

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

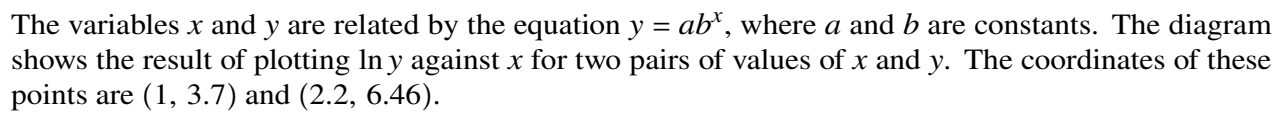
- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1** Find the exact coordinates of the points on the curve $y = \frac{x^2}{1-3x}$ at which the gradient of the tangent is equal to 8. [5]

[illegible]

- 2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 2i| \leq |z + 2 - i|$ and $0 \leq \arg(z + 1) \leq \frac{1}{4}\pi$. [4]



[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 The complex number u is defined by $u = \frac{3 + 2i}{a - 5i}$, where a is real.

(a) Express u in the Cartesian form $x + iy$, where x and y are in terms of a . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(b) Given that $\arg u = \frac{1}{4}\pi$, find the value of a . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 (a) Given that

$$\sin\left(x + \frac{1}{6}\pi\right) - \sin\left(x - \frac{1}{6}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right) - \cos\left(x - \frac{1}{3}\pi\right),$$

find the exact value of $\tan x$.

[4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(b) Hence find the exact roots of the equation

$$\sin\left(x + \frac{1}{6}\pi\right) - \sin\left(x - \frac{1}{6}\pi\right) = \cos\left(x + \frac{1}{3}\pi\right) - \cos\left(x - \frac{1}{3}\pi\right)$$

for $0 \leq x \leq 2\pi$.

[2]

[illegible]

- 6 The parametric equations of a curve are

$$x = \sqrt{t} + 3, \quad y = \ln t,$$

for $t > 0$.

- (a) Obtain a simplified expression for $\frac{dy}{dx}$ in terms of t . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) Hence find the exact coordinates of the point on the curve at which the gradient of the normal is -2 . [3]

.....

.....

.....

.....

.....

.....

.....

.....

7 The variables x and θ satisfy the differential equation

$$\frac{x}{\tan \theta} \frac{dx}{d\theta} = x^2 + 3.$$

It is given that $x = 1$ when $\theta = 0$.

Solve the differential equation, obtaining an expression for x^2 in terms of θ . [7]

[illegible]

- 8 (a) By sketching a suitable pair of graphs, show that the equation

$$\sqrt{x} = e^x - 3$$

has only one root.

[2]

- (b) Show by calculation that this root lies between 1 and 2.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Show that, if a sequence of values given by the iterative formula

$$x_{n+1} = \ln(3 + \sqrt{x_n})$$

converges, then it converges to the root of the equation in (a).

[1]

.....

.....

.....

.....

.....

.....

.....

- (d) Use the iterative formula to calculate the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

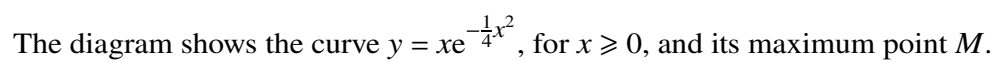
.....

.....

.....

.....

.....



- [4]

[illegible]

- (b)** Using the substitution $x = \sqrt{u}$, or otherwise, find by integration the exact area of the shaded region bounded by the curve, the x -axis and the line $x = 3$. [5]

[illegible]

10 Let $f(x) = \frac{24x + 13}{(1 - 2x)(2 + x)^2}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) State the set of values of x for which the expansion in (b) is valid. [1]

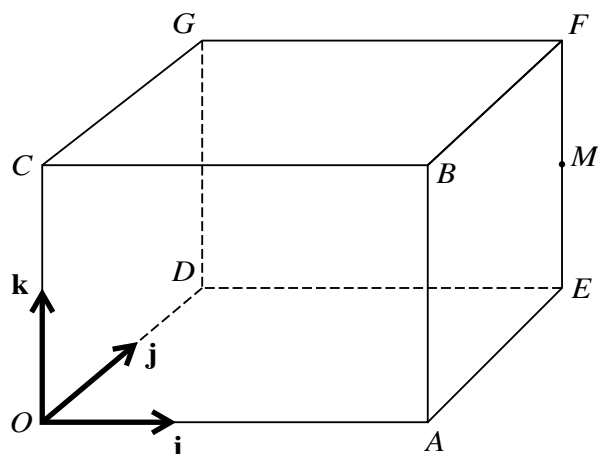
.....

