



Cambridge International AS & A Level

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MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.



- 1 Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $a + \ln b$, where a and b are integers. [4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 2 (a) Show that the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$ can be written as a quadratic equation in x . [3]

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- (b) Hence solve the equation $\log_4(2x+1) = 2\log_4(3x-1) - 2$. [2]

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3 (a) Express $3\sqrt{2}\sin(x+45^\circ) + \cos x$ in the form $R\cos(x-\alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. [4]

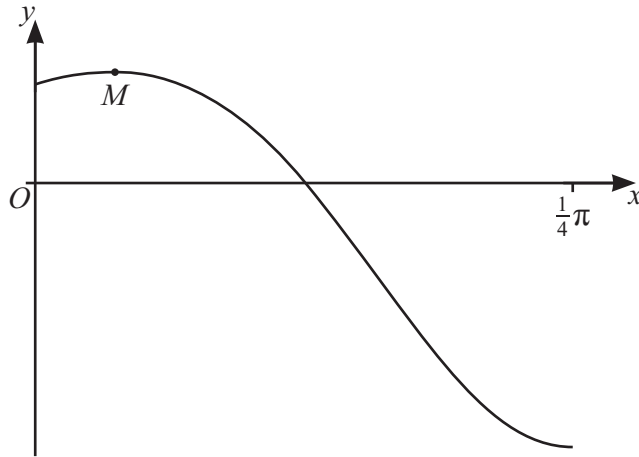
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(b) Hence solve the equation $3\sqrt{2}\sin(3\theta+45^\circ)+\cos 3\theta=-4$ for $0^\circ < \theta < 180^\circ$.

[4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dashed lines, creating a series of uniform gaps for letter height. The lines are evenly spaced across the entire page, providing a guide for consistent letter formation. There is no text or other markings on the page.



The diagram shows the graph of $y = e^{\sin 2x} \cos 4x$ for $0 \leq x \leq \frac{1}{4}\pi$, and its maximum point M .

Find the x -coordinate of M .

[5]

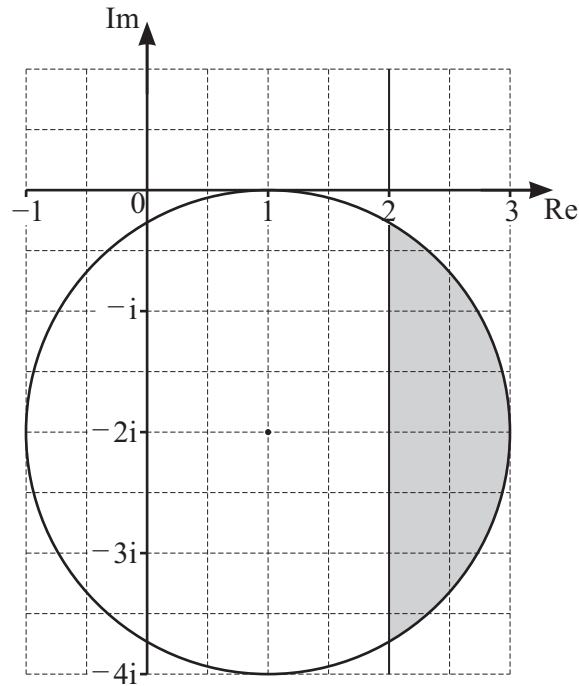
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The shaded region on the Argand diagram shows points representing complex numbers z defined by two inequalities. The shaded region is bounded by a circle and a line parallel to the imaginary axis. The boundaries of the region are included in the shaded region.

- (a) Find two inequalities in terms of z that define the shaded region. [3]

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- (b) Calculate the least value of $\arg z$ for points in this region. [3]

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- 6** Solve the quadratic equation $(2+i)w^2 + 4w + 2-i = 0$. Give your answers in the form $x+iy$, where x and y are real. [5]

This image shows a full page of white paper with horizontal dotted lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



7 The parametric equations of a curve are

$$x = t^2 - \ln(2t + 1), \quad y = \frac{t}{2t + 1}.$$

Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t .

