

Cambridge International A Level

MATHEMATICS**9709/31**

Paper 3 Pure Mathematics 3

October/November 2024**MARK SCHEME**Maximum Mark: 75

Published

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This document consists of **15** printed pages.

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GENERIC MARKING PRINCIPLE 1:

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- the specific skills defined in the mark scheme or in the generic level descriptors for the question
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GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
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- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
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- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
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 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

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CWO	Correct Working Only
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SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Substitute $x = -\frac{1}{2}$ and equate the result to zero	M1	
	Obtain a correct equation, e.g. $-\frac{4}{8} + \frac{a}{4} - \frac{5}{2} + b = 0$	A1	$(\frac{a}{4} + b = 3)$ Any equivalent form.
	Substitute $x = 2$ and $x = 4$ and use $p(4) = 3p(2)$	M1	If using long division, M1 is for correct use of two constant remainders. Condone if 3 is on the wrong side.
	Obtain a correct equation, e.g. $3(32 + 4a + 10 + b) = 256 + 16a + 20 + b$	A1	$(-2a + b = 75)$ Any equivalent form.
	Obtain $a = -32$ and $b = 11$	A1	
		5	

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Question	Answer	Marks	Guidance
2	Integrate to obtain $px^3 \ln 3x + q \int x^2 dx$	M1*	
	Obtain $\frac{1}{3}x^3 \ln 3x - \frac{1}{3} \int x^2 dx$	A1	Or unsimplified equivalent.
	Complete integration to obtain $\frac{1}{3}x^3 \ln 3x - \frac{1}{9}x^3$	A1	
	Use correct limits correctly in an expression of the form $rx^3 \ln 3x + sx^3$	DM1	$9 \ln 9 - 3 - \frac{1}{3} \ln 3 + \frac{1}{9}$ An exact expression for <i>their</i> integral.
	Obtain $\frac{53}{3} \ln 3 - \frac{26}{9}$	A1	Or 2-term equivalent.
		5	

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Question	Answer	Marks	Guidance
3	State or imply $\frac{1 + \frac{dy}{dx}}{x + y}$ as the derivative of $\ln(x + y)$	B1	
	State or imply $6xy + 3x^2 \frac{dy}{dx}$ as the derivative of $3x^2y$	B1	
	Substitute $(1, 0)$ and solve for $\frac{dy}{dx}$	M1	Having the correct form for at least one of the above.
	Obtain $\frac{dy}{dx} = \frac{1}{2}$ or 0.5	A1	
	Alternative Method for Question 3:		
	Rewrite as $x + y = e^{3x^2y}$ and state or imply $1 + \frac{dy}{dx}$ as the derivative of the LHS	B1	
	State or imply $\left(6xy + 3x^2 \frac{dy}{dx}\right)e^{3x^2y}$ as the derivative of e^{3x^2y}	B1	
	Substitute $(1, 0)$ and solve for $\frac{dy}{dx}$	M1	Having the correct form for at least one of the above.
	Obtain $\frac{dy}{dx} = \frac{1}{2}$ or 0.5	A1	
		4	

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Question	Answer	Marks	Guidance
4(a)	Factorise Or obtain an expression in $\cos^2 \theta$ or $\sec^2 \theta$	M1	$(\sec^2 \theta - \tan^2 \theta)(\sec^2 \theta + \tan^2 \theta)$ Or $-1 + \frac{2}{\cos^2 \theta}$, OE.
	Use of $1 + \tan^2 \theta = \sec^2 \theta$ (anywhere)	M1	The 2 method marks can appear in either order, in which case the 2 nd M1 is for expanding $(1 + \tan^2 \theta)^2 - \tan^4 \theta$.
	Obtain $1 \times (1 + \tan^2 \theta + \tan^2 \theta) = 1 + 2 \tan^2 \theta$	A1	Obtain given answer from full and correct working.
		3	
4(b)	Form an equation in $\tan 2\alpha$. Or multiply through by $\cos^4 2\alpha$ to form an equation in $\sin 2\alpha$ or $\cos 2\alpha$	M1	$1 + 2 \tan^2 2\alpha = 2 \tan^2 2\alpha (1 + \tan^2 2\alpha)$ $\cos^4 2\alpha + 2 \sin^2 2\alpha \cos^2 2\alpha = 2 \sin^2 2\alpha$ $\Rightarrow \sin^4 2\alpha + 2 \sin^2 2\alpha - 1 = 0$ or $\cos^4 2\alpha - 4 \cos^2 2\alpha + 2 = 0$
	Solve for $\tan 2\alpha$ or equivalent	M1	$\left(\tan 2\alpha = \pm \sqrt{\frac{1}{\sqrt{2}}} \right)$
	Obtain one correct solution for α , e.g. $20.0(30..)^{\circ}$	A1	
	Obtain a second correct value for α , e.g. 70.0° ($69.9698..^{\circ}$)	A1	
	Obtain solutions 110° (110.0) and 160° (160.0) for α , and no others in range	A1	
		5	

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Question	Answer	Marks	Guidance
5(a)	Sketch a relevant graph, e.g. $y = 2 + e^{-0.2x}$	B1	
	For the sketches: Correct curvature Intersections with the y -axis approximately correct Horizontal asymptote approximately correct – need not draw in Allow scale not marked and implied by their sketch		
	Sketch a second relevant graph, e.g. $y = \ln(1+x)$ and justify the given statement	B1	
		2	
5(b)	Calculate the value of a relevant expression or values of a relevant pair of expressions at $x = 7$ and $x = 9$	M1	
	Complete the argument correctly with correct calculated values	A1	E.g. $2.079 < 2.246$ and $2.302 > 2.165$, or $0.167 > 0$ and $-0.137 < 0$.
		2	
5(c)	Use the iterative process correctly at least once	M1	I.e., obtain one value and substitute that value back into the formula.
	Obtain final answer 8.03	A1	
	Show sufficient iterations to at least 4 decimal places to justify 8.03 to 2 decimal places, or show that there is a sign change in the interval (8.025, 8.035)	A1	E.g. 7, 8.4555, 7.8846, 8.0849, 8.0115, 8.0380, 8.0283, 8.0318 8, 8.0421, 8.0268, 8.0324, 9, 7.7172, 8.1490, 7.9887, 8.0463, 8.0253, 8.0329
		3	

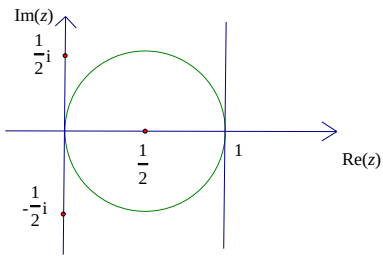
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Question	Answer	Marks	Guidance
6(a)	Correct use of product rule to differentiate	*M1	$2\cos 2x(1 + \sin 2x) + \sin 2x \times 2\cos 2x$, or $2\cos^2 x - 2\sin^2 x + 8\sin x \cos^3 x - 8\cos x \sin^3 x$. All terms needed but could have errors in the coefficients.
	Obtain $2\cos 2x + 4\cos 2x \sin 2x$	A1	OE
	Equate derivative to zero and solve for $2x$ or x	DM1	$2\cos 2x(1 + 2\sin 2x) = 0 \Rightarrow x = \frac{1}{2}\sin^{-1}\left(-\frac{1}{2}\right)$ Condone if they only consider $\cos 2x = 0$.
	Obtain $x = \frac{7}{12}\pi, y = -\frac{1}{4}$	A1	Mark degrees as a misread. The Q asks for an exact answer.
		4	
6(b)	Use correct double angle formula to integrate Or use integration by parts and correct double angle formula	M1	$\int \sin 2x + \frac{1 - \cos 4x}{2} dx$ Or $-\frac{1}{2}\cos 2x(1 + \sin 2x) + \int \frac{1 + \cos 4x}{2} dx$.
	Obtain $-\frac{1}{2}\cos 2x + \frac{x}{2} - \frac{1}{8}\sin 4x (+C)$	A1	Or $-\frac{1}{2}\cos 2x - \frac{1}{2}\cos 2x \sin 2x + \frac{x}{2} + \frac{1}{8}\sin 4x (+C)$
	Use limits 0 and $\frac{1}{2}\pi$ correctly in a solution containing $p\cos 2x$ and $q\sin 4x$	M1	$\frac{1}{2} + \frac{1}{4}\pi - 0 + \frac{1}{2} - 0 + 0$
	Obtain $\frac{1}{4}\pi + 1$	A1	
		4	

