



Cambridge International AS & A Level

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MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2024**1 hour 50 minutes**

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.





- Find the values of a and b .

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- [illegible]



[4]

This image shows a full page of primary-ruled paper. It features approximately 20 horizontal dotted lines spaced evenly down the page, providing a guide for handwriting practice. The paper is otherwise blank, with no margins, text, or other markings.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



11

to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.



[4]

[illegible]

Let $f(x) = \frac{5x^2 + 8x + 5}{(1 + 2x)(2 + x^2)}$.

[5]

[illegible]



[4]

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- [illegible]

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- (c) On a single Argand diagram, sketch the loci given by the equations $\text{Re}(z) = 1$ and $\left|z - \frac{1}{2}\right| = \frac{1}{2}$, where z is a complex number. [3]

- (d) The complex number z is such that $\text{Re}(z) = 1$. Use your answer to part (b) to give a geometrical description of the locus of $\frac{1}{z}$. [1]

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- 9 The position vector of point A relative to the origin O is $\overrightarrow{OA} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k}$.
The line l passes through A and is parallel to the vector $2\mathbf{i} + \mathbf{j} + 4\mathbf{k}$.

(a) State a vector equation for l .

[2]

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- (b) The position vector of point B relative to the origin O is $\overrightarrow{OB} = -t\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$, where t is a constant.
The line l also passes through B .

Find the value of t .

[3]

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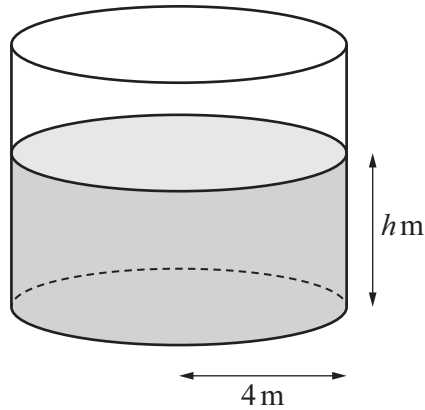




- Find the possible values of a .

[5]

[illegible]



A large cylindrical tank is used to store water. The base of the tank is a circle of radius 4 metres. At time t minutes, the depth of the water in the tank is h metres. There is a tap at the bottom of the tank. When the tap is open, water flows out of the tank at a rate proportional to the square root of the volume of water in the tank.

- (a) Show that $\frac{dh}{dt} = -\lambda\sqrt{h}$, where λ is a positive constant. [4]

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[6]

[illegible]

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MATHEMATICS**9709/32**

Paper 3 Pure Mathematics 3

October/November 2024**1 hour 50 minutes**

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[2]

$$x_{n+1} = \frac{1}{2} \tan^{-1}(\cos x_n)$$

[1]

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[3]

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5 (a) The complex number u is given by

$$u = \frac{(\cos \frac{1}{7}\pi + i \sin \frac{1}{7}\pi)^4}{\cos \frac{1}{7}\pi - i \sin \frac{1}{7}\pi}.$$

Find the exact value of $\arg u$.

[2]

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(b) The complex numbers u and u^* are plotted on an Argand diagram.

Describe the single geometrical transformation that maps u onto u^* and state the exact value of $\arg u^*$.

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[illegible]

[illegible]

$$x = \tan^2 2t, \quad y = \cos 2t,$$

(a) Show that $\frac{dy}{dx} = -\frac{1}{2}\cos^3 2t$.

[4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.



(b) Hence find the equation of the normal to the curve at the point where $t = \frac{1}{8}\pi$. Give your answer in the form $y = mx + c$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.



9 With respect to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 0 \\ 4 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} -3 \\ -2 \\ 2 \end{pmatrix}.$$

(a) The point D is such that $ABCD$ is a trapezium with $\overrightarrow{DC} = 3\overrightarrow{AB}$.

Find the position vector of D .

[2]

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(b) The diagonals of the trapezium intersect at the point P .

Find the position vector of P .

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(c) Using a scalar product, calculate angle ABC . [4]

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- 10 A balloon in the shape of a sphere has volume V and radius r . Air is pumped into the balloon at a constant rate of 40π starting when time $t = 0$ and $r = 0$. At the same time, air begins to flow out of the balloon at a rate of $0.8\pi r$. The balloon remains a sphere at all times.

(a) Show that r and t satisfy the differential equation

$$\frac{dr}{dt} = \frac{50-r}{5r^2}. \quad [3]$$

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(b) Find the quotient and remainder when $5r^2$ is divided by $50 - r$. [3]

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[6]

[illegible]

[1]

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(a) Find $f'(x)$ and hence find the exact coordinates of the stationary point of the curve with equation $y = f(x)$. [5]

[illegible]



[illegible]

If you use the following page to complete the answer to any question, the question number must be clearly shown.

[illegible]