[5]





8 The variables x and y satisfy the differential equation

$$(x^2 + 1) \frac{dy}{dx} = kxe^{2y},$$

where k is a constant. It is given that $y = 0$ when $x = 0$ and that $y = -\frac{1}{2}$ when $x = 1$.

Solve the differential equation and find the exact value of y when $x = \sqrt{3}$.

[8]

[illegible]

[illegible]



- 9 (a)** By sketching a suitable pair of graphs, show that the equation

$$\sec 2x = -e^x$$

has only one root in the interval $0 < x < \frac{1}{2}\pi$.

[2]

- (b)** Show by calculation that this root lies between 0.9 and 1.

[2]

[illegible]



(c) Show that if a sequence of values given by the iterative formula

$$x_{n+1} = \frac{1}{2} \cos^{-1}(-e^{-x_n})$$

converges, then it converges to the root of the equation in part (a).

[1]

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(d) Use the iterative formula given in part (c) to calculate x correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

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10 Let $f(x) = \frac{x^3 + 2x - 11}{(3+x)(2+x^2)}$.

(a) Express $f(x)$ in partial fractions.

[6]

[illegible]



(b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]



- 11** With respect to the origin O , the points A, B, C and D have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix}.$$

The line m passes through the points A and B .

- (a)** Find a vector equation for m .

[2]

[illegible]

- (b) Find the position vector of the point of intersection of m and the line passing through the points C and D . [4]

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



[4]

[illegible]



Additional page

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MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

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1 (a) Sketch the graph of $y = |x + 3a|$, where a is a positive constant.

[1]

(b) Hence or otherwise solve the inequality $|x + 3a| > a - 2x$.

[2]

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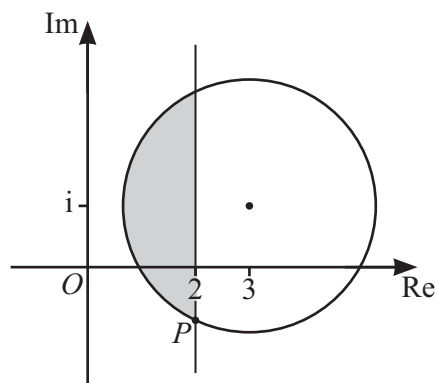
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2 Solve the equation $3 \times 2^{x+1} = 4 \times 3^{2x-3}$. Give your answer correct to 3 significant figures. [4]

[illegible]



The shaded region in the Argand diagram, bounded by a line and a circle, represents the complex numbers z satisfying

$$\operatorname{Re} z \leq 2 \text{ and } |z - (3 + i)| \leq 2.$$

The point P shown on the diagram is one of the points of intersection of the line and the circle.

- (a) Find the complex number represented by the point P . Give your answer in the form $x + iy$, where x and y are real and exact. [2]

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- (b) Find the greatest value of $\arg z$ for points in the shaded region. [3]

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4 Find the exact value of $\int_0^1 x \tan^{-1} x \, dx$.

[6]





- 5 (a)** It is given that $f(x) = (x-a)^2g(x)$, where $f(x)$ and $g(x)$ are polynomials.

Show that $(x - a)$ is a factor of $f'(x)$.

[2]

- (b)** It is given that $(x-3)^2$ is a factor of $2x^3 - 4x^2 + px + q$, where p and q are constants.

Find the values of p and q .

[5]



- 6 (a) By sketching a suitable pair of graphs, show that the equation $\cot 2x = 2 \sin 2x - 1$ has exactly one root in the interval $0 < x < \frac{1}{2}\pi$. [2]

- (b) Show by calculation that the root is in the interval $0.4 < x < 0.6$. [2]

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- (c) Use the iterative formula $x_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{1}{2 \sin 2x_n - 1} \right)$ to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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7 The equation of a curve is $2y^3 - 3x^2y - x^3 = 16$.