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Question	Answer	Marks	Guidance
7(a)	State or imply the form $\frac{A}{1+2x} + \frac{Bx+C}{2+x^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A=1$, $B=2$ and $C=3$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
7(b)	State $\frac{-1.-2.-3}{3!}(2x)^3$ or -8	B1 FT	Correct term in x^3 or coefficient of x^3 in the expansion of $A(1+2x)^{-1}$. Any equivalent form.
	Use a correct method to obtain the coefficient of x^2 in the expansion of $(2+x^2)^{-1}$ or the coefficient of x^2 in the expansion of $(1+\frac{x^2}{2})^{-1}$.	M1	Do not need to deal with 2^{-1} at this stage.
	Obtain $(Bx+C) \times \frac{1}{2} \times -\frac{1}{2}x^2$ or $-\frac{B}{4}x^3$ or $-\frac{B}{4}$	A1 FT	Follow <i>their</i> B (and C).
	Obtain final answer $-8\frac{1}{2}$ or $-8\frac{1}{2}x^3$	A1	Or simplified equivalent. Ignore additional terms for other powers of x .
		4	

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Question	Answer	Marks	Guidance
8(a)	Multiply numerator and denominator by $1 - yi$	M1	OE
	Obtain $\frac{1}{1+y^2} + \frac{-y}{1+y^2}i$	A1	OE
		2	
8(b)	Express $\left(a - \frac{1}{2}\right)^2 + b^2$ in terms of y and expand the bracket	M1	$\left(\frac{1}{1+y^2} - \frac{1}{2}\right)^2 + \left(\frac{(-)y}{1+y^2}\right)^2$
	Obtain $\left(\frac{1}{(1+y^2)^2} - \frac{2}{2(1+y^2)} + \frac{1}{4}\right) + \frac{y^2}{(1+y^2)^2}$	A1FT	Follow <i>their</i> answer from (a) provided it gives an expression in y .
	Obtain $\frac{1}{4}$ from full and correct working	A1	AG
		3	
8(c)	Show a vertical straight line through $1 + 0i$	B1	
	Show a circle centre $\frac{1}{2} + 0i$	B1	
	Show a circle with radius $\frac{1}{2}$ and centre not at the origin	B1	
		3	
8(d)	circle centre $\frac{1}{2} + 0i$ with radius $\frac{1}{2}$	B1	OE Condone inclusion of the origin.
		1	

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Question	Answer	Marks	Guidance
9(a)	Use a correct method to form a vector equation	M1	Allow in column vectors.
	Obtain $\mathbf{r} = 8\mathbf{i} - 5\mathbf{j} + 6\mathbf{k} + \lambda(2\mathbf{i} + \mathbf{j} + 4\mathbf{k})$	A1	Need $\mathbf{r} = \dots$
		2	
9(b)	State the position vector of a point on l in component form Or at least 2 correct components seen	B1 FT	Follow <i>their</i> equation $(8 + 2\lambda)\mathbf{i} + (-5 + \lambda)\mathbf{j} + (6 + 4\lambda)\mathbf{k}$. Might see the correct equation for the first time in (b) .
	Equate to $-\mathbf{i} + 4t\mathbf{j} + 3t\mathbf{k}$ and solve for t	M1	
	Obtain $t = -2$	A1	
		3	
9(c)	Evaluate the scalar product of a pair of relevant vectors	M1	$(2\mathbf{i} + \mathbf{j} + 4\mathbf{k}) \cdot (a\mathbf{i} - \mathbf{j} + 3\mathbf{k}) = 2a + 11$ OE, SOI
	Complete the process for finding the cosine of θ	*M1	Divide the scalar product by the product of the moduli and equate to $\cos \theta$.
	Obtain $\frac{2a + 11}{\sqrt{21}\sqrt{10 + a^2}} = \pm \frac{1}{\sqrt{6}}$	A1	OE
	Form a 3-term quadratic equation in a and solve for a	DM1	$a^2 + 88a + 172 = 0$ OE
	Obtain $a = -2, a = -86$	A1	Correct only (both values).
		5	

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Question	Answer	Marks	Guidance
10(a)	$\frac{dV}{dt} = \pm k\sqrt{V}$ or $\frac{dV}{dt} = 16\pi \frac{dh}{dt}$	B1	SOI
	Correct use of chain rule and $V = 16\pi h$	M1	OE, e.g. $\frac{dV}{dt} = 16\pi \frac{dh}{dt}$.
	Obtain $\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} = \frac{-k\sqrt{16\pi h}}{16\pi}$	A1	Any equivalent form in terms of h .
	$= -\left(\frac{k}{4\sqrt{\pi}}\right)\sqrt{h} = -\lambda\sqrt{h}$ since $\frac{k}{4\sqrt{\pi}}$ is constant	A1	Obtain given answer from full and correct working.
		4	
10(b)	Separate variables correctly and commence integration	*M1	$\int \frac{1}{\sqrt{h}} dh = \int -\lambda dt$ OE
	Obtain $-\lambda t = 2\sqrt{h} (+C)$	A1	
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form an equation in λ and/or C	DM1	OE, e.g. $0 = 4 + C$ or $-20\lambda = 2 \times 1.5 + C$.
	Use the boundary conditions in an equation containing pt and $q\sqrt{h}$ to form a second equation in λ and/or C and solve	DM1	$C = -4, \lambda = \frac{1}{20}$ OE, e.g. $-\frac{t}{20} = 2\sqrt{h} - 4$.
	Hence $t = 80 - 40\sqrt{h}$	A1	Must be seen.
	Time to empty the tank is 80 minutes	A1	
		6	

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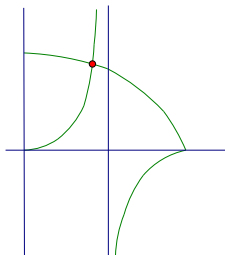
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1	Obtain a correct unsimplified version of the x or x^2 term of the expansion of $(9 - 3x)^{\frac{1}{2}}$ or $\left(1 - \frac{1}{3}x\right)^{\frac{1}{2}}$	M1	E.g. $-\frac{1}{2} \times \frac{1}{3}x$ or $-\frac{\frac{1}{2} \times \frac{1}{2}}{2} \times \frac{1}{9}x^2$ or $\frac{1}{2}9^{\frac{-1}{2}}(-3x)^1$ or $\frac{\frac{1}{2} \times \frac{-1}{2}}{2}9^{\frac{-3}{2}}(-3x)^2$. Not for symbolic coefficients in the form ${}^nC_r$.
	State correct first term 3	B1	
	Obtain the next two terms $-\frac{1}{2}x - \frac{1}{24}x^2$	A1 A1	A1 for each term correct. Do not ISW.
			SC M1A1 for $1 - \frac{1}{6}x - \frac{1}{72}x^2$ seen on its own or as a factor.
		4	

