of A , B and M . [5]

(ii) Find the volume obtained when the shaded region is rotated through 360° about the x -axis. [6]

10 The function $f : x \mapsto 2x^2 - 8x + 14$ is defined for $x \in \mathbb{R}$.

(i) Find the values of the constant k for which the line $y + kx = 12$ is a tangent to the curve $y = f(x)$. [4]

(ii) Express $f(x)$ in the form $a(x + b)^2 + c$, where a , b and c are constants. [3]

(iii) Find the range of f . [1]

The function $g : x \mapsto 2x^2 - 8x + 14$ is defined for $x \geq A$.

(iv) Find the smallest value of A for which g has an inverse. [1]

(v) For this value of A , find an expression for $g^{-1}(x)$ in terms of x . [3]