

Cambridge International A Level

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

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This document consists of **22** printed pages.

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
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- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

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- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
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Annotations

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	More information required
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	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Commence integration by parts and reach $Ax \ln 3x + \int B dx$	*M1	Accept $Ax \ln 3x - B \int x \times \frac{1}{x} dx$. Allow unsimplified.
	Obtain $x \ln 3x - x$	A1	
	Substitute limits correctly in an expression of the form $Ax \ln 3x + Bx$	DM1	$2 \ln 6 - 2 - \ln 3 + 1$
	Obtain $-1 + \ln 12$	A1	If no working seen, no marks are available.
		4	

Question	Answer	Marks	Guidance
2(a)	State or imply $2 = \log_4 4^2$	B1	
	Use log power, product or quotient law e.g. $2 \log_4 (3x - 1) = \log_4 (3x - 1)^2$	M1	E.g. $\log_4 (3x - 1)^2 - \log_4 4^2 = \log_4 \frac{(3x - 1)^2}{4^2}$
	Obtain a correct equation in any form, free of logs	A1	E.g. $16(2x + 1) = (3x - 1)^2$ or $9x^2 - 38x - 15 = 0$
		3	
2(b)	Solve 3-term quadratic	M1	
	Obtain answer 4.59 only	A1	$\frac{19 + 4\sqrt{31}}{9}$ OE. Accept AWR 4.59.
		2	

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Question	Answer	Marks	Guidance
3(a)	Obtain $3\sin x + 4\cos x$	B1	
	State $R = 5$	B1 FT	FT <i>their</i> 3 and 4.
	Use trig formula to find α e.g. $\alpha = \tan^{-1}\left(\frac{3}{4}\right)$	M1	Using <i>their</i> 3 and 4. If using $\cos\alpha = 4$ and $\sin\alpha = 3$ seen, this scores M0A0.
	Obtain $\alpha = 36.9^\circ$ (or awrt 36.9°)	A1	
		4	
3(b)	Evaluate $\cos^{-1}\left(\frac{-4}{R}\right)$	B1 FT	FT <i>their R</i> from (a). Allow $\cos^{-1}\left(\frac{-4}{5}\right)$ for B1 143.13.....
	Carry out a correct method to find a value of θ in the given interval	M1	
	Obtain answer, e.g. $60(.0)^\circ$	A1	Answers may be more accurate for A marks. Accept AWR T 60.0.
	Obtain a second answer, e.g. 84.6° and no other in the interval	A1	Ignore answers outside the given interval. Accept AWR T 84.6.
		4	

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Question	Answer	Marks	Guidance
4	Use correct product rule: $p \sin 4x e^{\sin 2x} + q \cos 2x \cos 4x e^{\sin 2x}$	M1	Condone sign error. Condone if no attempt at chain rule seen.
	Obtain derivative $-4 \sin 4x e^{\sin 2x} + 2 \cos 2x \cos 4x e^{\sin 2x}$	A1	OE
	Equate derivative to zero and obtain an equation in one trig function	M1	Allow arithmetical errors.
	Obtain $2 \sin^2 2x + 4 \sin 2x - 1 = 0$	A1	OE
	Obtain $x = 0.113$ and no other root	A1	May be more accurate.
		5	

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Question	Answer	Marks	Guidance
5(a)	Obtain $\operatorname{Re} z \geq 2$	B1	Allow $2 \leq \operatorname{Re}(z) \leq 3$ Condone strict inequalities throughout (a) . The shading is to the RHS of the diagram.
	Obtain answer of the form $ z - a \leq b$	M1	Accept equation or any inequality sign. Must have a complex and b real. Condone brackets in place of modulus for this mark.
	Obtain answer $ z - 1 + 2i \leq 2$	A1	OE $ z - (1 - 2i) \leq 2$
		3	
5(b)	Identify the correct point	B1	May be clearly indicated as a point on the diagram. May be $(2, -\sqrt{3} - 2)$. May be by attempt to solve $(x - 1)^2 + (y + 2)^2 = 2^2$ with $x = 2$.
	Carry out a correct method for finding the least value of $\arg z$	M1	
	Obtain answer -61.8° or 298.2° or -1.08° or $5.2(0)^\circ$	A1	
		3	

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Question	Answer	Marks	Guidance
6	Use quadratic formula to solve for w	M1	$w = \frac{-4 \pm \sqrt{16 - 4(2-i)(2+i)}}{4+2i}$
	Use $i^2 = -1$	M1	
	Obtain one of the answers $w = \frac{-4-2i}{4+2i}$ or $w = \frac{-4+2i}{4+2i}$	A1	OE
	Multiply numerator and denominator by conjugate of <i>their</i> denominator	M1	Need some evidence of multiplying.
	Obtain final answers $-\frac{3}{5} + \frac{4}{5}i$ and $-1 [+ 0i]$	A1	
	Alternative Method for Question 6		
	Multiply the equation by $2 - i$	M1	
	Use $i^2 = -1$	M1	
	Obtain $5w^2 + 4(2-i)w + (2-i)^2$	A1	OE
	Use quadratic formula or factorise to solve for w	M1	
	Obtain final answers $-\frac{3}{5} + \frac{4}{5}i$ and $-1 [+ 0i]$	A1	

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Question	Answer	Marks	Guidance
	Alternative Method 2 for Question 6		
	Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	Two separate equations.
	Use $i^2 = -1$	M1	
	Obtain $2(x^2 - y^2) - 2xy + 4x + 2 = 0$ and $x^2 - y^2 + 4xy + 4y - 1 = 0$	A1	OE
	Form equation in x only or y only and solve	M1	
	Obtain final answers $-\frac{3}{5} + \frac{4}{5}i$ and $-1 [+ 0i]$	A1	
		5	

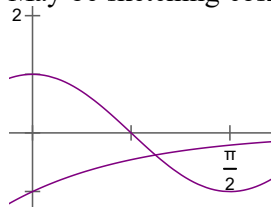
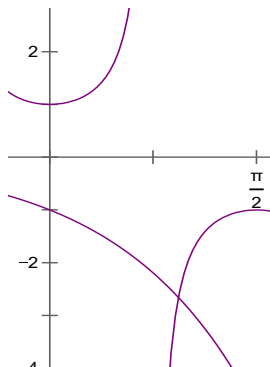
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Question	Answer	Marks	Guidance
7	State $\frac{dx}{dt} = 2t - \frac{2}{2t+1}$	B1	
	Use correct quotient rule or correct product rule	M1	Might split first as $\frac{1}{2} - \frac{1}{4t+2}$.
	Obtain $\frac{dy}{dt} = \frac{(2t+1) \cdot 1 - t(2)}{(2t+1)^2}$ oe	A1	Simplifies to $\frac{1}{(2t+1)^2}$
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	Allow for $\frac{dx}{dy}$.
	Obtain answer $\frac{1}{2(2t+1)(2t-1)(t+1)}$	A1	OE. Must have 1 in numerator. May have, e.g., $\frac{1}{(2t+1)(4t^2+2t-2)}$. ISW.
	Obtain $x = \left(\frac{y}{1-2y}\right)^2 - \ln\left(\frac{2y}{1-2y} + 1\right)$	B1	OE, e.g. $x = \left(\frac{y}{1-2y}\right)^2 - \ln\left(\frac{1}{1-2y}\right)$.
	Use correct quotient rule to differentiate	M1	
	Obtain $\frac{dx}{dy} = \left(\frac{2y}{1-2y}\right) \left(\frac{(1-2y)+2y}{(1-2y)^2}\right) - \frac{2}{1-2y}$	A1	OE, $\frac{dy}{dx} = \frac{(1-2y)^3}{2y-2(1-2y)^2} = \frac{(1-2y)^3}{-2(1-4y)(1-y)}$
	$\frac{dy}{dx} = \frac{\left(1 - \frac{2t}{2t+1}\right)^3}{-2\left(1 - \frac{4t}{2t+1}\right)\left(1 - \frac{t}{2t+1}\right)} = \frac{1}{-2(2t+1)(1-2t)(t+1)}$	M1	Express in terms of t .
	Obtain answer $\frac{1}{2(2t+1)(2t-1)(t+1)}$	A1	OE
		5	

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Question	Answer	Marks	Guidance
8	Separate variables correctly	B1	$\int \frac{kx}{x^2+1} dx = \int e^{-2y} dy$ Condone missing integral signs or missing dx, dy, but not both.
	Obtain term Ae^{-2y}	B1	OE $-\frac{1}{2}e^{-2y}$
	Obtain term $B\ln(x^2+1)$	B1	OE $\frac{k}{2}\ln(x^2+1)$
	Obtain $-\frac{1}{2}e^{-2y} = \frac{k}{2}\ln(x^2+1) (+C)$	B1	OE
	Use $y=0$ when $x=0$ to evaluate c or as limits in a solution containing terms of the form e^{-2y} and $\ln(x^2+1)$	M1	
	Use $y=-\frac{1}{2}$ when $x=1$ to evaluate k or as limits in a solution containing terms of the form e^{-2y} and $\ln(x^2+1)$	M1	
	Obtain $c = -\frac{1}{2}$ and $k = \frac{1-e}{\ln 2}$	A1	OE, e.g. $-\frac{1}{2}e^{-2y} = \frac{1-e}{2\ln 2}\ln(x^2+1) - \frac{1}{2}$.
	Obtain final answer $y = -\frac{1}{2}\ln(2e-1)$	A1	Or exact equivalent.
		8	

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Question	Answer	Marks	Guidance
9(a)	Sketch a relevant graph, e.g. $y = \sec 2x$ May be sketching $\cos 2x$ and $-e^{-x}$  For $y = \sec 2x$ correct shape, asymptote in correct position. Cuts at (0, 1) For $-e^{-x}$ correct shape, cuts at (0, 1) For $y = \cos 2x$ correct shape, correct max/min, cuts axis at $\frac{\pi}{4}$ For $-e^{-x}$ correct shape, cuts at (0, -1)	B1	 Complete scale not needed, but should have enough to imply the key points for both graphs.
	Sketch a second relevant graph, e.g. $y = -e^x$ and justify the given statement	B1	Needs to mark intersection with a dot, a cross, or say roots at points of intersection, OE.
		2	
9(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.9$ and $x = 1$	M1	E.g. $-4.401 < -2.460$ or $-1.94 < 0$ $-2.403 > -2.718$ or $0.315 > 0$ (or $0.179 > 0$ for the reciprocal graphs). $-0.048 < 0$
	Complete the argument correctly with correct calculated values	A1	
		2	