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Question	Answer	Marks	Guidance
2(a)	Sketch a relevant graph, e.g. $y = \cot 2x$	B1	Alt: use $\tan 2x$ and $\cos x$. And only one root in range. (also cross at $\frac{\pi}{2}$) 
	Sketch a second relevant graph on the same axes, e.g. $y = \sec x$ and justify the given statement	B1	Need to mark intersection with a dot, a cross, or say roots at points of intersection, OE.
		2	
2(b)	State $x = \frac{1}{2} \tan^{-1}(\cos x)$ and rearrange to the given equation $\cot 2x = \sec x$ Note: If using the alternative approach in (a) , can stop at $\tan 2x = \cos x$	B1	Should see $\tan 2x = \cos x$ before the given conclusion. Or rearrange $\cot 2x = \sec x$ to $x = \frac{1}{2} \tan^{-1}(\cos x)$ and state iterative formula $x_{n+1} = \frac{1}{2} \tan^{-1}(\cos x_n)$.
		1	

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Question	Answer	Marks	Guidance
3	Square $x + iy$ and equate real and imaginary parts to 6 and -8 respectively	*M1	Condone +8 in place of -8 and/or $i^2 = 1$.
	Obtain equations $x^2 - y^2 = 6$ and $2xy = -8$	A1	OE
	Eliminate one variable and find an equation in the other (from 2 equations each in 2 unknowns)	DM1	Condone a slip but not seriously incorrect algebra, e.g. use of $x = -4y$ is M0.
	Obtain $x^4 - 6x^2 - 16 = 0$ or $y^4 + 6y^2 - 16 = 0$	A1	Accept 3-term equivalents e.g. $x^4 = 6x^2 + 16$. Condone missing '= 0' if implied by subsequent work.
	Obtain answers $\pm(2\sqrt{2} - \sqrt{2}i)$ or exact equivalents	A1	Allow if values of x and y stated separately but the pairing is clear. Ignore additional correct solutions for x and y not real, but A0 if any additional incorrect answers.
		5	

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Question	Answer	Marks	Guidance
4	Use laws of indices correctly and solve for 5^x	*M1	E.g. obtain $5^x = \frac{5}{12}$ OE. Allow for $y = \dots$ if they have previously stated $y = 5^x$. Could be implied if they have a correct simplified equation in 5^x , e.g. $12 \times 5^x = 5$.
	Use a correct method for solving an equation of the form $5^x = a$, where $a > 0$	DM1	Allow $x \ln 5 = \ln\left(\frac{10}{24}\right)$.
	Obtain answer -0.544	A1	CWO. If no working shown, 0/3. Note: 3 dp required.
		3	
5(a)	$\frac{4\pi}{7}$ and/or $-\frac{\pi}{7}$	M1	SOI Allow $\frac{4}{7}\pi - \left(-\frac{1}{7}\pi\right)$ or $\frac{4}{7}\pi - \frac{1}{7}\pi$ Note: Many multiply top and bottom by the conjugate, which is fine, but to score the M1 they need to state or imply the argument of a complex number.
	Obtain $\arg u = \frac{5}{7}\pi$	A1	Do not accept degrees.
		2	

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Question	Answer	Marks	Guidance
5(b)	Reflection (in the) real axis	B1	Correct non-contradictory statement. Condone x -axis or horizontal axis. Need 'reflection'. Not 'mirror', 'flip'.
	$\arg u^* = -\frac{5}{7}\pi$	B1FT	FT <i>their</i> exact (a) . Accept $2\pi -$ <i>their</i> exact (a) . Accept an 'exact' expression in place of an exact value. Need to see a value or an expression. Do not accept $\arg u^* = -\arg u$ without a value seen.
		2	
6	Form a pair of equations in a and b	*M1	Condone sign slips but must be using the given coordinates correctly. e.g. $\begin{cases} \ln a + 8.27 = 3.4 \ln b \\ \ln a + 2.24 = 0.5 \ln b \end{cases}$ or $\begin{cases} ae^{2.24} = b^{0.5} \\ ae^{8.27} = b^{3.4} \end{cases}$
	Carry out a correct method for finding $\ln a$ or $\ln b$ or a or b	DM1	Condone sign slip.
	Obtain value $a = 0.3$	A1	(0.30109...)
	Obtain value $b = 8$	A1	(7.99895...) Allow A0A1 if both values 'correct' but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.

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Question	Answer	Marks	Guidance
6	Alternative Method for Question 6:		
	Carry out a correct method for finding $\ln b$ or b (Need to link the gradient to $\ln b$ at some point)	*M1	Condone sign slips but must be using the given coordinates correctly. $\ln b = \frac{8.27 - 2.24}{3.4 - 0.5} (= 2.079\dots)$
	Obtain value $b = 8$	A1	
	Correct method to find $\pm \ln a$ or a	DM1	Condone sign slip ($\ln a = -1.200\dots$).
	Obtain value $a = 0.3$	A1	Allow A0A1 if both values ‘correct’ but not rounded to 1 sf. Allow 4/4 for $0.3y = 8^x$ with correct working shown.
		4	

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Question	Answer	Marks	Guidance
7(a)	Use correct double angle formula to obtain an equation in $\tan x$	M1	e.g. $\tan^3 x + \frac{2 \times 2 \tan x}{1 - \tan^2 x} - \tan x (= 0)$. Allow if the correct formula is quoted but then they lose the 2 from the numerator when they use the formula.
	Obtain a correct equation in $\tan x$ in any form without fractions	A1	E.g. $\tan^3 x - \tan^5 x + 4 \tan x - \tan x + \tan^3 x (= 0)$. Condone if ‘= 0’ is missing here.
	Reduce to the given answer of $\tan^4 x - 2 \tan^2 x - 3 = 0$ correctly	A1	Obtain given answer from correct working but condone if never mention $\tan x \neq 0$. Condone the right terms in a different order ‘Show that’ so each line must be correct.
		3	
7(b)	A complete correct method to solve the equation to obtain a value for θ	M1	$(\tan 2\theta = \pm\sqrt{3})$ Allow if they make a slip in copying the equation but do have a complete method to obtain a value of θ . M0 if they get a value for 2θ but never halve it.
	Obtain two of $(\theta =) \frac{1}{6}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$ and $\frac{5}{6}\pi$	A1	
	Obtain the other two of $(\theta =) \frac{1}{6}\pi, \frac{1}{3}\pi, \frac{2}{3}\pi$ and $\frac{5}{6}\pi$ and no others in the interval	A1	Exact, ignore any answers outside interval Accept $\frac{2}{6}\pi$ for $\frac{1}{3}\pi$ and $\frac{4}{6}\pi$ for $\frac{2}{3}\pi$. Do not need to see $\theta = \frac{\pi}{2}$ (from $\tan 2\theta = 0$).
		3	