.....

.....

.....

11



In the diagram, $OABCDEFG$ is a cuboid in which $OA = 3$ units, $OC = 2$ units and $OD = 2$ units. Unit vectors $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are parallel to OA , OD and OC respectively. M is the midpoint of EF .

- (a) Find the position vector of M . [1]

.....

.....

.....

The position vector of P is $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.

- (b) Calculate angle PAM . [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

© UCLES 2023

[illegible]

BLANK PAGE

Permission to reproduce items where third-party owned material protected by copyright is included has been sought and cleared where possible. Every reasonable effort has been made by the publisher (UCLES) to trace copyright holders, but if any items requiring clearance have unwittingly been included, the publisher will be pleased to make amends at the earliest possible opportunity.

To avoid the issue of disclosure of answer-related information to candidates, all copyright acknowledgements are reproduced online in the Cambridge Assessment International Education Copyright Acknowledgements Booklet. This is produced for each series of examinations and is freely available to download at www.cambridgeinternational.org after the live examination series.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

BLANK PAGE

- 1 (a) Sketch the graph of $y = |4x - 2|$.

[1]

- (b) Solve the inequality $1 + 3x < |4x - 2|$.

[4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

2 The parametric equations of a curve are

$$x = (\ln t)^2, \quad y = e^{2-t^2},$$

for $t > 0$.

Find the gradient of the curve at the point where $t = e$, simplifying your answer.

[4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 3** The polynomial $2x^3 + ax^2 - 11x + b$ is denoted by $p(x)$. It is given that $p(x)$ is divisible by $(2x - 1)$ and that when $p(x)$ is divided by $(x + 1)$ the remainder is 12.

Find the values of a and b .

[5]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 4 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 4 - 3i| \leq 2$ and $\operatorname{Re} z \leq 3$. [4]

- (b) Find the greatest value of $\arg z$ for points in this region. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

5 Find the exact value of $\int_0^6 \frac{x(x+1)}{x^2+4} dx$.

[6]

[illegible]

- 6 (a) By sketching a suitable pair of graphs, show that the equation

$$\cot x = 2 - \cos x$$

has one root in the interval $0 < x \leq \frac{1}{2}\pi$.

[2]

- (b) Show by calculation that this root lies between 0.6 and 0.8.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Use the iterative formula $x_{n+1} = \tan^{-1} \left(\frac{1}{2 - \cos x_n} \right)$ to determine the root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

[illegible]

- 7 (a) By expressing 3θ as $2\theta + \theta$, prove the identity $\cos 3\theta \equiv 4\cos^3\theta - 3\cos\theta$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

(b) Hence solve the equation

$$\cos 3\theta + \cos \theta \cos 2\theta = \cos^2 \theta$$

for $0^\circ \leq \theta \leq 180^\circ$.

[5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 8** It is given that $\frac{2+3ai}{a+2i} = \lambda(2-i)$, where a and λ are real constants.

(a) Show that $3a^2 + 4a - 4 = 0$.

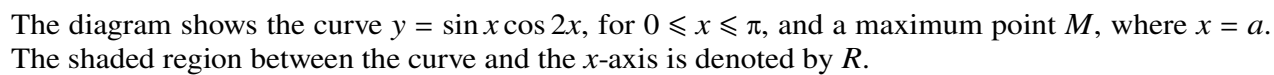
[4]

[illegible]

(b) Hence find the possible values of a and the corresponding values of λ .

[3]

[illegible]



- [illegible]

- (b)** Find the exact area of the region R , giving your answer in simplified form. [4]

[illegible]

where c is a positive constant. It is given that the angle between l and m is 60° .