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Question	Answer	Marks	Guidance
9(c)	State the equation $x = \frac{1}{2} \cos^{-1}(-e^{-x})$ and rearrange to obtain $\sec 2x = -e^x$ Or rearrange $\sec 2x = -e^x$ as $x = \frac{1}{2} \cos^{-1}(-e^{-x})$ and state $x_{n+1} = \frac{1}{2} \cos^{-1}(-e^{-x_n})$	B1	
		1	
9(d)	Use the iterative process correctly at least once	M1	
	Obtain final answer 0.978	A1	
	Show sufficient iterations to at least 5 d.p. to justify 0.978 to 3 d.p. or show there is a sign change in the interval (0.9775, 0.9785)	A1	E.g. 1, 0.97376, 0.97903, 0.97796, 0.97813 0.95, 0.98395, 0.97697, 0.97838, 0.97809 0.9, 0.99475, 0.97480, 0.97882, 0.97800, 0.97817
		3	

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Question	Answer	Marks	Guidance
10(a)	State or imply the form $A + \frac{B}{3+x} + \frac{Cx+D}{2+x^2}$	B1	
	Use a correct method for finding a constant	M1	Might be working from $1 + \frac{-3x^2 - 17}{(3+x)(2+x^2)}$.
	Obtain one of $A = 1$, $B = -4$, $C = 1$ and $D = -3$	A1	SC: If B0 scored due to missing term(s), then a maximum of M1A1 is available for a correct method leading to a correct value.
	Obtain a second value	A1	SC if obtaining $A = 1$ and then scoring B0, they can score maximum M1A1 + A1 for a correct value for one other constant.
	Obtain a third value	A1	
	Obtain all four correct values	A1	
		6	

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Question	Answer	Marks	Guidance
10(b)	Use a correct method to find the first two terms in the expansion of $(3 + x)^{-1}$, $\left(1 + \frac{x}{3}\right)^{-1}$, $(2 + x^2)^{-1}$ or $\left(1 + \frac{x^2}{2}\right)^{-1}$	M1	Symbolic binomial coefficients not sufficient for the M1.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A1 FT	FT B , C and D .
		A1 FT	E.g. $\frac{B}{3}\left(1 - \frac{x}{3} + \frac{x^2}{9}\right)$ and $\frac{Cx + D}{2}\left(1 - \frac{x^2}{2}\right)$.
	Multiply $(Cx + D)$ by the expansion of $\left(1 + \frac{x^2}{2}\right)^{-1}$ up to the term in x^2 where $CD \neq 0$	M1	Expansion of $\left(1 + \frac{x^2}{2}\right)^{-1}$. Must be of the form $1 + Qx^2$. Multiplication must include 3 relevant terms.
	Obtain final answer $-\frac{11}{6} + \frac{17}{18}x + \frac{65}{108}x^2$	A1	Or exact equivalent. Do not ISW attempts to multiply expression by, e.g., 108.
		5	

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Question	Answer	Marks	Guidance
11(a)	Carry out a correct method for finding a vector equation for m	M1	
	Obtain $\mathbf{r} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	A1	Or $\mathbf{r} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$ OE. Must have $\mathbf{r} = \dots$
		2	
11(b)	Express general point of the second line in component form, i.e. $(1 + 2\mu, -3 - 2\mu, 1 + 3\mu)$	B1	Or $(3 + 2\mu, -5 - 2\mu, 4 + 3\mu)$.
	Equate at least two pairs of corresponding components and solve for λ or for μ	M1	
	Obtain $\lambda = -4$ or $\mu = -2$	A1	Note if m has direction vector $\begin{pmatrix} -1 \\ -1 \\ -2 \end{pmatrix}$, then $\lambda = 4$. If line through CD has direction vector $\begin{pmatrix} -2 \\ 2 \\ -3 \end{pmatrix}$, then $\mu = 2$.
	Obtain position vector of point of intersection is $\begin{pmatrix} -3 \\ 1 \\ -5 \end{pmatrix}$	A1	Accept coordinates.
		4	

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Question	Answer	Marks	Guidance
11(c)	Find $\vec{r}$ for a general point P on m , e.g. $\begin{pmatrix} 0 \\ 8 \\ 2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$	B1	Or $\begin{pmatrix} -1 \\ 7 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$.
	Calculate scalar product of $\vec{r}$ and a direction vector for m and equate the result to zero	M1	
	Obtain $\lambda = -2$	A1	OE
	Obtain answer $\begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}$	A1	OE
		4	

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	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

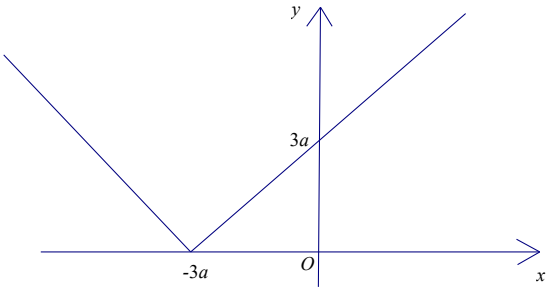
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

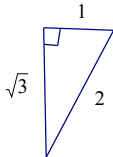
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Question	Answer	Marks	Guidance
1(a)		B1	Roughly symmetrical. Condone some inaccuracy, but needs to look as if they intended symmetry and straight lines. If not, then B0. Needs to be in the correct position. Needs to have two solid line segments. Needs to exist in both quadrants above the axis. Ignore dotted lines below the axis. Solid line below the axis is B0. Condone if no scale shown, but need to see $3a$ and $-3a$ marked. Ignore $y = a - 2x$ if seen.
		1	
1(b)	Obtain critical value $-\frac{2a}{3}$ from $x + 3a = a - 2x$	B1	Ignore $x = 4a$ if seen.
	State final answer $x > -\frac{2a}{3}$	B1	Need a clear conclusion – must imply rejection of $x = 4a$. B0 if using $\geq$.
	Alternative Method for Question 1(b)		
	Obtain critical value $-\frac{2a}{3}$ from $(x + 3a)^2 = (a - 2x)^2$	B1	Ignore $x = 4a$ if seen.
	State final answer $x > -\frac{2a}{3}$	B1	Need a clear conclusion – must imply rejection of $x = 4a$. B0 if using $\geq$.
		2	

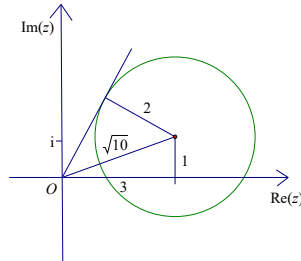
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Question	Answer	Marks	Guidance
2	Use a correct law for the logarithm of a product or the logarithm of a power	*M1	Not available after incorrect manipulation, e.g. $6^{x+1} = 12^{2x-3}$
	Obtain $\ln 3 + (x+1)\ln 2 = \ln 4 + (2x-3)\ln 3$	A1	Or equivalent with no powers
	Solve for x	DM1	E.g. $x = \frac{\ln \frac{2}{81}}{\ln \frac{2}{9}}$ Allow with logarithms evaluated.
	Obtain 2.46 only	A1	
	Alternative Method for Question 2		
	Use a correct law of indices for a product or a power	*M1	E.g. $2^{x+1} = 2 \times 2^x$ Not available after incorrect manipulation, e.g. $6^{x+1} = 12^{2x-3}$. $4 = 2^2$ on its own is not enough.
	Obtain $\left(\frac{2}{9}\right)^x = \frac{2}{81}$	A1	OE with powers of x only, e.g. $6 \times 2^x = \frac{4}{27} \times 9^x$ Note: $(3^x)^2$ is not far enough.
	Solve for x	DM1	E.g. $x = \frac{\ln \frac{2}{81}}{\ln \frac{2}{9}}$ or $x = \ln_{\frac{2}{9}} \frac{2}{81}$. Allow with logarithms evaluated.
	Obtain 2.46 only	A1	
		4	

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Question	Answer	Marks	Guidance
3(a)	Complete method to obtain equation in y only or Obtain the y coordinate of P	M1	E.g. $(2-3)^2 + (y-1)^2 = 4$ Or consider triangle and state $\pm(\sqrt{3}-1)$. 
	Obtain $2 + i(1 - \sqrt{3})$	A1	Or exact equivalent in the form $x + iy$. Allow $2 - i(\sqrt{3} - 1)$. Must not be coordinates, and not x, y stated separately.
		2	

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Question	Answer	Marks	Guidance
3(b)	State one relevant angle, e.g. $\tan^{-1} \frac{1}{3}$ or $\sin^{-1} \frac{2}{\sqrt{10}}$	B1	 Note: $\tan^{-1} \frac{1}{3} = \sin^{-1} \frac{1}{\sqrt{10}}$
	Complete method to obtain the required angle	M1	$\tan^{-1} \frac{1}{3} + \sin^{-1} \frac{2}{\sqrt{10}}$
	Obtain 1.01 radians or 57.7°	A1	Accept AWRT
	Alternative Method for Question 3(b)		
	If $y = mx$ is a tangent to the circle $\Rightarrow (x-3)^2 + (mx-1)^2 = 4$ $\Rightarrow ((1+m^2)x^2 + (-6-2m)x + 6 = 0),$ then $(-6-2m)^2 - 24(1+m^2) = 0$ and $m = \frac{3+2\sqrt{6}}{5}$	B1	Must be choosing the positive root.
	Substitute into the quadratic and solve for x, or use gradient to obtain tangent Required angle is $\frac{\pi}{2} - \sin^{-1} \left(\frac{9-\sqrt{6}}{5\sqrt{6}} \right)$ OE	M1	Note $x = \frac{9-\sqrt{6}}{5}$
	Obtain 1.01 radians or 57.7°	A1	Accept AWRT
		3	