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Question	Answer	Marks	Guidance
8(a)	Use correct product rule or chain rule to find derivative of x with respect to t	M1	Obtain $k \tan 2t \sec^2 2t$.
	Obtain $\frac{dx}{dt} = 4 \tan 2t \sec^2 2t$ oe	A1	
	$\frac{dy}{dt} = -2 \sin 2t$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$ to obtain given answer $\frac{dy}{dx} = -\frac{1}{2} \cos^3 2t$	B1	Condone if $\frac{dy}{dx}$ missing.
		4	
8(b)	Obtain $x = 1$ and $y = \frac{\sqrt{2}}{2}$	B1	Accept $y = 0.707 \dots$
	State or imply gradient of tangent is $\frac{-\sqrt{2}}{8}$ or gradient of normal is $4\sqrt{2}$	B1	Any equivalent form, e.g. $2^{\frac{5}{2}}$. Accept -0.177 or 5.66 .
	Use correct method to find equation of normal using <i>their</i> values	M1	Need a fully substituted equation for the normal (in any form) or to get at least as far as finding value for m and expression for c .
	Obtain equation of normal is $y = 4\sqrt{2}x - \frac{7\sqrt{2}}{2}$ or equivalent 3 term equation	A1	E.g., $y = 5.66x - 4.95$. Must be $y = \dots$
		4	

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Question	Answer	Marks	Guidance
9(a)	Use a correct method to find $\overrightarrow{OD}$	M1	E.g. $\overrightarrow{OC} + 3(\overrightarrow{OA} - \overrightarrow{OB}) =$ $(-3\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}) + 3((2\mathbf{i} + \mathbf{j} - 3\mathbf{k}) - (4\mathbf{j} + \mathbf{k}))$ $(\overrightarrow{AB} = -2\mathbf{i} + 3\mathbf{j} + 4\mathbf{k})$ Accept column vectors throughout.
	Obtain position vector of D is $3\mathbf{i} - 11\mathbf{j} - 10\mathbf{k}$	A1	Accept coordinates.
		2	
9(b)	Carry out correct method for finding a vector equation for $\overrightarrow{AC}$ or $\overrightarrow{BD}$	*M1	E.g. $2\mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(5\mathbf{i} + 3\mathbf{j} - 5\mathbf{k})$ or $4\mathbf{j} + \mathbf{k} + \mu(3\mathbf{i} - 15\mathbf{j} - 11\mathbf{k})$. Condone missing $\mathbf{r} = \dots$
	Both diagonal equations correct.	A1ft	Seen or implied. Follow their D . Condone missing $\mathbf{r} = \dots$
	Equate at least two pairs of corresponding components and solve for λ or for μ	DM1	Dependent on using relevant lines and two different parameters.
	Obtain $\lambda = -\frac{1}{4}$ or $\mu = \frac{1}{4}$	A1	The values will depend on the directions of their lines
	Obtain position vector of P is $\frac{3}{4}\mathbf{i} + \frac{1}{4}\mathbf{j} - \frac{7}{4}\mathbf{k}$	A1	OE Accept coordinates. Do not ISW.

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Question	Answer	Marks	Guidance
9(b)	Alternative Method for Question 9(b):		
	State or imply $\overrightarrow{AC} = 5\mathbf{i} - 3\mathbf{j} + 5\mathbf{k}$	B1 FT	Or $\overrightarrow{BD} = 3\mathbf{i} - 15\mathbf{j} - 11\mathbf{k}$ Follow <i>their D</i> if used.
	Identify similar triangles with ratio 1 : 3	M1	
	Use similar triangles to obtain $\overrightarrow{OP}$, e.g. $\overrightarrow{OP} = \overrightarrow{OA} + \frac{1}{4}\overrightarrow{AC}$	M1	Must be correct fraction.
	Obtain position vector of <i>P</i> is $\frac{3}{4}\mathbf{i} + \frac{1}{4}\mathbf{j} - \frac{7}{4}\mathbf{k}$	A2	OE Allow A1A0 if any two values are correct.
		5	
9(c)	Find direction vector $\overrightarrow{BA} = 2\mathbf{i} - 3\mathbf{j} - 4\mathbf{k}$ and $\overrightarrow{BC} = -3\mathbf{i} - 6\mathbf{j} + \mathbf{k}$ or equivalent	B1FT	Or $\overrightarrow{AB}$ and $\overrightarrow{CB}$. FT if using an incorrect $\overrightarrow{AB}$ from earlier work.
	Carry out correct process for evaluating the scalar product of two relevant vectors	M1	Allow if one is going in the negative direction, e.g. $\overrightarrow{AB}$ and $\overrightarrow{BC}$.
	Using the correct process for the moduli, divide <i>their</i> scalar product by the product of their moduli and evaluate the inverse cosine of the result to obtain an angle	M1	Independent of the first M1. For their two vectors $\theta = \cos^{-1} \frac{8}{\sqrt{29}\sqrt{46}} = \dots$
	Obtain answer 77.3° (or 1.35 radians)	A1	77.347... Correctly rounded to more than 3 sf or AWRT 77.3.
		4	

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Question	Answer	Marks	Guidance
10(a)	Obtain $\frac{dV}{dt} = 40\pi - 0.8\pi r$ or equivalent	B1	Need a complete correct statement seen or implied.
	Obtain $\frac{dV}{dr} = 4\pi r^2$ or equivalent e.g. $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$	B1	Need a complete correct statement seen or implied.
	Use the chain rule to obtain given answer (including the derivative)	B1	Allow if $\frac{dr}{dt} = \frac{50-r}{5r^2}$ follows $\frac{dr}{dt} = \frac{40-0.8r}{4r^2}$ without further explanation (π already cancelled) and no incorrect statements seen.
		3	
10(b)	Commence division and reach quotient of the form $-5r \pm 250$ or $5r^2 = (50 - r)(Ar + B) + C$ and reach $A = -5$ and $B = \pm 250$	M1	Allow M1 if divide by $r - 50$ to obtain $5r \pm 250$.
	Obtain quotient $-5r - 250$	A1	Do not need to state which is quotient and which is remainder. However, if clearly muddled, then M1A1A0 for both expressions correct.
	Obtain remainder 12 500	A1	Note: 12 500 following division by $r - 50$ is correct and scores this A1 ISW.
			SC B1 only for correct use of remainder theorem to obtain correct remainder.
		3	

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Question	Answer	Marks	Guidance
10(c)	Prepare to integrate e.g. separate variables correctly Or express in the form $\frac{dt}{dr} = \frac{5r^2}{50-r} \left(= -(5r+250) + \frac{12500}{50-r} \right)$	B1FT	$\int \frac{5r^2}{50-r} dr = \int 1 dt$ Condone missing dr , dt or missing integral signs, but not both. Follow their division in (b) if substitute before separating.
	Obtain term t	DB1	
	Obtain terms $\frac{A}{2}r^2 + Br - C\ln(50-r)$	M1	From <i>their</i> $Ar + B + \frac{C}{50-r}$ in (b) where $ABC \neq 0$. Allow a single slip in the coefficients.
	Obtain terms $-\frac{5}{2}r^2 - 250r - 12500\ln(50-r)$	A1FT	FT <i>their</i> (b) , provided of the correct form.
	Use $t = 0$, $r = 0$ to evaluate a constant or as limits in a solution containing terms of the form r^2 , r , $\ln(50-r)$ and t	M1	
	Obtain final answer $t = -\frac{5}{2}r^2 - 250r - 12500\ln(50-r) + 12500\ln 50$	A1	OE Must be $t = \dots$ Allow with $12500\ln 50 = 48900$ or better.
		6	
10(d)	Obtain $t = 70.5$	B1	May be more accurate (70.4605...).
		1	

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Question	Answer	Marks	Guidance
11(a)	Use correct quotient rule NB the question asks for $f'(x)$ so need complete form	M1	Or correct product rule.
	Obtain correct derivative in any form, e.g. $\frac{4e^{2x}(e^{2x} - 3e^x + 2) - 2e^{2x}(2e^{2x} - 3e^x)}{(e^{2x} - 3e^x + 2)^2}$	A1	
	Equate <i>their</i> derivative to zero	*M1	Can be implied by numerator equated to zero for quotient rule. $(8 = 6e^x)$
	Solve for x to obtain $x = \ln a$	DM1	a positive.
	Obtain $x = \ln \frac{4}{3}$ and $y = -16$	A1	No errors seen. Accept equivalent exact forms, e.g. $x = \ln \frac{8}{6}$.