(a) Show that $\frac{dy}{dx} = \frac{x^2 + 2xy}{2y^2 - x^2}$. [4]

[illegible]



(b) Hence find the coordinates of the points on the curve at which the normal is parallel to the y -axis. [4]

[illegible]



8 (a) Prove the identity $\sin 4x \equiv 4 \sin x (2 \cos^3 x - \cos x)$.

[3]

[illegible]



(b) Hence find the exact value of $\int_0^{\frac{1}{4}\pi} \cos^3 x \sin 4x \, dx$.

[5]

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- 9 Let $f(x) = \frac{x^2 + 4ax + 6a^2}{(x+2a)(x+3a)}$, where a is a positive constant.

(a) Express $f(x)$ in partial fractions.

[5]

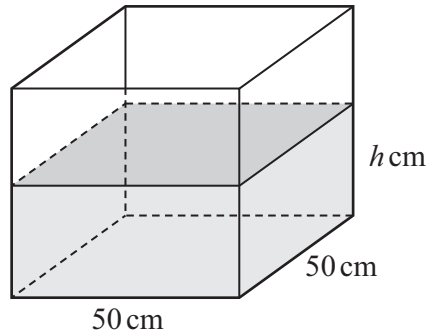
[illegible]



- (b) Hence find the exact value of $\int_{-a}^a f(x) dx$. Give your answer in the form $a(p + \ln q)$, where p and q are rational. [4]

[4]





The diagram shows a tank for holding water. The tank is in the shape of a cube of side 50 cm. At time t seconds, the depth of water in the tank is h cm. Water is poured into the tank at a rate of $5000 \text{ cm}^3 \text{ s}^{-1}$. Water pours out of the tank through a hole in the bottom at a rate proportional to h^2 .

When $h = 20$, the depth of the water is increasing at a rate of 0.4 cm s^{-1} .

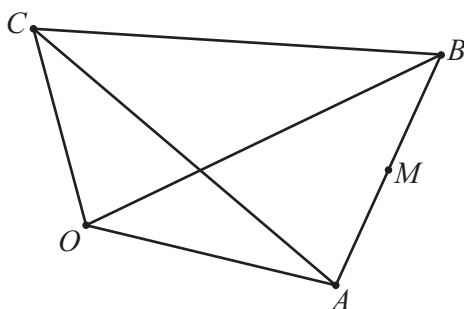
- (a) Show that $\frac{dh}{dt} = \frac{500 - h^2}{250}$. [4]

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- (b)** Given that $h = 0$ when $t = 0$, find the time taken for the depth of the water in the tank to reach 20 cm. [5]

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Relative to the origin O , the position vectors of the points A , B and C are

$$\overrightarrow{OA} = 4\mathbf{i} - 2\mathbf{j}, \quad \overrightarrow{OB} = 2\mathbf{i} + 8\mathbf{j} + 4\mathbf{k}, \quad \text{and} \quad \overrightarrow{OC} = -2\mathbf{i} + 6\mathbf{k}.$$

The midpoint of AB is M , as shown in the diagram.

- (a) Find the vectors $\overrightarrow{MB}$ and $\overrightarrow{MC}$. [2]

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- (b) Calculate the exact value of the cosine of angle CMB . [3]

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(c) Hence or otherwise find the exact area of triangle ABC .

[4]

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MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2025

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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[4]

[illegible]



2 Find the quotient and the remainder when $3x^4 - 2x^2$ is divided by $x + 1$.

[3]

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- 3 Solve the equation $2^{3x-4} = \frac{3}{5^x}$. Give your answer in the form $\frac{\ln m}{\ln n}$, where m and n are integers. [4]

[illegible]

- 4 On an Argand diagram shade the region whose points represent complex numbers z which satisfy both the inequalities $|z + 2i| \leq 3$ and $|z + 2i| \leq |z - 2 + 4i|$. [5]





5 (a) Show that $\cos 4x + 2 \sin^2 x - 1 \equiv 8 \sin^4 x - 6 \sin^2 x$.

[4]

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(b) Hence solve the equation $\cos 4x + 2 \sin^2 x - 1 = 0$ for $-180^\circ \leq x \leq 180^\circ$.