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Question	Answer	Marks	Guidance
4	Begin integration by parts and obtain $px^2 \tan^{-1} x + q \int \frac{x^2}{1+x^2} dx$	M1*	Condone sign error in formula for integration by parts.
	Obtain $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} \int \frac{x^2}{1+x^2} dx$	A1	OE Allow with $\arctan x$ or $\tan^{-1} x$.
	Use $\int \frac{x^2}{1+x^2} dx = \int 1 - \frac{1}{1+x^2} dx$ and integrate to obtain $\beta x + \gamma \tan^{-1} x$	DM1	Split the fraction and integrate.
	Obtain $\frac{x^2}{2} \tan^{-1} x - \frac{1}{2} (x - \tan^{-1} x)$	A1	OE
	Substitute correct limits correctly in an expression of the form $\alpha x^2 \tan^{-1} x + \beta x + \gamma \tan^{-1} x$	DM1	$\frac{\pi}{8} - 0 - \frac{1}{2} + \frac{\pi}{8} + 0(-0)$ Dependent on both previous M marks. Need to see evidence of the use of the lower limit at least once. Must evaluate the trigonometry using radians.
	Obtain $\frac{\pi}{4} - \frac{1}{2}$	A1	ISW Or exact two term equivalent
		6	

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Question	Answer	Marks	Guidance
5(a)	Use correct product rule	M1	$f'(x) = (x-a)^2 g'(x) + 2(x-a)g(x)$ Allow incorrect chain rule.
	Obtain correct derivative and state a clear conclusion	A1	AG E.g. take out a factor of $(x-a)$ and show the factorised form as far as $(x-a)h(x)$ correctly (no further comment needed), or show that $f'(a) = 0$ and state that ' $(x-a)$ is a factor'.
		2	

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Question	Answer	Marks	Guidance
5(b)	Use $f(3) = 0$	M1	$2 \times 3^3 - 4 \times 3^2 + 3p + q = 0$
	Obtain $3p + q = -18$	A1	OE Powers should be evaluated but do not need to be simplified.
	Use $f'(3) = 0$	M1	$54 - 24 + p = 0$
	Obtain $p = -30$	A1	OE
	Obtain $p = -30, q = 72$	A1	Correct only.
	Alternative Method for Question 5(b)		
	Attempt division of $f(x)$ by $(x-3)^2$ as far as $(2x + Q)$	M1	$Q \neq 0$
	Obtain quotient $(2x + 8)$	A1	
	Equate linear remainder to zero and compare coefficients	M1	Or expand $(2x + 8)(x - 3)^2$ and compare coefficients. Method to obtain an equation in p or q .
	obtain one of $p = -30, q = 72$	A1	
	Obtain $p = -30, q = 72$	A1	Correct answers only.

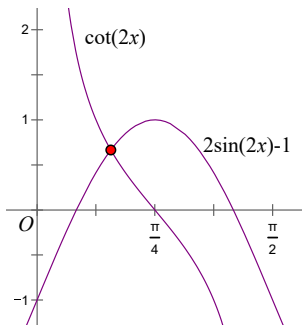
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Question	Answer	Marks	Guidance
5(b)	Alternative Method 2 for Question 5(b)		
	Use $f(3) = 0$	M1	$54 - 36 + 3p + q = 0$
	Obtain $3p + q = -18$	A1	OE Powers evaluated.
	Divide $f(x)$ by $(x-3)^2$ as far as $(2x+Q)$ and form an equation in p only or q only or in p and q	M1	Using this as part of a hybrid method they need to be working towards a second equation by considering coefficients in the remainder or use $f(-4) = 0$ or equivalent for <i>their</i> $(2x+8)$.
	Obtain correct equation in p or q	A1	
	Obtain $p = -30, q = 72$	A1	Correct answer only.
	Alternative Method 3 for Question 5(b)		
	$f'(x) = 6x^2 - 8x + p$	B1	
	$= (x-3)(6x+10)$	M1	Use the factor $x-3$.
	Obtain $p = -30$	A1	
	$f(x) = (x-3)^2(2x+Q) \quad (= 2x^3 - 4x^2 - 30x + q)$	M1	Use the factor $(x-3)^2$.
	$\Rightarrow Q = 8, \quad q = 72$	A1	

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Question	Answer	Marks	Guidance
5(b)	Alternative Method 4 for Question 5(b)		
	Expand $f(x) = (x-3)^2(Ax+B)$ and compare at least one coefficient other than for x^3	M1	$A=2$ can be found by inspection.
	Obtain $q=9B$ and $p=9A-6B$ or obtain $B=8$	A1	
	Use <i>their</i> A and B to solve for p or q	M1	$A=2 \quad B=8$
	Obtain one of $p=-30, q=72$	A1	
	Obtain $p=-30, q=72$	A1	Correct answer only.
		5	

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Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph, e.g. $y = \cot 2x$ Scales must be correct if seen. Do not need a full scale, but do need to be able to verify the key points mentioned below: For $y = \cot 2x$, correct intercept on x -axis, asymptotes at $x = 0$, $x = \frac{1}{2}\pi$ implied (condone large gap between the curve and the asymptote on the RHS). Must be continuous. For $2\sin 2x - 1$, correct shape and position. No incorrect curvature. Correct maximum implied. Intercept at $(0, -1)$.	B1	
	Sketch a second relevant graph, e.g. $y = 2\sin 2x - 1$ and justify the given statement	B1	Need to see enough of both graphs to confirm there is not a second intersection in the given interval. Need to mark the intersection with a dot or a cross, or say that the root lies at the point of intersection (or equivalent).
		2	
6(b)	Calculate the values of a relevant expression or pair of expressions at $x = 0.4$ and $x = 0.6$ Allow working on a smaller interval contained within the given interval	M1	E.g. $0.9712 > 0.435$ and $0.389 < 0.864$ $0.537 > 0$ and $-0.475 < 0$ Or using the iterative formula, $0.18 > 0$, $-0.17 < 0$ Or using $\tan 2x(2\sin 2x - 1) - 1 = 0$ $-0.55 < 0$, and $1.22 > 0$. M0 if working in degrees.
	Justify the given statement with correct calculated values	A1	Values to 2 sf or better. Accept values with statement $f(0.4) \times f(0.6) < 0$.
		2	

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Question	Answer	Marks	Guidance
6(c)	Use the iterative process $x_{n+1} = \frac{1}{2} \tan^{-1} \left(\frac{1}{2 \sin 2x_n - 1} \right)$ correctly at least once	M1	M0 if working in degrees. Need to get as far as a second iteration completed.
	Obtain final answer 0.49	A1	
	Show sufficient iterations to 4 d.p. to justify 0.49 to 2 d.p. or show that there is a sign change in (0.485, 0.495) –0.0157 < 0, 0.355 > 0	A1	E.g. 0.5, 0.4858, 0.4967, 0.4883, 0.4947, ... or 0.4, 0.5804, 0.4378, 0.5395, 0.4595, 0.5188, 0.4726, 0.5075, 0.4804, 0.5009, 0.4851, 0.4972, 0.4879, 0.4950, 0.4895, 0.4937 or 0.6, 0.4291, 0.5483, 0.4544, 0.5235, 0.4695, 0.5101, 0.4786, 0.5025, 0.4840, 0.4981, 0.4872, 0.4955, 0.4891, 0.4940 The last 2 options are a lot of iterations. Allow the marks if they start correctly and finish correctly. Condone if there are some missing in the middle.
		3	

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Question	Answer	Marks	Guidance
7(a)	State or imply $6y^2 \frac{dy}{dx}$ as the derivative of $2y^3$	B1	
	State or imply $3x^2 \frac{dy}{dx} + 6xy$ as the derivative of $3x^2y$	B1	
	Complete the differentiation and equate the derivative of the LHS to zero and solve for $\frac{dy}{dx}$ (the $= 0$ can be implied)	M1	Needs to be clear how the value of $\frac{dy}{dx}$ was obtained, e.g. collecting like terms and using brackets.
	Obtain $\frac{dy}{dx} = \frac{x^2 + 2xy}{2y^2 - x^2}$ from correct working	A1	AG Accept $\frac{dy}{dx} = \frac{2xy + x^2}{2y^2 - x^2}$. No incorrect statements, e.g. ‘cancel by 3’ or ‘divide top and bottom by 3’ are acceptable, but to have $\frac{dy}{dx} = \frac{3x^2 + 6xy}{6y^2 - 3x^2}$ and say “divide by 3” at the final step scores A0.
		4	
7(b)	Equate derivative to 0 and solve for x or for x in terms of y .	*M1	E.g., $x^2 + 2xy = 0 \Rightarrow x = 0$ or $x = -2y$. Must be using the numerator.
	Obtain $(0, 2)$	B1	Allow if the values are stated separately. Do not ISW.
	Use <i>their</i> $x = -2y$ to form an equation in one unknown Or equivalent e.g. substitute $y = -\frac{x^2}{2x}$	dM1	E.g., $2y^3 - 12y^3 + 8y^3 = 16$ or $-2\left(\frac{1}{8}x^3\right) + 3\left(\frac{1}{2}x^3\right) - x^3 = 16$.
	Obtain $(4, -2)$	A1	Allow if the values are stated separately. Do not ISW
		4	

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Question	Answer	Marks	Guidance
8(a)	Use correct double angle formula to expand $\sin 4x$	*M1	$2\sin 2x \cos 2x$ OE
	Use correct double angle formulae to obtain an expression in $\sin x$ and $\cos x$	dM1	E.g., $4\sin x \cos x (2\cos^2 x - 1)$ OE.
	Obtain $\sin 4x \equiv 4\sin x (2\cos^3 x - \cos x)$ from fully correct working	A1	AG
	Alternative Method for Question 8(a)		
	Use correct double angle formula to rearrange the RHS	*M1	E.g. $2\sin 2x (2\cos^2 x - 1)$
	Use correct double angle formula to obtain an expression in $\sin 2x$ and $\cos 2x$	dM1	E.g. $2\sin 2x \cos 2x$
	Obtain $\sin 4x \equiv 4\sin x (2\cos^3 x - \cos x)$ from fully correct working	A1	AG
	Alternative Method 2 for Question 8(a)		
	Use correct angle sum formulae to expand $\sin 4x$ as far as an expression in $\sin x$, $\cos x$, $\sin 2x$ and $\cos 2x$	*M1	E.g. $\sin x (\cos x \cos 2x - \sin x \sin 2x)$ $+ \cos x (\sin x \cos 2x + \cos x \sin 2x)$
	Use correct double angle formulae to obtain an expression in $\sin x$ and $\cos x$	dM1	E.g., $\sin x \cos x (2\cos^2 x - 1) - 2\sin^3 x \cos x$ $+ \sin x \cos x (2\cos^2 x - 1) + 2\sin x \cos^3 x$
	Obtain $\sin 4x \equiv 4\sin x (2\cos^3 x - \cos x)$ from fully correct working	A1	
		3	