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Question	Answer	Marks	Guidance
11(a)	Alternative Method for Question 11(a)		
	Complete method to express $f(x)$ in partial fractions	M1	As far as $p + \frac{q}{e^x - 2} + \frac{r}{e^x - 1}$ with values for p , q and $r \left(2 + \frac{8}{e^x - 2} - \frac{2}{e^x - 1} \right)$. Allow in $u \left(u = e^x \right)$.
	Differentiate to obtain $f'(x) = \frac{se^x}{(e^x - 2)^2} + \frac{te^x}{(e^x - 1)^2}$	*M1	Note: the question requires $f'(x)$ so if they have substituted for e^x , they will also need chain rule.
	Obtain $f'(x) = \frac{-8e^x}{(e^x - 2)^2} + \frac{2e^x}{(e^x - 1)^2}$	A1	From correct work.
	Equate derivative to zero and solve for x to obtain $x = \ln a$	DM1	Must follow correctly to give a positive value of a .
	Obtain $x = \ln \frac{4}{3}$ and $y = -16$	A1	No errors seen. Accept $x = \ln \frac{8}{6}$, or equivalent.
		5	

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Question	Answer	Marks	Guidance
11(b)	State or imply $\frac{du}{dx} = e^x$	B1	
	Obtain $\int \frac{2u}{u^2 - 3u + 2} du$ or equivalent Or ₁ $\int \left(\frac{2}{u} + \right) \frac{8}{u(u-2)} - \frac{2}{u(u-1)} du$	B1	Correct expression in u . Condone missing du or missing integral but not both. Allow FT if using their partial fractions from (a).
	State or imply partial fractions of the form $\frac{A}{u-1} + \frac{B}{u-2}$ Or ₁ $\frac{C}{u} + \frac{D}{u-2} + \frac{E}{u-1}$ Or ₂ $\frac{2u-3}{u^2-3u+2} + \frac{3}{u^2-3u+2} = \frac{2u-3}{u^2-3u+2} + \frac{F}{u-2} + \frac{G}{u-1}$	B1 FT	Complete reduction to partial fractions. Correct form for <i>their</i> integrand.
	Use a correct method for finding a constant	M1	Available if they have incorrect form.
	Obtain correct $\frac{-2}{u-1} + \frac{4}{u-2}$ Or ₂ $\frac{2u-3}{u^2-3u+2} + \frac{3}{u-2} - \frac{3}{u-1}$	A1	
	Integrate to obtain $a \ln(u-1) + b \ln(u-2)$ or equivalent	*M1	M0 if they have additional terms that do not cancel out.
	Obtain correct $-2 \ln(u-1) + 4 \ln(u-2)$ or equivalent	A1FT	FT values of <i>their</i> partial fraction coefficients.
	Correctly use limits $u = 5$ and 3 in an expression of the form $a \ln(u-1) + b \ln(u-2)$ or $x = \ln 5$ and $\ln 3$ in an expression of the form $a \ln(e^x - 1) + b \ln(e^x - 2)$	DM1	
	Obtain $\ln \frac{81}{4}$	A1	Accept $\ln 20.25$.
		9	

Cambridge International A Level

MATHEMATICS**9709/33**

Paper 3 Pure Mathematics 3

October/November 2024**MARK SCHEME**Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

This document consists of **26** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics-Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(a)	For all 4 marks, scales must be approximately equal, dashes can replace numbers. Arcs don't have to be perfectly circular, mark intention.		
	Show an arc of a circle, centre the origin and radius 2. Only need 2 on Re(z) or 2i on Im(z) or $r = 2$ to show correct radius	B1	
	Show an arc centre the origin for $0 \leq \arg z \leq \frac{1}{4}\pi$ with any radius	B1	
	Max B1 if sector shaded		
		2	
1(b)	Show an arc of a circle, centre the origin and radius 4. Only need 4 on Re(z) or 4i on Im(z) or $r = 4$ to show correct radius	B1	
	Show an arc centre the origin for $0 \leq \arg z \leq \frac{1}{2}\pi$ with any radius	B1	
	Max B1 if sector shaded		
		2	

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Question	Answer	Marks	Guidance
2(a)	State or imply the equation $x = \sqrt{\frac{4}{5-2x}}$ and square the equation	*B1	Could work with x_{n+1} throughout or with x_n throughout instead of x .
	Rearrange this with at least one intermediate step in the form $2x^3 - 5x^2 + 4 = 0$	DB1	
	Alternative Method 1 for Question 2(a)		
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to either $x^2 = \frac{4}{5-2x}$ or $x = \sqrt{\frac{4}{5-2x}}$ and then obtain the iterative formula $x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$	B2	
	Alternative Method 2 for Question 2(a)		
	Rearrange $2x^3 - 5x^2 + 4 = 0$ to $x^2(5 - 2x) = 4$ (or a different intermediate step) and to $x^2 = \frac{4}{5-2x}$ and to $x_{n+1}^2 = \frac{4}{5-2x_n}$	*B1	Must have introduced x_{n+1} and x_n .
	Obtain the iterative formula $x_{n+1} = \sqrt{\frac{4}{5-2x_n}}$	DB1	
		2	

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Question	Answer	Marks	Guidance
2(b)	Use the iterative process correctly at least once	M1	The question specifies initial value 1.2, so must use the formula to obtain a value and then use this value in the formula.
	Obtain final answer 1.28	A1	Can gain this mark even if less than 4 dp shown in iteration.
	Show sufficient iterations to at least 4 dp to justify 1.28 to 2 dp or show that there is a sign change in the interval (1.275, 1.285)	A1	1.2, 1.2403, 1.2601, 1.2700, 1.2752, 1.2778, ... Allow small errors, truncation and recovery.
		3	

Question	Answer	Marks	Guidance
3(a)	State or imply that $\ln P = \ln a + kt$ or $\ln P = \ln a + k(\ln e)t$	B1	Can be implied by both a and k correct.
	$\ln P = \frac{1}{20}t + 3$ B0 until associated with a and /or k		$P = e^3 e^{\frac{1}{20}t}$ gets B1B1.
	State $k = \frac{1}{20}$, not from $\frac{dP}{dt} = k$	B1	OE. Can be embedded in $P = ae^{kt}$.
	$\ln a = 3 \Rightarrow a = 20$ to 2 sf	B1	Must be 2 sf, can be embedded in $P = ae^{kt}$.
		3	
3(b)	Form a correct equation in t using a and k , or <i>their</i> a and k where a will cancel (or are both numerical)	M1	E.g. $2a = ae^{kt}$, $2 = e^{kt}$, $kt = \ln 2$.
	Obtain $t = 14$ [hours]	A1	Allow 13.75 [hrs] (13 hrs 45 min) to 14 [hrs]. ISW
		2	