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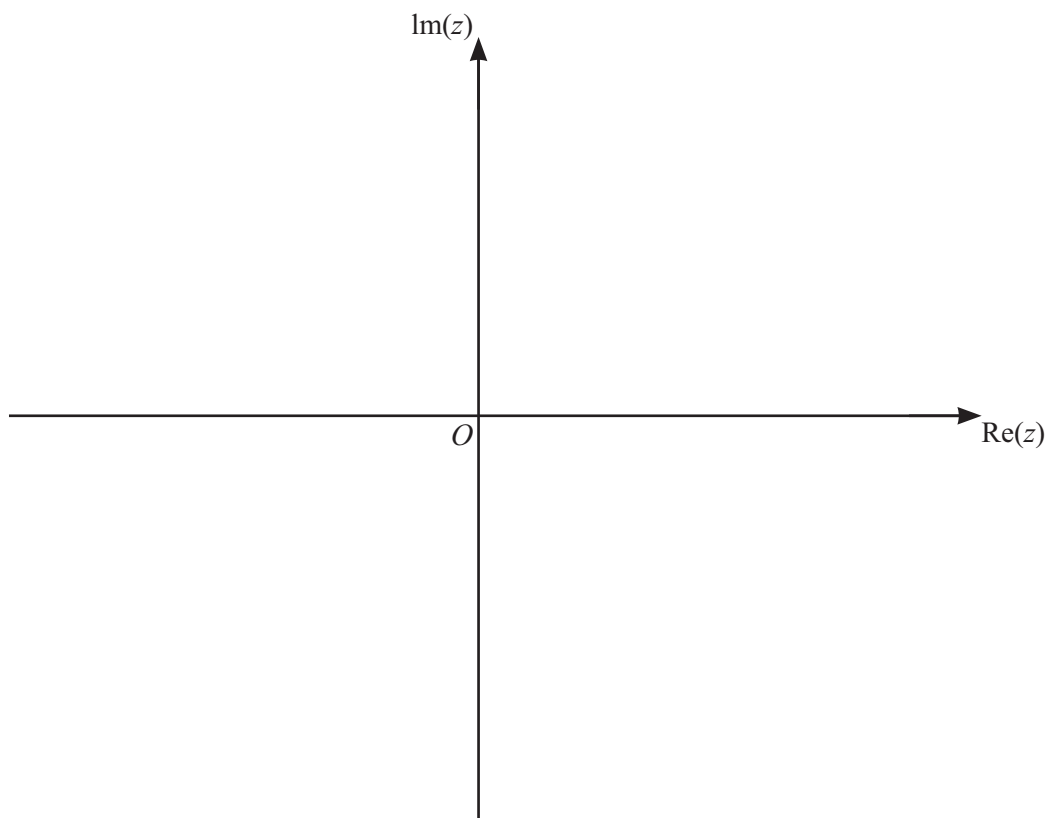
1 The complex number z satisfies $|z| = 2$ and $0 \leq \arg z \leq \frac{1}{4}\pi$.

(a) On the Argand diagram below, sketch the locus of the points representing z .

[2]

(b) On the **same diagram**, sketch the locus of the points representing z^2 .

[2]



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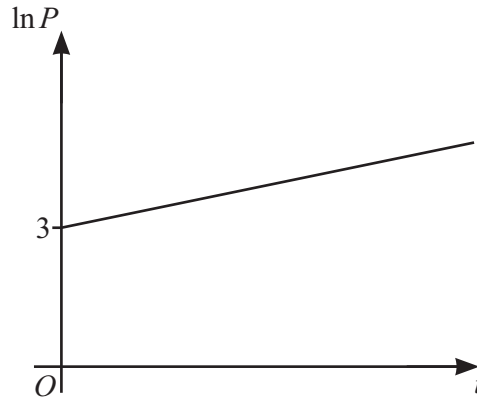
- $$x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$$

This image shows a blank sheet of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page. There are no margins, text, or other markings on the paper.

- [illegible]



3



The number of bacteria in a population, P , at time t hours is modelled by the equation $P = ae^{kt}$, where a and k are constants. The graph of $\ln P$ against t , shown in the diagram, has gradient $\frac{1}{20}$ and intersects the vertical axis at $(0, 3)$.

- (a) State the value of k and find the value of a correct to 2 significant figures. [3]

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- (b) Find the time taken for P to double. Give your answer correct to the nearest hour. [2]

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$$\frac{z-3i}{z+3i} = \frac{2-9i}{5}.$$

[5]

[illegible]

[illegible]

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$$l: \mathbf{r} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + 2\mathbf{k}) \quad \text{and} \quad m: \mathbf{r} = 2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + 5\mathbf{k}).$$

(a) State the coordinates of P .

[1]

[illegible]

[3]

This image shows a full page of white paper with horizontal blue or grey ruling lines. The lines are evenly spaced and run across the width of the page, providing a template for handwriting practice. There are no margins, text, or other markings on the page.



[3]

[illegible]

$$x = 3 \sin 2t, \quad y = \tan t + \cot t,$$

for $0 < t < \frac{1}{2}\pi$.

(a) Show that $\frac{dy}{dx} = \frac{-2}{3 \sin^2 2t}$.

[5]

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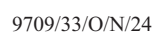
(a) Express $f(x)$ in partial fractions.

[3]

This image shows a full page of a document template designed for handwriting practice or general note-taking. It consists of approximately 28 evenly spaced horizontal dotted lines across the entire width of the page. The background is plain white, and there are no margins, headers, footers, or other markings present.

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

[illegible]

[illegible]



[illegible]

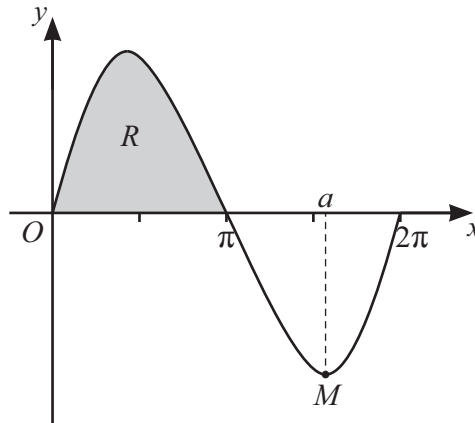
- (a) Show that $200 \frac{dh}{dt} = 250 - 3h$. [3]

[illegible]



[5]

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- (a) Find the value of a correct to 2 decimal places.

[6]

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