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Question	Answer	Marks	Guidance
8(b)	Use the identity to obtain $p \int \sin x \cos^6 x \, dx + q \int \sin x \cos^4 x \, dx$	B1	$8 \int \sin x \cos^6 x \, dx - 4 \int \sin x \cos^4 x \, dx$ Accept terms of the correct form but without the integral signs or the dx.
	Obtain $r \cos^7 x + s \cos^5 x$	B1	
	Obtain $-\frac{8}{7} \cos^7 x + \frac{4}{5} \cos^5 x$	B1	If using the substitution $u = \cos x$, accept the form $-\frac{8}{7}u^7 + \frac{4}{5}u^5$.
	Use the correct limits correctly in an expression of the form $r \cos^7 x + s \cos^5 x$	M1	OE Must be evaluated, e.g. $-\frac{8}{7} \times \frac{1}{8\sqrt{2}} + \frac{4}{5} \times \frac{1}{4\sqrt{2}} + \frac{8}{7} - \frac{4}{5}$.
	Obtain $\frac{1}{35}(\sqrt{2} + 12)$	A1	Or exact simplified equivalent (e.g. as 2 terms).
		5	

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Question	Answer	Marks	Guidance
9(a)	State or imply the form $A + \frac{B}{x+2a} + \frac{C}{x+3a}$	B1	
	Use a correct method for finding a coefficient	M1	
	Obtain one of $A=1$, $B=2a$ and $C=-3a$	A1	SC: If B0 is scored because of an omission of 'A', then maximum M1A1 is available for one constant correct. SC: If they substitute a value for a , or if their working implies the use of $a=1$, they can score maximum B1M1.
	Obtain a second value	A1	
	Obtain the third value	A1	
	Alternative Method for Question 9(a)		
	State or imply $1 + \frac{\text{linear expression}}{(x+2a)(x+3a)}$	B1	$1 + \frac{-ax}{(x+2a)(x+3a)}$
	State or imply the form $1 + \frac{B}{x+2a} + \frac{C}{x+3a}$	B1	
	Use a correct method for finding B or C	M1	
	Obtain one of $B=2a$ and $C=-3a$	A1	
	Obtain the second value	A1	
		5	

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Question	Answer	Marks	Guidance
9(b)	Integrate and obtain terms $Ax + B \ln(x + 2a) + C \ln(x + 3a)$	B2FT	B1 for any two terms correct, B2 for all three terms correct. The FT is on A , B and C : $x + 2a \ln(x + 2a) - 3a \ln(x + 3a)$. Allow for FT on a split completed in (b) .
	Substitute limits correctly in an integral containing at least 2 terms from the form $rx + s \ln(x + 2a) + t \ln(x + 3a)$	M1	E.g. $2a + 2a \ln\left(\frac{3}{1}\right) - 3a \ln\left(\frac{4}{2}\right)$
	Obtain $a\left(2 + \ln\left(\frac{9}{8}\right)\right)$ from correct working	A1	Accept equivalent fractions with integers. A0 XP if $a\left(2 + \ln\left(\frac{9}{8}\right)\right)$ is following an error in (a) .
		4	

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Question	Answer	Marks	Guidance
10(a)	State $\frac{dV}{dt} = 5000 - kh^2$	B1	SOI
	Use $V = 2500h$ and chain rule to obtain $\frac{\left(\text{their } \frac{dV}{dt}\right)}{2500}$	*M1	OE $\frac{dh}{dt} = \frac{5000 - kh^2}{2500}$
	Use $h = 20$, $\frac{dh}{dt} = 0.4$ to obtain k	dM1	$0.4 = \frac{5000 - 400k}{2500} \Rightarrow k = 10$
	Obtain $\frac{dh}{dt} = \frac{500 - h^2}{250}$ from correct working	A1	AG There needs to be a complete statement for A1. Condone ‘divide by 10’ in place of showing the cancelling.
		4	

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Question	Answer	Marks	Guidance
10(b)	Separate variables correctly	B1	E.g. $\int \frac{1}{500-h^2} dh = \frac{1}{250} \int 1 dt$ Condone missing integral signs or missing dh , dt , but not both.
	Obtain $\frac{1}{250}t$	B1	OE (depending on how they use the 250).
	Obtain $\frac{1}{2\sqrt{500}} \ln \left \frac{\sqrt{500}+h}{\sqrt{500}-h} \right $	B1	OE, e.g. $\frac{1}{20\sqrt{5}} \ln \left \frac{10\sqrt{5}+h}{10\sqrt{5}-h} \right $. Coefficients will depend on where the 250 is. Condone missing modulus signs.
	Use $t=0, h=0$ in an expression of the form $pt+q\ln(a+h)+r\ln(b-h)$ to obtain the constant of integration or as limits on a definite integral	M1	Requires a value for the constant. M0 if <i>their</i> equation has no solution. M0 if they never had a constant. Need some indication that the conclusion is based on using $t=0, h=0$, e.g. " $t=0, h=0 \Rightarrow C=0$ ".
	Obtain $t=16.1$ (s)	A1	Accept AWRT.
		5	

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Question	Answer	Marks	Guidance
11(a)	$\vec{} -\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$	B1	Do not ISW. Accept as a column vector.
	$\vec{} -5\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$	B1	Do not ISW. Accept as a column vector. SC: B0B1 for column vectors with i, j, k (or coordinates) (allow one slip).
		2	
11(b)	$\vec{}, \vec{}$ correct process for evaluating the scalar product of	M1	Or $\vec{}, \vec{}$ Need some evidence of an attempt to evaluate the scalar product.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli for their two vectors.	M1	$\pm \left(\frac{5 - 15 + 8}{\sqrt{30} \cdot \sqrt{50}} \right)$ The scalar product could be incorrect, but it must be a scalar. Condone a sign change.
	Obtain $-\frac{\sqrt{15}}{75}$	A1	Or exact equivalent, e.g. $-\frac{1}{5\sqrt{15}}, \frac{-2}{\sqrt{30}\sqrt{50}}$. A0 if they never have a statement " $\cos \theta = \dots$ ". ISW if they have correct " $\cos \theta = \dots$ " and then state the angle.

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Question	Answer	Marks	Guidance
	Alternative Method for Question 11(b)		
	Carry out the correct process for finding the lengths of the sides of triangle <i>BMC</i>	M1	$MB = \sqrt{30}, MC = \sqrt{50}, BC = \sqrt{84}$
	Correct use of the cosine rule as far as $\cos \theta = \dots$	M1	$\cos M = \frac{\text{their } MB ^2 + \text{their } MC ^2 - 84}{2\text{their } MB \times \text{their } MC }$ $\left(\cos M = \frac{50 + 30 - 84}{2\sqrt{50}\sqrt{30}} \right)$
	Obtain $\frac{-1}{\sqrt{375}}$	A1	Or exact equivalent. A0 if they never have a statement " $\cos \theta = \dots$ ". ISW if they have correct " $\cos \theta = \dots$ " and then state the angle.
		3	

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Question	Answer	Marks	Guidance
11(c)	Working with triangle MBC : Correct method to calculate the exact value of $\sin(\angle CMB)$	M1	
	$\sin(\angle CMB) = \sqrt{\frac{374}{375}}$	A1	Or equivalent, e.g. $\sin(\angle CMB) = \sqrt{\frac{5400}{5625}}$. A0 XP if they had changed the sign in their working for the cosine in (b) and for the final answer.
	Correct method to calculate the area of triangle ABC using MB and MC	M1	$2 \times \text{area of } CMB$.
	$2 \times \frac{1}{2} \sqrt{30} \sqrt{50} \times \sqrt{\frac{374}{375}} = 2\sqrt{374}$	A1	Or exact equivalent, e.g. $2\sqrt{2}\sqrt{11}\sqrt{17}$ or $\frac{\sqrt{3366000000}}{1500}$.
	Alternative Method for Question 11(c)		
	Working with triangle ABC : correct method to calculate the exact sine of an angle in the triangle ABC	M1	$\cos A = \frac{56}{\sqrt{120 \times 76}}$, $\cos B = \frac{64}{\sqrt{120 \times 84}}$, $\cos C = \frac{20}{\sqrt{76 \times 84}}$
	Obtain $\sin A = \frac{4\sqrt{374}}{\sqrt{120}\sqrt{76}}$ or $\sin B = \frac{4\sqrt{374}}{\sqrt{120}\sqrt{84}}$ or $\sin C = \frac{4\sqrt{374}}{\sqrt{76}\sqrt{84}}$	A1	Or exact equivalent.
	Correct method to calculate the area of triangle ABC	M1	Must have a matching set of sides for their angle.
	Area = $\frac{1}{2} \sqrt{120} \sqrt{76} \frac{4\sqrt{374}}{\sqrt{120}\sqrt{76}}$ or area = $\frac{1}{2} \sqrt{120} \sqrt{84} \frac{4\sqrt{374}}{\sqrt{120}\sqrt{84}}$ or area = $\frac{1}{2} \sqrt{76} \sqrt{84} \frac{4\sqrt{374}}{\sqrt{76}\sqrt{84}} = 2\sqrt{374}$	A1	Or exact equivalent.
		4	

Cambridge International A Level

MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **24** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Annotations guidance for centres

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $(3x+2)^2 < 3^2(2x-1)^2$, or pair of linear equations $(3x+2) = \pm 3(2x-1)$	B1	Allow '=', or any inequality sign.
	Make reasonable attempt at solving a 3-term quadratic, or solve two linear equations for x	M1	E.g. $27x^2 - 48x + 5 = 0$, $(9x-1)(3x-5) = 0$, $x = \dots$ Allow even if quadratic not given in a 3-term form. See guidelines document for solving a quadratic.
	Obtain critical values $x = \frac{1}{9}$ and $x = \frac{5}{3}$	A1	Allow '=', or any inequality sign.
	State final answer $x < \frac{1}{9}$, $x > \frac{5}{3}$ only	A1	Allow 'OR' but not 'AND'. Allow ' $\cup$ ' but not ' $\cap$ '. No marks can be scored if no working is seen.
	Alternative Method for Question 1		
	Obtain critical value $x = \frac{5}{3}$ from a graphical method, or by inspection, or by solving a linear equation or an inequality	B1	Allow '=', or any inequality sign.
	Obtain critical value $x = \frac{1}{9}$ similarly	B2	Allow '=', or any inequality sign.
	State final answer $x < \frac{1}{9}$, $x > \frac{5}{3}$ only	B1	Allow 'OR' but not 'AND'. Allow ' $\cup$ ' but not ' $\cap$ '. No marks can be scored if no working is seen.
		4	