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Question	Answer	Marks	Guidance
4	Substitute $z = x + iy$ and obtain a horizontal equation Do not allow if this would lead to an equation containing xy terms which do not cancel	*M1	E.g. $5(x + (y - 3)i) = (2 - 9i)(x + (y + 3)i)$
	Use $i^2 = -1$ anywhere	M1	
	Obtain e.g. $5x + 5(y - 3)i = (2x + 9y + 27) + i(2y + 6 - 9x)$ or e.g. $3x^2 + 3y^2 - 12y - 63 + (9x^2 + 9y^2 - 30x + 54y + 81)i = 0$	A1	Or equivalent expression free of products of complex numbers. Terms can be in any order.
	Obtain simultaneous equations by equating real and imaginary parts	DM1	E.g. $3x - 9y = 27$ and $3y + 9x = 21$ $3x^2 + 3y^2 - 12y - 63 = 0$ and $9x^2 + 9y^2 - 30x + 54y + 81 = 0$
	Obtain $[z =] 3 - 2i$ only	A1	

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Question	Answer	Marks	Guidance
4	Alternative Method for Question 4:		
	Obtain a horizontal equation in z Do not allow if it would lead to an equation containing z^2 where the xy terms do not cancel	*M1	E.g. $5z - 15i = 2z + 6i - 27i^2 - 9iz$. Allow errors, but no brackets.
	Use $i^2 = -1$ anywhere	M1	
	Obtain $z = \frac{9+7i}{1+3i}$	A1	OE (might have an uncanceled factor of 3)
	Multiply top and bottom by $1-3i$ or equivalent for <i>their</i> z	DM1	Must see working for numerator or denominator, e.g. $9 - 27i + 21 + 7i$ or $1 + 9$ or 10 . If $\frac{9+7i}{1+3i} = 3 - 2i$ M0A0. If $\frac{9+7i}{1+3i} \times \frac{1-3i}{1-3i} = 3 - 2i$ M0A0 SC B1 . If $\frac{9+7i}{1+3i} \times \frac{1-3i}{1-3i}$ and working in numerator or denominator and $3 - 2i$ M1A1.
	Obtain $[z =] 3 - 2i$ only	A1	
		5	

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Question	Answer	Marks	Guidance
5(a)	Rewrite $\cos^4 \theta$ as $\left(\frac{1+\cos 2\theta}{2}\right)^2$ or $\sin^4 \theta$ as $\left(\frac{1-\cos 2\theta}{2}\right)^2$ or $4\sin^2 \theta \cos^2 \theta$ as $\sin^2 2\theta$	B1	Starting on left. Double angle for one term.
	Obtain $\left(\frac{1+\cos 2\theta}{2}\right)^2 - \left(\frac{1-\cos 2\theta}{2}\right)^2 - \sin^2 2\theta$	B1	OE, e.g. $1 \times \cos 2\theta - \sin^2 2\theta$.
	Expand to $\frac{1}{4} + \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos^2 2\theta - \left(\frac{1}{4} - \frac{1}{2}\cos 2\theta + \frac{1}{4}\cos^2 2\theta\right) - (1 - \cos^2 2\theta)$ and simplify to obtain $\cos^2 2\theta + \cos 2\theta - 1$	B1	AG
	Alternative Method 1 for Question 5(a):		
	Express $\cos^4 \theta - \sin^4 \theta$ as $(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta)$ or rewrite $4\sin^2 \theta \cos^2 \theta$ as $\sin^2 2\theta$	B1	Starting on left.
	Simplify to $\cos 2\theta - \sin^2 2\theta$	B1	If $\cos^4 \theta - \sin^4 \theta = \cos 2\theta$ instead of $(\cos^2 \theta + \sin^2 \theta)(\cos^2 \theta - \sin^2 \theta) = \cos 2\theta$, B0.
	Use $\sin^2 2\theta = 1 - \cos^2 2\theta$ to obtain $\cos^2 2\theta + \cos 2\theta - 1$	B1	AG

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Question	Answer	Marks	Guidance
5(a)	Alternative Method 2 for Question 5(a):		
	Use correct double angle formulae once e.g. replace $\cos 2\theta$ with $\cos^2 \theta - \sin^2 \theta$ $(\cos^2 \theta - \sin^2 \theta)^2 + (\cos^2 \theta - \sin^2 \theta) - 1$	B1	Starting on right. Double angle for one term.
	Expand to obtain $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta - \sin^4 \theta + 2\sin^4 \theta + \cos^2 \theta - \sin^2 \theta - 1$ * or $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta + \cos^2 \theta - \sin^2 \theta - 1$ leading to $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta - 2\sin^2 \theta$ leading to $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta - 2\sin^2 \theta (\cos^2 \theta + \sin^2 \theta)$ **	B1	Write $\sin^4 \theta$ as $-\sin^4 \theta + 2\sin^4 \theta$. Write $2\sin^2 \theta$ as $2\sin^2 \theta (\cos^2 \theta + \sin^2 \theta)$.
	Rewrite as * $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta - \sin^4 \theta + 2\sin^4 \theta - 2\sin^2 \theta$ leading to $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta - \sin^4 \theta + 2\sin^2 \theta (\sin^2 \theta - 1)$ leading to $\cos^4 \theta - 4\sin^2 \theta \cos^2 \theta - \sin^4 \theta$ ** $\cos^4 \theta - 2\sin^2 \theta \cos^2 \theta + \sin^4 \theta - 2\sin^2 \theta \cos^2 \theta - 2\sin^4 \theta$ leading to $\cos^4 \theta - 4\sin^2 \theta \cos^2 \theta - \sin^4 \theta$	B1	
		3	

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Question	Answer	Marks	Guidance
5(b)	State a quadratic equation in $\cos 2\alpha$ and solve for α $(\cos^2 2\alpha + \cos 2\alpha - 1 = 0)$	M1	Alternative: form a quadratic in $\tan^2 \alpha$ and solve for α $(\tan^4 \alpha + 4 \tan^2 \alpha - 1 = 0)$.
	Obtain $\alpha = 25.9^\circ$ or $\alpha = 154.1^\circ$	A1	May be more accurate. Allow 154 for 154.1.
	Obtain $\alpha = 25.9^\circ$ and $\alpha = 154.1^\circ$ and no others in range	A1	May be more accurate. Allow 154 for 154.1. Mark answers in radians as a misread (0.452, 2.69).
		3	

Question	Answer	Marks	Guidance
6(a)	$P(2, 1, -3)$	B1	Accept $x = 2, y = 1, z = -3$. Do not accept $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ or $\begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.
		1	