[4]

[illegible]



6 Find the exact value of $\int_0^{\frac{1}{6}\pi} x^2 \sin 2x \, dx$.

[6]

[illegible]

[illegible]



- 7 Solve the equation $\frac{5z}{2-i} - zz^* + 20 + 8i = 0$. Give your answers in the form $x + iy$, where x and y are real. [6]

[illegible]

[illegible]

8 The curve with equation $y = e^{-5x} \ln 5x$ has a stationary point at $x = p$.

(a) Show that p satisfies the equation $\ln 5p = \frac{1}{5p}$. [3]

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(b) By sketching a suitable pair of graphs, show that the equation in part (a) has only one root. [2]





(c) Show by calculation that $0.2 < p < 0.6$.

[2]

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(d) It is given that the equation in part (a) can be written in the form $p = \frac{1}{5} \exp\left(\frac{1}{5p}\right)$, where $\exp(x)$ denotes e^x .

Use an iterative formula based on this rearrangement to calculate p correct to 2 decimal places.
Give the result of each iteration to 4 decimal places. [3]

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- 9** The line l_1 passes through the point $(3, 1, -6)$ and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

The line l_2 passes through the point $(-1, 3, -6)$ and is perpendicular to the vector $3\mathbf{i} - 2\mathbf{j} + \mathbf{k}$. The direction vector for l_2 has no component in the x -direction.

- (a) Write down a vector equation for l_1 and find a vector equation for l_2 . [3]

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- (b)** Calculate the acute angle between l_1 and l_2 . [3]

[illegible]



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(c) Find the position vector of the point of intersection of l_1 and l_2 . [3]

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- 10 (a) Express $\frac{2}{1-9y^2}$ in partial fractions.

[2]

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- (b) The variables x and y satisfy the differential equation

$$2 \cos^2 3x \frac{dy}{dx} = 1 - 9y^2,$$

and $y = 0$ when $x = \frac{1}{12}\pi$.

Solve the differential equation and obtain an expression for y in terms of x .

[6]

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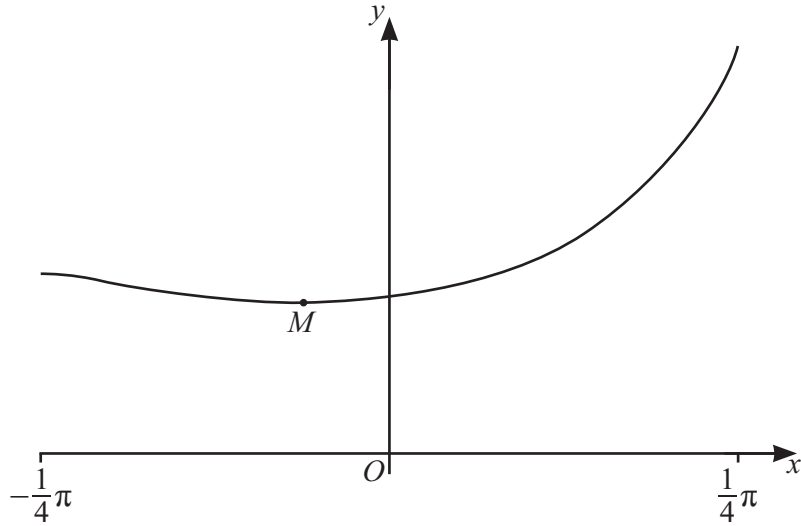
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[illegible]



The diagram shows the graph of $y = \sec^2 x \sqrt{3 + 2 \tan x}$ for $-\frac{1}{4}\pi \leq x \leq \frac{1}{4}\pi$, and its minimum point M .

- (a) Find the x -coordinate of M .

[6]

[illegible]



- (b) Using the substitution $u = 3 + 2 \tan x$, find the exact value of the area of the region bounded by the curve, the x -axis and the lines $x = -\frac{1}{4}\pi$ and $x = \frac{1}{4}\pi$. [6]





Additional page

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MATHEMATICS**9709/35**

Paper 3 Pure Mathematics 3

October/November 2025**1 hour 50 minutes**

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1 (a) Sketch the graph of $y = |3x - 6|$.