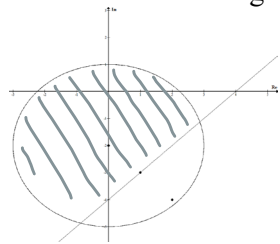
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Question	Answer	Marks	Guidance
2	Commence division and reach partial quotient of the form $3x^3 \pm 3x^2$ or $3x^4 - 2x^2 \equiv (x+1)(Ax^3 + Bx^2 + Cx + D) + Ex + F$, and reach $A = 3$ and $B = \pm 3$	M1	May be seen in synthetic division.
	Obtain quotient $3x^3 - 3x^2 + x - 1$ Do not ISW	A1	Don't need to state which is the quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct.
	Obtain remainder of 1	A1	Do not ISW. Allow e.g. $3x^3 - 3x^2 + x - 1 + \frac{1}{x+1}$ but NOT remainder = $\frac{1}{x+1}$.
	Alternative Method for Question 2		
	$f(-1) = 3 - 2 = 1 = \text{remainder}$	B1	Do not ISW.
	Use division or inspection or compare coefficients	M1	$3x^4 - 2x^2 - 1 \equiv (x+1)(3x^3 - 3x^2 + x - 1)$
	Obtain quotient $3x^3 - 3x^2 + x - 1$	A1	Do not ISW. Allow e.g. $3x^3 - 3x^2 + x - 1 + \frac{1}{x+1}$ but NOT remainder = $\frac{1}{x+1}$.
		3	

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Question	Answer	Marks	Guidance
3	Use log quotient law, e.g. $\ln \frac{3}{5^x} = \ln 3 - \ln 5^x$ or log product law, e.g. $\ln 5^x + \ln(2^{3x-4})$	M1	May work in log to any base for first 3 marks.
	Use log power law, e.g. $\ln 5^x = x \ln 5$ or $\ln(2^{3x-4}) = (3x-4) \ln 2$	M1	Condone missing brackets if recovered at some point.
	Obtain correct expression for x in any exact form, e.g. $\frac{\ln 3 + 4 \ln 2}{3 \ln 2 + \ln 5}$	A1	
	Obtain final answer $\frac{\ln 48}{\ln 40}$	A1	Do not ISW. Final answer of $\log_{40} 48$ scores 3 marks only. No working award 0 marks.
	Alternative Method for Question 3		
	Rearrange to obtain $(2^3 \times 5)^x = 3 \times 2^4$	B1	OE
	Use log power law on an equation of the form $a^{bx} = c$ e.g. $x \ln(2^3 \times 5) = \ln(3 \times 2^4)$	M1	May work in log to any base for first 3 marks.
	Obtain correct expression for x in any exact form, e.g. $x = \frac{\ln(3 \times 2^4)}{\ln(2^3 \times 5)}$	A1	
	Obtain final answer $\frac{\ln 48}{\ln 40}$	A1	Do not ISW. Final answer of $\log_{40} 48$ scores 3 marks only. No working award 0 marks.
		4	

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Question	Answer	Marks	Guidance
4	Show a circle centre $(0, -2)$	B1	For all marks: Accept a scale or dashes representing a scale or points labelled. If scale only on one axis, allow this to imply the same scale on the other axis. Condone dashed circle and dashed perpendicular bisector for all marks
	Show a circle with radius 3	B1FT	FT centre not at the origin. Allow circle with 3 radii correct out of 4 ‘compass directions’. If no indication of scale, allow SC B1FT only for circle.
	Show the point representing $(2, -4)$ or the midpoint $(1, -3)$	B1	May be implied by a correct perpendicular bisector. Condone if $(2, -4)$ on the circle.
	Show the perpendicular bisector of the line joining $(2, -4)$ and $(0, -2)$ or the perpendicular bisector of the line joining $(2, -4)$ and centre of their circle or the perpendicular bisector going through $(1, -3)$	B1FT	FT is on the positions of $(2, -4)$ and $(0, -2)$ or on the position of $(2, -4)$ and centre of their circle or on the position of $(1, -3)$. Cuts (or would cut) x -axis between 3 and 5 and y -axis between -5 and -3 if everything else correct. If no indication of scale, allow SC B1FT only for perpendicular bisector.
	Shade the correct region 	B1	Dependent on all previous marks. Allow SC B1 for correct shading if the perpendicular bisector looks correct and the only error is that it is slightly out when crossing the axes. If no indication of scale, allow SC B1 for shading if everything is relatively correct.
		5	

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Question	Answer	Marks	Guidance
5(a)	Use correct double angle formula to express $\cos 4x$ in terms of $\sin^2 2x$	B1	Need to see $1 - 2 \sin^2 2x$ or $(1 - \sin^2 2x) - \sin^2 2x$. May be implied by further work.
	Use correct double angle formula to express $\cos 4x$ in terms of single angles	M1	Allow with $\cos x$ but not $\cos 2x$. E.g. $1 - 2(2 \sin x \cos x)^2$ or $2(1 - 2 \sin^2 x)^2 - 1$.
	Obtain a correct expression in $\sin x$ in any form	A1	E.g. $1 - 2[4 \sin^2 x(1 - \sin^2 x)] + 2 \sin^2 x - 1$, or $2(1 - 2 \sin^2 x)^2 - 1 + 2 \sin^2 x - 1$.
	Obtain the given answer $8 \sin^4 x - 6 \sin^2 x$	A1	AG Must show at least one intermediate line of working including $\sin^4 x$ between first A1 and AG. Allow, e.g., A for x except in the final answer. Allow recovery on the next line after a slip. Allow recovery on the next line after missing x .
	Alternative Method for Question 5(a)		
	Use correct double angle formula to express whole expression in terms of $\cos 2x$	M1	E.g. $(1 - \cos 2x)(2 - 2 \cos 2x - 3)$.
	Use correct double angle formula to express $2 \cos^2 2x - 1$ as $\cos 4x$	B1	
	Use correct double angle formula to express whole expression as $\cos 4x - \cos 2x$	A1	
	Obtain the given answer $\cos 4x + 2 \sin^2 x - 1$	A1	AG Allow, e.g., A for x except in the final answer. Allow recovery on the next line after a slip. Allow recovery on the next line after missing x .
		4	

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Question	Answer	Marks	Guidance
5(b)	Obtain answers 0° , 180° and -180°	B1	
	Carry out a correct method to find a value of x in the given interval for $8 \sin^2 x - 6 = 0$	M1	Condone a wrong value of x if $\sin x = \sqrt{\frac{3}{4}}$ OE seen. Allow M1A1A1 if dividing by $\sin^2 x$, but B1 is not scored.
	Obtain answer, e.g. 60°	A1	In radians, would be $\frac{1}{3}\pi$.
	Obtain remaining answers, e.g. -60° , 120° and -120° and no other in the interval	A1	Ignore answers outside the given interval.
		4	

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Question	Answer	Marks	Guidance
6	Commence integration by parts and reach $Ax^2 \cos 2x - \int Bx \cos 2x dx$	*M1	OE Condone sign error in formula.
	Obtain $-\frac{1}{2}x^2 \cos 2x + \int x \cos 2x dx$	A1	OE Unsimplified.
	Complete integration by parts and reach $Ax^2 \cos 2x + Bx \sin 2x + C \cos 2x$	*M1	OE $\int Bx \cos 2x dx = Bx \sin 2x + C \cos 2x$ may be written separately to $Ax^2 \cos 2x$ for M1, but not for A1.
	Obtain $-\frac{1}{2}x^2 \cos 2x + \frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x$	A1	OE Unsimplified.
	Substitute limits correctly in an expression of the form $Ax^2 \cos 2x + Bx \sin 2x + C \cos 2x$ and evaluate the trig expressions	DM1	$A\left(\frac{\pi}{6}\right)^2 \left(\frac{1}{2}\right) + B\left(\frac{\pi}{6}\right) \left(\frac{\sqrt{3}}{2}\right) + C\left(\frac{1}{2}\right) - C$ Allow one slip, including omitting the ‘– C’. Do not allow only decimals
	Obtain answer $-\frac{1}{144}\pi^2 + \frac{\sqrt{3}}{24}\pi - \frac{1}{8}$ or exact three-term equivalent	A1	ISW Allow equivalent fractions. Allow 0.125 for $\frac{1}{8}$.
		6	

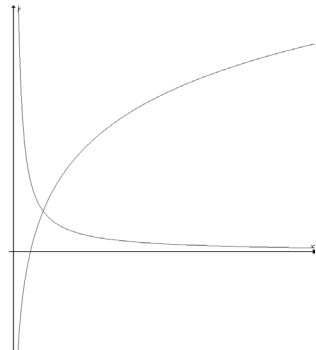
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Question	Answer	Marks	Guidance
7	Attempt to remove fraction by multiplying throughout by $2 - i$ or multiply fraction by $\frac{2+i}{2+i}$	*M1	Allow if still includes brackets and/or i^2 . E.g. $5z - zz^*(2 - i) + (20 + 8i)(2 - i) = 0$ or better. E.g. $\frac{5z}{2-i} \times \frac{2+i}{2+i} - zz^* + 20 + 8i = 0$ or better.
	Substitute $z = x + iy$ and $z^* = x - iy$ throughout the equation	B1	May see $5x + 5iy - 2(x^2 + y^2) + i(x^2 + y^2) + 48 - 4i = 0$, or $2x + ix + 2iy - y - x^2 - y^2 + 20 + 8i = 0$.
	Use $i^2 = -1$ correctly at least once and equate real and imaginary parts to zero	*DM1	OE, e.g. $5x - 2x^2 - 2y^2 + 48 = 0$ and $5y + x^2 + y^2 - 4 = 0$, or $2x - y - x^2 - y^2 + 20 = 0$ and $x + 2y + 8 = 0$. For <i>their</i> horizontal equation.
	Obtain two correct equations e.g. $5x - 2(x^2 + y^2) + 48 = 0$ and $5y + (x^2 + y^2) - 4 = 0$	A1	E.g. $2x - y - x^2 - y^2 + 20 = 0$ and $x + 2y + 8 = 0$. Allow $5iy + i(x^2 + y^2) - 4i = 0$ or $ix + 2iy + 8i = 0$.
	Solve a quadratic and a linear equation for x or for y	DM1	
	Obtain answers $2 - 5i$ and $-\frac{16}{5} - \frac{12}{5}i$ only	A1	Accept $x = 2$ $y = -5$ and $x = -\frac{16}{5}$ $y = -\frac{12}{5}$ only OE, or $(2, -5)$ and $\left(-\frac{16}{5}, -\frac{12}{5}\right)$ only OE. Allow decimals. Do not ISW.
		6	