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Question	Answer	Marks	Guidance
6(b)	Use the correct method to find the scalar product of the direction vectors	M1	$\pm(-1 \times 2 + 2 \times 5) = \pm 8$ Allow error of $0 \times -1 = -1$.
	Divide the scalar product by the product of the moduli to obtain $\pm \cos \theta$ using consistent vectors throughout	M1	$\frac{\text{their } 8}{\sqrt{\text{their } 5} \sqrt{\text{their } 30}}$
	Obtain $\cos \theta = \frac{8}{5\sqrt{6}}$	A1	OE, e.g. $\frac{8}{\sqrt{150}}$ or $\frac{4\sqrt{6}}{15}$. If no $\frac{8}{5\sqrt{6}}$ seen, just 49.2, then A0. Decimal only seen, A0. ISW
6(b)	Alternative Method for Question 6(b):		
	Use of cosine rule: e.g. sides of $\sqrt{5}, \sqrt{30}$ and $\sqrt{19}$ found	B1	Could use other points.
	e.g. $\cos \theta = \frac{5 + 30 - 19}{2\sqrt{5}\sqrt{30}}$	M1	
	Obtain $\cos \theta = \frac{8}{5\sqrt{6}}$	A1	OE, e.g. $\frac{8}{\sqrt{150}}$ or $\frac{4\sqrt{6}}{15}$ or $\frac{8}{\sqrt{5}\sqrt{30}}$. If no $\frac{8}{5\sqrt{6}}$ seen, just 49.2, then A0. Decimal only seen, A0. ISW
		3	

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Question	Answer	Marks	Guidance
6(c)	Any two of $ PA = 2\sqrt{5}$ $ PB = \sqrt{30}$ or $ AB = \sqrt{82}$ seen	B1	May be seen by stating or implying that $\lambda = 2$ and $\mu = -1$.
	Area = $\frac{1}{2} \times 2\sqrt{5} \times \sqrt{30} \times \sqrt{1 - \frac{64}{150}}$	M1	Correct method for the exact area of the triangle. Note that: $\sin APB = \frac{\sqrt{129}}{15}$ $\sin ABP = \sqrt{\frac{86}{615}}$ $\cos ABP = \frac{46}{\sqrt{2460}}$ Perp A to BP = $\frac{\sqrt{2580}}{15}$ Perp B to AP = $\frac{\sqrt{430}}{5}$
	$= \sqrt{86}$	A1	Or simplified exact equivalent. ISW
	Alternative Method for Question 6(c)		
	$\vec{PA} \times \vec{PB} = -4\mathbf{i} - 18\mathbf{j} - 2\mathbf{k}$	B1	$\vec{PA} = -2\mathbf{i} + 4\mathbf{k}$, $\vec{PB} = -2\mathbf{i} + \mathbf{j} - 5\mathbf{k}$.
	Area = $\frac{1}{2} \vec{PA} \times \vec{PB} = \frac{1}{2} \sqrt{16 + 324 + 4}$	M1	Correct method for the exact area of the triangle.
	$= \sqrt{86}$	A1	Or simplified exact equivalent. ISW
		3	

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Question	Answer	Marks	Guidance
7(a)	Obtain $\frac{dx}{dt} = 6 \cos 2t$	B1	Allow $3.2 \cos 2t$. $dx = 6 \cos 2t$ is B0.
	Obtain $\frac{dy}{dt} = \sec^2 t - \operatorname{cosec}^2 t$	B1	Any equivalent form. $dy = \sec^2 t - \operatorname{cosec}^2 t$ is B0, but $\frac{dy}{dx} = \frac{\sec^2 t - \operatorname{cosec}^2 t}{6 \cos 2t}$ can go on to gain M1M1A1, so 3/5 possible.
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	*M1	$\frac{dy}{dx} = \frac{\sec^2 t - \operatorname{cosec}^2 t}{6 \cos 2t}$
	Express as a single fraction With $\frac{dy}{dt}$ correctly simplified as a single fraction in terms of $\sin t$ and $\cos t$ Allow with $6 \cos 2t$ expressed as $\frac{1}{6 \cos 2t}$ outside bracket	DM1	Allow $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \times \frac{1}{6 \cos 2t}$ or $\frac{\sin^2 t - \cos^2 t}{\cos^2 t \sin^2 t} \div 6 \cos 2t$ or $\frac{\sin^2 t - \cos^2 t}{\frac{\cos^2 t \sin^2 t}{6 \cos 2t}}$
	Obtain $\left(\frac{-4 \cos 2t}{6 \cos 2t \times \sin^2 2t} = \right) \frac{-2}{3 \sin^2 2t}$ from full and correct working Numerator and denominator must have identical terms before cancelling, both $\cos 2t$ or both + and – ($\sin^2 t - \cos^2 t$)	A1	AG Allow slips in θ and x for t to recover earlier marks, provided these are corrected before the final line. Do not allow serious errors in working for the final mark.

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Question	Answer	Marks	Guidance
7(a)	Alternative Method for Question 7(a):		
	$y = \frac{6}{x}$	B2	Using $y = \frac{\tan^2 t + 1}{\tan t} = \frac{\sec^2 t}{\frac{\sin t}{\cos t}} = \frac{1}{\sin t \cos t}$
	$\frac{dy}{dx} = -6x^{-2}$	B1	
	$\frac{dy}{dx} = -\frac{6}{9\sin^2 2t}$	M1	
	Obtain $\frac{dy}{dx} = \frac{-2}{3\sin^2 2t}$ from full and correct working	A1	
		5	
7(b)	Gradient of normal = $\frac{3}{2}$	B1	
	Use correct method to find the equation of the normal	M1	E.g. $(y - 2) = \frac{3}{2}(x - 3)$ or find c in $y = \frac{3}{2}x + c$. Allow a wrong value for x or y but not both, with <i>their</i> normal gradient.
	Obtain $2y - 3x + 5 = 0$	A1	Or $k(2y - 3x + 5) = 0$, where k is an integer.
		3	

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Question	Answer	Marks	Guidance
8(a)	State or imply the form $\frac{A}{a-2x} + \frac{B}{3a+x}$ and use a correct method to find a constant	M1	
	Obtain $A=2a$ or $B=a$	A1	
	Obtain $A=2a$ and $B=a$	A1	
		3	
8(b)	Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	M1	
	Obtain $+2\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+\dots\right)$	A1ft	OE. May be unsimplified. Follow <i>their</i> A, B for an expansion involving a .
	Obtain $+\frac{1}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+\dots\right)$	A1ft	OE. May be unsimplified. Follow <i>their</i> A, B for an expansion involving a .
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	A1	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x . Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.

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Question	Answer	Marks	Guidance
8(b)	Alternative Method for Question 8(b)		
	Expanding $7a^2(a-2x)^{-1}(3a+x)^{-1}$ from the original question. Use a correct method to obtain the first two terms of the expansion of $(a-2x)^{-1}$ or $\left(1-\frac{2x}{a}\right)^{-1}$ or $(3a+x)^{-1}$ or $\left(1+\frac{x}{3a}\right)^{-1}$	M1	
	Obtain $+7a\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+\dots\right)$ or $+\frac{7a}{3}\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+\dots\right)$	A1	OE. May be unsimplified. May be implied by the expression shown for the next A1.
	Obtain $+\frac{7}{3}\left(1+\frac{2x}{a}+\frac{4x^2}{a^2}+\dots\right)\left(1-\frac{x}{3a}+\frac{x^2}{9a^2}+\dots\right)$	A1	OE. May be unsimplified.
	Obtain $+\frac{7}{3}+\frac{35x}{9a}+\frac{217x^2}{27a^2}$	A1	Or simplified equivalent. Final answer. Ignore any terms in higher powers of x . Do not ISW, e.g. multiplying by $27a^2$. Condone different order of terms.
		4	
8(c)	$ x < \frac{a}{2}$	B1	Or $-\frac{a}{2} < x < \frac{a}{2}$. Mark final answer. Must make a clear statement.
		1	