- [4]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (b)** Show that the length of the perpendicular from $(6, -3, 6)$ to l is $\sqrt{11}$. [5]

[illegible]

- 11 The variables x and y satisfy the differential equation

$$x^2 \frac{dy}{dx} + y^2 + y = 0.$$

It is given that $x = 1$ when $y = 1$.

- (a) Solve the differential equation to obtain an expression for y in terms of x . [8]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (b) State what happens to the value of y when x tends to infinity. Give your answer in an exact form. [1]

.....

.....

.....

.....

.....

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.

Cambridge International AS & A Level

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2023

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

BLANK PAGE

- 1** Find the set of values of x satisfying the inequality $|2^{x+1} - 2| < 0.5$, giving your answer to 3 significant figures. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

- 2 On an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 1 + 2i| \leq |z|$ and $|z - 2| \leq 1$. [5]

- 3** The polynomial $2x^3 + ax^2 + bx + 6$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $(x + 2)$ the remainder is -38 and when $p(x)$ is divided by $(2x - 1)$ the remainder is $\frac{19}{2}$.

Find the values of a and b .

[5]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 Solve the quadratic equation $(3 + i)w^2 - 2w + 3 - i = 0$, giving your answers in the form $x + iy$, where x and y are real. [5]

[illegible]

- 5** Find the exact coordinates of the stationary points of the curve $y = \frac{e^{3x^2-1}}{1-x^2}$. [6]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 6 (a)** Show that the equation $\cot^2 \theta + 2 \cos 2\theta = 4$ can be written in the form

$$4 \sin^4 \theta + 3 \sin^2 \theta - 1 = 0. \quad [3]$$

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[Turn over

7 The equation of a curve is $x^3 + y^2 + 3x^2 + 3y = 4$.

(a) Show that $\frac{dy}{dx} = -\frac{3x^2 + 6x}{2y + 3}$. [3]

[illegible]

- (b)** Hence find the coordinates of the points on the curve at which the tangent is parallel to the x -axis. [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- 8 The variables x and y satisfy the differential equation

$$e^{4x} \frac{dy}{dx} = \cos^2 3y.$$

It is given that $y = 0$ when $x = 2$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

9 Let $f(x) = \frac{17x^2 - 7x + 16}{(2 + 3x^2)(2 - x)}$.

(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b) Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^3 . [5]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) State the set of values of x for which the expansion in (b) is valid. Give your answer in an exact form. [1]

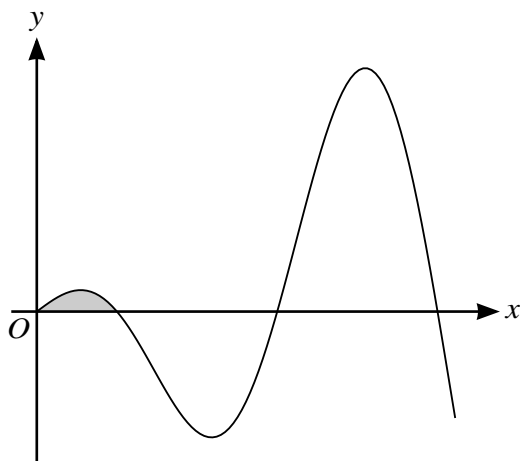
.....

.....

.....

.....

.....



(a) Find the equation of the tangent to the curve at the point where $x = \frac{1}{2}\pi$. [4]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for practicing various letters and words. There is no text or other markings on the page.

- (b)** Find the exact area of the shaded region shown in the diagram, bounded by the curve and the x -axis. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 11** The line l has equation $\mathbf{r} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$. The points A and B have position vectors $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + \mathbf{k}$ respectively.

(a) Find a unit vector in the direction of l . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

The line m passes through the points A and B .

(b) Find a vector equation for m . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (c) Determine whether lines l and m are parallel, intersect or are skew. [5]

[illegible]

[illegible]

Cambridge Assessment International Education is part of Cambridge Assessment. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which is a department of the University of Cambridge.