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Question	Answer	Marks	Guidance
8(a)	Use the correct product or quotient rule, e.g. $e^{-5x} \frac{d}{dx}(\ln 5x) + \ln 5x \frac{d}{dx}(e^{-5x})$	M1	M0 if $y = e^{-5p} \ln 5p$ seen prior to differentiation. Accept if only seen in actual derivative = 0.
	Obtain the correct derivative in any form e.g. $\frac{1}{x}e^{-5x} - 5e^{-5x} \ln 5x$	A1	
	Obtain the given answer $\ln 5p = \frac{1}{5p}$ after full and correct working	A1	AG May go from $\frac{1}{x}e^{-5x} - 5e^{-5x} \ln 5x = 0$, or $\frac{1}{x}e^{-5x} = 5e^{-5x} \ln 5x$ to the given answer without intermediate working.
		3	

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Question	Answer	Marks	Guidance
8(b)	Sketch an acceptable graph, e.g. $y = \ln 5x$ or $y = \frac{1}{5x}$ 	M1	For both marks: Note: Allow without scale on either axis, but if $y = \ln 5x = 0$ identified to be not $x = 0.2$, then 0 marks for $y = \ln 5x$. Allow graphs not labelled, or labelled with p instead of x . Allow $\ln 5x$ starting at the x -axis. If either graph shown in other quadrants, must be correct. For $y = \frac{1}{5x}$, asymptotic behaviour needed for at least one axis. Must not touch axes.
	Sketch a second acceptable graph, e.g. $y = \frac{1}{5x}$ or $\ln 5x$, and justify the given statement by dot, cross or statement only one intersection.	A1	
		2	

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Question	Answer	Marks	Guidance
8(c)	Calculate the values of a relevant expression or pair of expressions at $p = 0.2$ and $p = 0.6$ Note can use, e.g., $p = 0.3$ and $p = 0.5$, or any smaller interval which works	M1	$f(p) = \ln 5p - \frac{1}{5p}$ $f(0.2) = -1 < 0$, $f(0.6) = 0.765 > 0$ At least one correct value to at least 2sf. Or comparing $\ln 5p$ and $\frac{1}{5p}$. At least 3 correct values to at least 2sf.
	Complete the argument correctly with correct calculated values	A1	
		2	
8(d)	Use the iterative formula correctly at least twice	M1	M0 for 0.3526, 0.3526, 0.3526...
	Obtain final answer $p = 0.35$, Answer = 0.35, or just 0.35 stated	A1	Allow, e.g., $a_1, a_2, a_3 \dots$ or $x_1, x_2, x_3 \dots$ or answer ₁ , answer ₂ , answer ₃ ... for M1 and second A1. For first A1, must be $p = 0.35$ or answer = 0.35 unless just 0.35 is stated, e.g. not $x = 0.35$, $p_1 = \dots$, $p_\infty = \dots$ etc.
	Show sufficient iterations to 4 dp to justify 0.35 to 2 dp or show there is a sign change in the interval (0.345, 0.355) 0.2, 0.5437, 0.2889, 0.3996, 0.3299, 0.3667, 0.3451, 0.3571, 0.3502, 0.3541 0.25, 0.4451, 0.3135, 0.3786, 0.3392, 0.3607, 0.3482, 0.3552, 0.3512, 0.3535 0.3, 0.3895, 0.3342, 0.3639, 0.3465, 0.3562, 0.3507, 0.3538, 0.3520, 0.3530 0.45, 0.3119, 0.3797, 0.3387, 0.3610, 0.3480, 0.3553, 0.3512, 0.3535 0.5, 0.2984, 0.3910, 0.3336, 0.3643, 0.3463, 0.3563, 0.3506, 0.3538 0.55, 0.2877, 0.4008, 0.3294, 0.3670, 0.3449, 0.3572, 0.3501, 0.3541 0.6, 0.2791, 0.4095, 0.3260, 0.3694, 0.3437, 0.3579, 0.3497, 0.3543	A1	E.g. 0.4, 0.3297, 0.3668, 0.3450, 0.3571, 0.3502, 0.3541. Allow M1(A1 or A0) A1 if more values are to at least 4dp than to 3dp. SC B1 for starting from either 0.3526 or 0.3527 and using iterative formula correctly at least twice if the sequence shows a correct change in the 4th decimal place (and SC DB1 for getting $p = 0.35$), but 0 marks otherwise.
		3	

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Question	Answer	Marks	Guidance
9(a)	Obtain $\mathbf{r} = 3\mathbf{i} + \mathbf{j} - 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	B1	OE Must have $\mathbf{r} = \dots$, but penalise missing $\mathbf{r} =$ only once in (a) . Do not allow column vectors with $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ included.
	Carry out a correct method for finding a direction vector for l_2 , $(3\mathbf{i} - 2\mathbf{j} + \mathbf{k}) \cdot (y\mathbf{j} + z\mathbf{k}) = 0$	M1	E.g. $-2y + z = 0$ SOI.
	Obtain $\mathbf{r} = -\mathbf{i} + 3\mathbf{j} - 6\mathbf{k} + \mu(\mathbf{j} + 2\mathbf{k})$	A1	OE Must have $\mathbf{r} = \dots$, but penalise missing $\mathbf{r} =$ only once in (a) . Do not allow column vectors with $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ included.
		3	
9(b)	Carry out correct process for evaluating the scalar product of the direction vectors of l_1 and l_2	*M1	Using <i>their</i> direction vectors from (a) . Ignore symbol if state e.g. ‘ $\times$ ’ in place of ‘ $\cdot$ ’. Allow the same parameter for both lines here.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and state $\cos \theta =$ the result	DM1	Using <i>their</i> direction vectors from (a) .
	Obtain answer 28.6° or 0.498°	A1	
		3	

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Question	Answer	Marks	Guidance
9(c)	Equate components of general points on <i>their</i> l_1 and <i>their</i> l_2 , provided these are both equations of lines and solve for λ or for μ	M1	$(3 + 2\lambda, 1 + \lambda, -6 + 4\lambda) = (-1, 3 + \mu, -6 + 2\mu)$ Do not allow the same parameter for both lines here if solving using two linear equations. Allow M1 for $3 + 2\lambda = -1 \Rightarrow \lambda = \dots$ even if the other equations are incorrect or not stated.
	Obtain correct answer for λ or μ , e.g. $\lambda = -2, \mu = -4$	A1	Allow M1A1 for $3 + 2\lambda = -1$ leading to $\lambda = -2$, even if the other equations are incorrect or not stated.
	Obtain position vector of point of intersection is $-\mathbf{i} - \mathbf{j} - 14\mathbf{k}$	A1	OE Do not accept coordinates. Allow even if $\mathbf{j}$ and $\mathbf{k}$ equations are incorrect or not stated.
		3	

Question	Answer	Marks	Guidance
10(a)	Carry out a relevant method to find A and B such that $\frac{2}{1-9y^2} = \frac{A}{1+3y} + \frac{B}{1-3y}$ o	M1	OE Allow M1 for finding A and B for $\frac{A}{1+3y} + \frac{B}{3y-1}$ and A1 for $A = 1, B = -1$ if $-2 = A(3y - 1) + B(1 + 3y)$. But, A0 for $A = -1, B = 1$ if $2 = A(3y - 1) + B(1 + 3y)$.
	Obtain $A = 1$ and $B = 1$	A1	If work with x and never see y , award M1A0, but allow M1A1 if y is seen anywhere on right hand side.
		2	

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Question	Answer	Marks	Guidance
10(b)	Separate variables correctly and attempt integration of at least one side	M1	Integrate to obtain at least one log term of the form $p \ln(a + by)$ OE, $q \ln \frac{a+by}{a-by}$ on one side, or a tan term on the other, and disregard the 2 if it appears.
	Integrate $\frac{2}{1-9y^2}$ to obtain $\frac{1}{3} \ln(1+3y) - \frac{1}{3} \ln(1-3y)$	A1FT	OE, e.g. $\frac{1}{3} \ln \left(\frac{1+3y}{1-3y} \right)$. FT $\frac{A}{3} \ln(1+3y) - \frac{B}{3} \ln(1-3y)$ or $\frac{A}{3} \ln(1+3y) + \frac{B}{3} \ln(3y-1)$ if <i>their</i> partial fractions used. The '2' must have been dealt with correctly for this mark (check right hand side for 2 appearing here)
	Obtain $r \tan 3x$	B1	
	Obtain term $\frac{1}{3} \tan 3x$	B1	Allow $\frac{1}{6} \tan 3x$ if '2' not dealt with correctly earlier.
	Use $y = 0$ when $x = \frac{1}{12} \pi$ to evaluate a constant or as limits in a solution of the form $p \ln(1+3y) + q \ln(1-3y) + r \tan 3x$ where $p, q, r \neq 0$	M1	$0 + 0 + 1/3 + C = 0$ No errors in substitution.
	$\frac{1}{3} \ln(1+3y) - \frac{1}{3} \ln(1-3y) = \frac{1}{3} \tan 3x - \frac{1}{3}$ Obtain answer $y = \frac{1}{3} - \frac{2}{3(1+e^{\tan 3x-1})}$ or $y = \frac{e^{\tan 3x-1} - 1}{3(e^{\tan 3x-1} + 1)}$	A1	OE ISW
		6	