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Question	Answer	Marks	Guidance
9(a)	Divide to obtain quotient $x^2 + k$	M1	k is a constant.
	Obtain quotient $x^2 - 4$	A1	If quotient stated separately, mark at this stage.
	Obtain remainder 32	A1	If remainder stated separately, mark at this stage. Need not state which is quotient and remainder, but if stated wrongly, max 2/3. After a correct division, still allow the marks if then written as $x^2 - 4 + \frac{32}{x^2 + 4}$.
	Alternative Method for Question 9(a)		
	Expands brackets to get $B = 0$	M1	$(x^2 + 4)(x^2 + Bx + C) + D =$ $x^4 + Bx^3 + (C + 4)x^2 + 4Bx + 4C + D$
	$C = -4$	A1	
	$D = 32$	A1	Need not state which is quotient and remainder, but if stated wrongly, max 2/3.
		3	

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Question	Answer	Marks	Guidance
9(b)	$\frac{1}{3}x^3 - 4x$	B1 FT	Follow <i>their</i> quotient of form $Ax^2 + B$.
	Obtain $p \tan^{-1} qx$ where $q = 2$ or $q = \frac{1}{2}$	M1	
	Obtain $16 \tan^{-1} \frac{1}{2}x$	A1 FT	Follow <i>their</i> constant remainder, i.e. $\left(\frac{\text{their constant remainder}}{2} \right) \tan^{-1} \frac{1}{2}x$.
	Use limits correctly in an expression containing $p \tan^{-1} qx$ where $q = 2$ or $q = \frac{1}{2}$ and $rx^3 + sx$	M1	Terms need not be evaluated, e.g. $[8\sqrt{3} - 8\sqrt{3}] + 16 \tan^{-1} \sqrt{3} - \left(\frac{8}{3} - 8 + 16 \tan^{-1} 1 \right)$ or $\frac{8}{3} - 8$ can be $-\frac{16}{3}$, $16 \tan^{-1} \sqrt{3}$ can be $\frac{16\pi}{3}$, $16 \tan^{-1} 1$ can be 4π .
	Obtain $\frac{4}{3}(\pi + 4)$ from full and correct working	A1	AG
		5	

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Question	Answer	Marks	Guidance
10(a)	State that $\frac{dV}{dt} = 50000 - 600h$	B1	May be seen as $\frac{dV}{dt} = 50000$ and $\frac{dV}{dt} = [-]600h$. When put together (may be in the chain rule) B1 can be awarded.
	[Use $V = 40000h$ to] obtain $\frac{dV}{dh} = 40000$ and use this and <i>their</i> $\frac{dV}{dt}$ in the correct chain rule to obtain $\frac{dh}{dt}$ or [Use $V = 40000h$ to] obtain $\frac{dV}{dt} = 40000 \frac{dh}{dt}$ and equate to <i>their</i> $\frac{dV}{dt}$	M1	$\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh}$ E.g. $\frac{50000 - 600h}{40000} = \frac{dh}{dt}$ E.g. $40000 \frac{dh}{dt} = 50000 - 600h$

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Question	Answer	Marks	Guidance
10(a)	Obtain $200 \frac{dh}{dt} = 250 - 3h$ from full and correct working	A1	AG $\frac{dh}{dt} = \frac{50000 - 600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen SC B1 (1/3). $\frac{dV}{dt} = 50000 - 600h$ B1 followed by $\frac{dh}{dt} = \frac{50000 - 600h}{40000}$ OE, leading to given answer with no other working, or no incorrect working seen, SC B1 2/3. $\frac{dV}{dt} = \frac{dV}{dh} \times \frac{dh}{dt}$ followed by $50000 - 600h = 40000 \frac{dh}{dt}$ OE, leading to given answer with no other working, or no incorrect working seen, B1 for implied $\frac{dV}{dt}$ and SC B1 2/3.
		3	

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Question	Answer	Marks	Guidance
10(b)	Separate variables correctly and integrate one side correctly	M1	E.g. $\int \frac{1}{250-3h} dh = \int \frac{1}{200} dt$. Integral signs may be omitted, 200 may be on opposite side.
	Obtain $-\frac{1}{3} \ln 250-3h = \frac{t}{200} (+C)$	A1	OE Condone missing “+C” and lack of modulus signs.
	Use $t = 0$, $h = 50$ in an expression containing $\ln(250-3h)$ or $\ln 250-3h $ to find the constant of integration.	M1	Or equivalent use of limits 50 and 80.
	Obtain $C = -\frac{1}{3} \ln 100$	A1	OE, e.g. $\frac{1}{3} \ln \frac{100}{250-3h} = \frac{t}{200}$, or $-\frac{200}{3} \ln\left(\frac{10}{100}\right)$. With or without modulus signs on the log terms.
	$t = 150$	A1	
		5	

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Question	Answer	Marks	Guidance
11(a)	Use of correct product rule and correct chain rule	M1	$\frac{dy}{dx} = A \cos x \sqrt{2 + \cos x} + \frac{B \sin x \sin x}{\sqrt{2 + \cos x}}$
	Obtain $\frac{dy}{dx} = 2 \cos x \sqrt{2 + \cos x} - \frac{2 \sin^2 x}{2 \sqrt{2 + \cos x}}$	A1	OE
	Equate the derivative to zero and obtain a horizontal 3 term quadratic equation or 4 term quartic equation in $\cos a$ If M0 earlier then needs that expression to be such that arrive at 3 term quadratic or 4 term quartic equation in $\cos x$ without further trig errors. The only error in the form of the differential allowed is for $(2 + \cos x)^{-\frac{1}{2}}$ to be $(2 + \cos x)^{+\frac{1}{2}}$ or $(2 + \cos x)^{-\frac{3}{2}}$	*M1	Accept in $\cos x$. E.g. $3 \cos^2 x + 4 \cos x - 1 = 0$. E.g. $3 \cos^4 x + 16 \cos^3 x + 18 \cos^2 x - 1 = 0$.
	Solve for $\cos a$	DM1	$\left(\cos a = \frac{-2 + \sqrt{7}}{3} \text{ or } 0.215 \right)$ Allow presence of other solution(s).
	Obtain $a = 4.93$	A1	Allow more accurate, e.g. 4.929... even though question states 2 dp. If $x = 1.35$ leads to $x = 4.93$ award A1 BOD. If $x = 1.35$ and $x = 4.93$ award A0.
		5	

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Question	Answer	Marks	Guidance
11(b)	State or imply $du = -\sin x dx$	B1	OE If B0, max M1M1M1.
	Substitute throughout for u and du	M1	
	Obtain $-\int 2\sqrt{u} du$	A1	OE. Ignore limits if $-\int 2\sqrt{u} du$, but if $+\int 2\sqrt{u} du$, then must have correct limits $\int_1^3 2\sqrt{u} du$. (See final M1)
	Integrate to obtain $ku^{\frac{3}{2}} (+C)$	M1	Constant of integration not required
	Use correct limits correctly in an expression of the form $ku^{\frac{3}{2}}$ or $k(2 + \cos x)^{\frac{3}{2}}$	M1	1 and 3 for u , or 0 and π for x .
	Obtain $\frac{4}{3}(3\sqrt{3}-1)$ or $4\sqrt{3}-\frac{4}{3}$ or $\frac{4}{3}\sqrt{27}-\frac{4}{3}$	A1	OE. Allow, e.g., $\sqrt{3}^3$ for $\sqrt{27}$. ISW but don't ignore e.g. multiplying throughout by 3. If the answer is changed from negative to positive value at end, then A0. Last M1A1 can use modulus, providing no errors seen.
		6	