[1]

(b) Solve the inequality $5x - 3 < |3x - 6|$.

[3]

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- 2 On a sketch of an Argand diagram, shade the region which represents complex numbers z satisfying both the inequalities $|z - 1 - 3i| \leq 2$ and $|z| \geq 4$. [4]





3 The variables x and y satisfy the equation $Ay = b^x$, where A and b are constants.

(a) Show that the graph of $\ln y$ against x is a straight line. [2]

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(b) When $x = 3.4$, $\ln y = 0.86$ and when $x = 5.7$, $\ln y = 2.56$.

Find the value of A and the value of b . Give your answers correct to 2 significant figures. [3]

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4 The equation of a curve is $x^2 \ln 2y - y \ln(2 + x^2) = \ln 6$.

Find the exact value of the gradient of the curve at the point $(2, 3)$. Give your answer in simplified form. [5]

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5 Find the complex numbers, z , which satisfy the equation

$$zz^* + 5iz + 2 - 10i = 0.$$

Give your answers in the form $x + iy$, where x and y are real.

[5]

[illegible]



6 A curve has equation $y = \frac{\tan x}{5 + 2 \sin x}$.

(a) Express $\frac{dy}{dx}$ as a simplified fraction in terms of $\sin x$. [4]

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(b) Show that the curve has no stationary points. [2]

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7 (a) Use the substitution $u = x^2 - 3$ to show that

$$\int_{\sqrt{7}}^{\sqrt{12}} \frac{4x^3}{\sqrt{x^2-3}} dx = \int_a^b \frac{2(u+3)}{\sqrt{u}} du,$$

where a and b are values to be found.

[4]

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(b) Hence, find the exact value of $\int_{\sqrt{7}}^{\sqrt{12}} \frac{4x^3}{\sqrt{x^2-3}} \, dx$.

[4]





- 8 (a)** Express $3\sqrt{3}\sin\left(\theta + \frac{1}{6}\pi\right) - 2\sin\theta$ in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and state the value of α correct to 3 decimal places. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



(b) Hence, solve the equation $3\sqrt{3}\sin\left(2x + \frac{1}{6}\pi\right) - 2\sin 2x = \sqrt{6}$ for $0 < x < \pi$.

[4]





9 The equations of two lines are given by

$$l_1: \mathbf{r} = (2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) + \lambda(\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}),$$

$$l_2: \mathbf{r} = (3\mathbf{i} - \mathbf{j} + 5\mathbf{k}) + \mu(2\mathbf{i} + 3\mathbf{j} + a\mathbf{k}).$$

(a) Find the value of a for which l_1 is perpendicular to l_2 . [2]

[illegible]

(b) Find the value of a for which l_1 and l_2 intersect. [4]

[illegible]



(c) Find the values of a for which the acute angle between l_1 and l_2 is equal to $\cos^{-1}\left(\frac{5}{14}\right)$. [4]





10 The constant a is such that $\int_0^a x e^{\frac{1}{2}x} dx = 6$.

(a) Show that $a = 2 + e^{-\frac{1}{2}a}$.

[5]

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(b) Verify by calculation that a lies between 2.2 and 2.4.

[2]

(c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[3]





- 11 A fungal disease is affecting some of the trees in a forest. The fraction of the trees affected after t years is denoted by x . The rate of increase of x is proportional to the product of the fraction of the trees affected and the fraction of the trees **not** affected.

(a) Explain why, after t years, $\frac{dx}{dt} = kx(1-x)$, where k is a constant. [1]

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- (b)** When the disease is first detected, one quarter of the trees are affected. Two years later, one third of the trees are affected.

Solve the differential equation to find the number of years from the time when the disease is first detected until the time when three quarters of the trees are affected. Give your answer correct to the nearest year. [8]

[illegible]



This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.





Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]





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