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Question	Answer	Marks	Guidance
11(a)	Differentiate $\sqrt{3+2\tan x}$ to obtain $\sec^2 x (3+2\tan x)^{-\frac{1}{2}}$	B1	OE (can be unsimplified). Not dependent on product rule, but must be convincing or seen in isolation, not as a derivative of the whole expression. B0B0 for e.g. $2\sec x \sec x \tan x \sec^2 x (3+2\tan x)^{-\frac{1}{2}}$.
	Differentiate $\sec^2 x$ to obtain $2\sec x \sec x \tan x$	B1	OE Not dependent on product rule but must be convincing or seen in isolation, not as a derivative of the whole expression. B0B0 for e.g. $2\sec x \sec x \tan x \sec^2 x (3+2\tan x)^{-\frac{1}{2}}$ OE, e.g. $\frac{2\sin x}{\cos^3 x}$.
	Use correct product (or quotient) rule	M1	$\sec^2 x \frac{d}{dx}(\sqrt{\dots\dots\dots}) + \sqrt{\dots\dots\dots} \frac{d}{dx}(\sec^2 x)$
	[Obtain derivative, if correct $2\sec^2 x \tan x (3+2\tan x)^{\frac{1}{2}} + \sec^4 x (3+2\tan x)^{-\frac{1}{2}}$] and equate derivative to zero and obtain an equation in one trig function	M1	Must include $(\dots)^{\frac{1}{2}}$ and $(\dots)^{-\frac{1}{2}}$ here. Arithmetical errors only for this M1. May work in terms of $\sin x$ and $\cos x$.
	Obtain $5\tan^2 x + 6\tan x + 1 = 0$	A1	OE, e.g. $5\tan^4 x + 6\tan^3 x + 6\tan^2 x + 6\tan x + 1 = 0$ $52\sin^4 x - 28\sin^2 x + 1 = 0$ $52\cos^4 x - 76\cos^2 x + 25 = 0$ $\cot^2 x + 6\cot x + 5 = 0$
	Obtain AWR $x = -0.197$ only	A1	ISW May be more accurate.
		6	

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Question	Answer	Marks	Guidance
11(b)	$\frac{du}{dx} = 2 \sec^2 x$	B1	SOI
	Reach an integral of the form $\int A\sqrt{u} \, du$	*M1	OE
	Obtain $\int \frac{1}{2} u^{\frac{1}{2}} \, du$	A1	OE
	Obtain $\frac{1}{3} u^{\frac{3}{2}}$	A1FT	OE FT <i>their</i> coefficient.
	Substitute correct limits correctly in an expression of the form $Bu^{\frac{3}{2}}$ or $B(3 + 2 \tan x)^{\frac{3}{2}}$ and obtain $c \left(5^{\frac{3}{2}} - 1^{\frac{3}{2}} \right)$	DM1	OE $u = 1$ and $u = 5$ $x = -\frac{1}{4}\pi$ and $x = \frac{1}{4}\pi$ Do not allow only decimals.
	Obtain answer $\frac{5\sqrt{5}}{3} - \frac{1}{3}$	A1	Or exact equivalent, e.g. $\frac{\sqrt{125}-1}{3}$. ISW
		6	

Cambridge International A Level

MATHEMATICS**9709/35**

Paper 3 Pure Mathematics 3

October/November 2025**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2025 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **21** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

PUBLISHED**Mathematics-Specific Marking Principles**

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Annotations guidance for centres**

Examiners use a system of annotations as a shorthand for communicating their marking decisions to one another. Examiners are trained during the standardisation process on how and when to use annotations. The purpose of annotations is to inform the standardisation and monitoring processes and guide the supervising examiners when they are checking the work of examiners within their team. The meaning of annotations and how they are used is specific to each component and is understood by all examiners who mark the component.

We publish annotations in our mark schemes to help centres understand the annotations they may see on copies of scripts. Note that there may not be a direct correlation between the number of annotations on a script and the mark awarded. Similarly, the use of an annotation may not be an indication of the quality of the response.

The annotations listed below were available to examiners marking this component in this series.

Annotations

Annotation	Meaning
	More information required
	Accuracy mark awarded zero
	Accuracy mark awarded one
	Independent accuracy mark awarded zero
	Independent accuracy mark awarded one
	Independent accuracy mark awarded two
	Benefit of the doubt
	Blank Page
	Incorrect
Dep	Used to indicate DM0 or DM1

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Annotation	Meaning
DM1	Dependent on the previous M1 mark(s)
	Follow through
	Indicate working that is right or wrong
Highlighter	Highlight a key point in the working
	Ignore subsequent work
	Judgement
	Judgement
	Method mark awarded zero
	Method mark awarded one
	Method mark awarded two
	Misread
	Omission or Other solution
Off-page comment	Allows comments to be entered at the bottom of the RM marking window and then displayed when the associated question item is navigated to.
On-page comment	Allows comments to be entered in speech bubbles on the candidate response.
	Judgment made by the PE
	Premature approximation
	Special case
	Indicates that work/page has been seen

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Annotation	Meaning
	Error in number of significant figures
	Correct
	Transcription error
	Correct answer from incorrect working

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

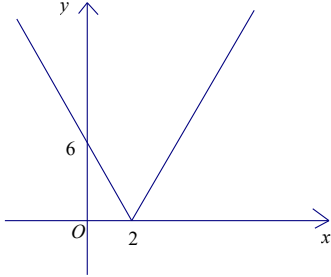
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

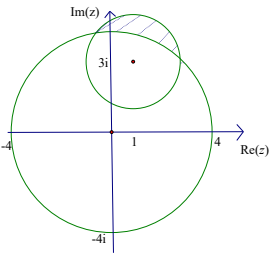
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)		B1	Straight lines. Symmetrical, and extending into the second quadrant. 2 and 6 marked correctly on the axes. Ignore $y = 3x - 6$ below the axis if intention is clear, e.g. dotted line or the required lines are clearly bolder. Ignore any attempt to sketch $y = 5x - 3$.
		1	
1(b)	Solve the linear equation $6 - 3x = 5x - 3$, or solve the quadratic equation $(5x - 3)^2 = (3x - 6)^2$	M1	Or corresponding inequality.
	Obtain critical value $x = \frac{9}{8}$	A1	Ignore $x = -\frac{3}{2}$ if seen.
	Obtain final answer $x < \frac{9}{8}$	A1	No other answer.
		3	

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Question	Answer	Marks	Guidance
2	Show a circle with centre $1+3i$ Some indication of scale seen or implied on both axes	B1	 If shaded region shown with no circles drawn, full marks available if correct with correct descriptions of the circles which formed it.
	Show a circle with correct radius 2, and centre not at the origin, so i.	B1	
	Show a circle or semi-circle in the first 2 quadrants, with radius 4 and centre at the origin	B1	
	Shade the correct region on a correct diagram	B1	
		4	

Question	Answer	Marks	Guidance
3(a)	Take logarithms of both sides to obtain $\ln A + \ln y = x \ln b$	M1	Or three-term equivalent with no indices.
	Compare with $Y = mX + c$, $pY + qX + r = 0$	A1	AG Condone use of $y = mx + c$ in comparison.
		2	
3(b)	Form equations in A and b and solve for A or b	M1	$\ln A + 0.86 = 3.4 \ln b$ and $\ln A + 2.56 = 5.7 \ln b$. Or $Ae^{0.86} = b^{3.4}$ and $Ae^{2.56} = b^{5.7}$.
	Obtain $b = 2.1$ or $A = 5.2$	A1	Allow for 2 sf or more.
	Obtain $b = 2.1$ and $A = 5.2$	A1	Accept $A = 5.3$.
		3	

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Question	Answer	Marks	Guidance
4	$\frac{d}{dx} x^2 \ln 2y = 2x \ln 2y + x^2 \times \frac{1}{y} \frac{dy}{dx}$	B1	SOI
	$\frac{d}{dx} y \ln(2 + x^2) = \frac{dy}{dx} \times \ln(2 + x^2) + \frac{2xy}{2 + x^2}$	B1	
	Equate derivative of right-hand side to zero and substitute $x = 2, y = 3$	M1	Must be using implicit differentiation and have attempted differentiation of products where appropriate. Need to see evidence of substitution.
	Obtain $4 \ln 6 + \frac{4}{3} \cdot \frac{dy}{dx} - \ln 6 \cdot \frac{dy}{dx} - 2 = 0$	A1	OE
	Simplify to obtain $\frac{6 - 12 \ln 6}{4 - 3 \ln 6}$ or $4 - \frac{10}{4 - 3 \ln 6}$	A1	OE Do not ISW. No fractions within fractions.
		5	

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Question	Answer	Marks	Guidance
5	Substitute $z = x + iy$ and z^* and expand	M1*	
	use $i^2 = -1$ twice	DM1	$x^2 + y^2 + 5ix - 5y + 2 - 10i = 0$
	Compare real and imaginary parts and equate to zero	M1	Must obtain at least one equation.
	Obtain $x^2 + y^2 - 5y + 2 = 0$ and $5(i)x - 10(i) = 0$	A1	OE
	Obtain $z = 2 + 3i$ and $z = 2 + 2i$	A1	
		5	

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Question	Answer	Marks	Guidance
6(a)	Use correct quotient rule (or correct product rule)	*M1	Allow one error.
	Obtain $\frac{dy}{dx} = \frac{(5 + 2 \sin x) \sec^2 x - \tan x \times 2 \cos x}{(5 + 2 \sin x)^2}$	A1	
	Express derivative in terms of $\sin x$ and $\cos x$, and simplify to an expression in $\sin x$ only	DM1	$\frac{dy}{dx} = \frac{(5 + 2 \sin x) \frac{1}{\cos^2 x} - \frac{\sin x}{\cos x} \times 2 \cos x}{(5 + 2 \sin x)^2}$
	$\frac{dy}{dx} = \frac{5 + 2 \sin^3 x}{(1 - \sin^2 x)(5 + 2 \sin x)^2}$	A1	OE without fractions within a fraction. Denominator must be in factorised form. ISW once correct form is seen.
	Alternative Method for Question 6(a)		
	Use correct quotient rule (or correct product rule)	*M1	Allow one error.
	Obtain $\frac{dy}{dx} = \frac{\cos x(5 + 2 \sin x) \cos x - \sin x(\cos x(2 \cos x) + (5 + 2 \sin x)(-\sin x))}{\cos^2 x(5 + 2 \sin x)^2}$	A1	
	Simplify to an expression in $\sin x$ only	DM1	
	$\frac{dy}{dx} = \frac{5 + 2 \sin^3 x}{(1 - \sin^2 x)(5 + 2 \sin x)^2}$	A1	OE without fractions within a fraction. Denominator must be in factorised form. ISW once correct form is seen.

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Question	Answer	Marks	Guidance
6(a)	Alternative Method 2 for Question 6(a)		
	Use correct quotient rule (or correct product rule)	*M1	Allow one error.
	$\frac{dy}{dx} = \frac{(5 \cos x + \sin 2x) \cos x - \sin x (-5 \sin x + 2 \cos 2x)}{\cos^2 x (5 + 2 \sin x)^2}$	A1	
	Simplify to an expression in $\sin x$ only	DM1	
	$\frac{dy}{dx} = \frac{5 + 2 \sin^3 x}{(1 - \sin^2 x)(5 + 2 \sin x)^2}$	A1	OE without fractions within a fraction. Denominator must be in factorised form. ISW once correct form is seen.
		4	
6(b)	Set numerator of <i>their</i> derivative equal to zero and attempt to solve for $\sin x$	M1	Must be a polynomial in $\sin x$.
	Show that there are no solutions and hence no stationary points.	A1	AG From correct working only using a correct numerator from part (a). Reference to $-1 \leq \sin x \leq 1$ OE must be made.
	Alternative Method for Question 6(b)		
	Use of $-1 \leq \sin x \leq 1$ leading to $3 \leq 5 + 2 \sin^3 x \leq 7$	M1	
	$\therefore 5 + 2 \sin^3 x \neq 0$	A1	OE
		2	

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Question	Answer	Marks	Guidance
7(a)	State or imply $\frac{du}{dx} = 2x$	B1	
	Obtain $x = \sqrt{7} \Rightarrow u = 4(=a)$ and $x = \sqrt{12} \Rightarrow u = 9(=b)$	B1	Allow if just written down.
	Use $\frac{du}{dx} = 2x$ and substitute u for x in the integral	M1	E.g., using $\int_{\sqrt{7}}^{\sqrt{12}} \frac{2x^2 \times 2x}{\sqrt{x^2 - 3}} dx$. Condone missing or incorrect limits.
	Obtain $\int_{\sqrt{7}}^{\sqrt{12}} \frac{4x^3}{\sqrt{x^2 - 3}} dx = \int_4^9 \frac{2(u+3)}{\sqrt{u}} du$ from full and correct working with sufficient detail	A1	AG Allow from working in reverse if fully correct.
		4	
7(b)	Integrate to obtain $cu^{\frac{3}{2}} + du^{\frac{1}{2}}$	*M1	May integrate by parts. Must be in terms of u as ‘Hence’.
	Obtain $\frac{4}{3}u^{\frac{3}{2}} + 12u^{\frac{1}{2}}$	A1	
	Use <i>their</i> limits correctly	DM1	For example: $\frac{4}{9}(27-8) + 12(3-2)$ or $72 - \frac{104}{3}$ or $88 - \frac{152}{3}$.
	Obtain $\frac{112}{3}$ or $37\frac{1}{3}$	A1	Exact answer only. Answer only is 0/4.
		4	

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Question	Answer	Marks	Guidance
8(a)	Expand $\sin\left(\theta + \frac{1}{6}\pi\right)$ and substitute trigonometric values	*M1	$\frac{\sqrt{3}}{2}\sin\theta + \frac{1}{2}\cos\theta$
	Obtain $\frac{5}{2}\sin\theta + \frac{3\sqrt{3}}{2}\cos\theta$	A1	
	Obtain $R = \sqrt{13}$	B1	From $\frac{1}{2}\sqrt{25+27}$.
	Correct use of trig to obtain α	DM1	E.g. $\cos\alpha = \frac{5}{2\sqrt{13}}$, $\sin\alpha = \frac{3\sqrt{3}}{2\sqrt{13}}$ or $\tan\alpha = \frac{3\sqrt{3}}{5}$. M0 if it comes from $\sin\alpha = \frac{3\sqrt{3}}{2}$, $\cos\alpha = \frac{5}{2}$ or $\frac{9}{2}\sin\theta + \frac{3\sqrt{3}}{2}\cos\theta$.
	$\alpha = 0.805$	A1	
		5	
8(b)	$2x + \alpha = \sin^{-1} \frac{\sqrt{6}}{R} \left(= \sin^{-1} \frac{\sqrt{6}}{\sqrt{13}} = 0.747 \right)$	B1FT	Follow <i>their</i> R (and α). Accept AWRT 0.747 (0.746898....).
	Solve for x to get an answer in the range	M1	$2x + 0.805 = 0.747, 2.395, 7.030$
	Obtain 0.795 or 3.11	A1	Accept AWRT 0.795 or 3.11 from correct work.
	Obtain 0.795 and 3.11 and no others in range	A1	Accept AWRT 0.795 or 3.11 from correct work.
		4	

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Question	Answer	Marks	Guidance
9(a)	Calculate the scalar product of the direction vectors to form an equation in a .	M1	$2+6-3a=0$
	$a = \frac{8}{3}$	A1	Exact answer only, but ISW if go on to give a decimal.
		2	
9(b)	Express general point of a line in component form	B1	$\begin{pmatrix} 2+\lambda \\ 1+2\lambda \\ 4-3\lambda \end{pmatrix}$ or $\begin{pmatrix} 3+2\mu \\ -1+3\mu \\ 5+a\mu \end{pmatrix}$
	Equate i and j components of l_2 and l_1 and solve for λ or μ	M1	
	Obtain $\lambda = -7, \mu = -4$	A1	SOI
	Substitute in the equation for the k components and obtain $a = -5$	A1	
		4	

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Question	Answer	Marks	Guidance
9(c)	Use the scalar product to obtain an expression for the cosine of the angle	*M1	$\cos \theta = \frac{2+6-3a}{\sqrt{14}\sqrt{13+a^2}}$
	Form a quadratic equation in a	DM1	E.g. $14(8-3a) = 5\sqrt{14}\sqrt{13+a^2}$.
	Obtain $101a^2 - 672a + 571 = 0$	A1	Or three-term equivalent, e.g. $1414a^2 - 9408a + 7994 = 0$.
	Obtain $a = 1, a = \frac{571}{101}$	A1	Accept 1 and 5.65. ISW once correct values seen. SC B1 if $a = 1$ spotted from $14(8-3a) = 5\sqrt{14}\sqrt{13+a^2}$ OE, 3/4 max.
		4	

Question	Answer	Marks	Guidance
10(a)	Use integration by parts to obtain $pxe^{\frac{1}{2}x} + q \int e^{\frac{1}{2}x} dx$	*M1	
	Obtain $2xe^{\frac{1}{2}x} - 2 \int e^{\frac{1}{2}x} dx$	A1	
	Complete the integration to obtain $2xe^{\frac{1}{2}x} - 4e^{\frac{1}{2}x}$	A1	
	Use limits 0 and a correctly and equate the answer to 6	DM1	$2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a}(-0) + 4 = 6$
	Obtain $a = 2 + e^{-\frac{1}{2}a}$ from full and correct working	A1	AG
		5	

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Question	Answer	Marks	Guidance
10(b)	Calculate the value of a relevant expression or values of a relevant pair of expressions at 2.2 and 2.4. Values to at least 2 sf. Need all relevant values but only one (pair) needs to be correct to score M1. Complete set of values for <i>their</i> expression. If not comparing with 0, then the comparison must be clear.	M1	Example 1 $2.2 < 2.33$ and $2.4 > 2.30$ Example 2 $f(a) = a - 2 - e^{-\frac{1}{2}a}$ OE $f(2.2) = -0.132...$ $f(2.4) = 0.0988...$ Example 3 $f(a) = 2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a} + 4$ $f(2.2) = 5.20...$ $f(2.4) = 6.65...$ Example 4 $f(a) = 2ae^{\frac{1}{2}a} - 4e^{\frac{1}{2}a} - 2$ $f(2.2) = -0.80...$ $f(2.4) = 0.65...$
	Complete the argument correctly with correct calculated values. Accept truncated values. Allow work on a smaller interval.	A1	Example 1 A clear explanation is needed. Example 2 $f(2.2) = -0.132... < 0$ $f(2.4) = 0.0988... > 0$ OE Example 3 $f(2.2) = 5.20... < 6$ $f(2.4) = 6.65... > 6$ Example 4 $f(2.2) = -0.80... < 0$ $f(2.4) = 0.65... > 0$
		2	

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Question	Answer	Marks	Guidance
10(c)	Use iterative process correctly at least once	M1	
	Obtain final answer 2.31	A1	
	Show sufficient iterations to at least 4 dp to justify 2.31 to 2 dp	A1	Allow recovery. Allow truncation. E.g. 2.3, 2.3166, 2.3140, 2.3144.
		3	

Question	Answer	Marks	Guidance
11(a)	Obtain $\frac{dx}{dt} = kx(1-x)$, stating where the factors come from	B1	AG Factor $1-x$ explained.
		1	

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Question	Answer	Marks	Guidance
11(b)	Separate variables correctly	B1	$\int \frac{1}{x(1-x)} dx = \int k dt$ Condone missing integral signs or dx and dt but not both.
	Correct method to express $\frac{1}{x(1-x)}$ in partial fractions	M1	Condone a sign error between the fractions.
	Obtain $\frac{1}{x} + \frac{1}{1-x}$	A1	
	Obtain general solution $\ln x - \ln(1-x) = kt + A$	A1	OE
	Use $t = 0, x = \frac{1}{4}$ in an expression containing $pt, q \ln x$ and $q \ln(1-x)$ to obtain the constant of integration	*M1	$A = \ln \frac{1}{3}$
	Use $t = 2, x = \frac{1}{3}$ in an expression containing $pt, q \ln x$ and $q \ln(1-x)$ to calculate the value of k	DM1	$k = \frac{1}{2} \ln \frac{3}{2}$
	Obtain $\ln \frac{x}{1-x} = \frac{1}{2} \ln \frac{3}{2} t + \ln \frac{1}{3}$	A1	Must be seen, as part of question demand. OE, e.g. $\ln \frac{x}{1-x} = (0.2027...)t - 1.0986...$
	Substitute $x = 0.75$ and obtain $t = 11$ years (to the nearest year).	A1	10.838... may be seen.
		8	SC M1DM1A0A1 for candidates who use $t = 1$ <u>and</u> $t = 3$ rather than $t = 0$ and $t = 2$ to obtain $t = 12$ (11.838...) years (to the nearest year).