



Cambridge International A Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2023

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

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PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

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Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

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Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

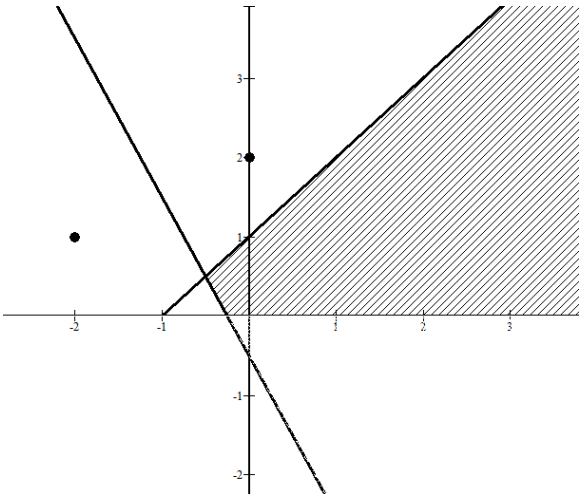
Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use correct quotient or product rule	*M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{(1-3x)2x - x^2(-3)}{(1-3x)^2} \left(= \frac{2x - 3x^2}{(1-3x)^2} \right)$ or $3x^2(1-3x)^{-2} + 2x(1-3x)^{-1}$.
	Equate derivative to 8 and solve for x	DM1	$75x^2 - 50x + 8 = (15x - 4)(5x - 2)$.
	Obtain answers $x = \frac{2}{5}$ and $\frac{4}{15}$	A1	Exact values required.
	Obtain answers $y = -\frac{4}{5}$ and $\frac{16}{45}$	A1	Allow A1 for one correct point.
		5	

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Question	Answer	Marks	Guidance
2	Show points representing $2i$ and $-2 + i$	B1	Can be implied if the correct perpendicular is drawn.
	Show perpendicular bisector of <i>their</i> ($2i$ and $-2 + i$)	B1FT	
	Show correct half–line of gradient 1 from point $(-1, 0)$	B1	Should pass through $(0, 1)$.
	Correct loci and shade correct region	B1	
		4	

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Question	Answer	Marks	Guidance
3	State or imply that $\ln y = \ln a + x \ln b$	B1	
	Carry out a completely correct method for finding $\ln a$ or $\ln b$	M1	$3.7 = \ln a + \ln b$ and $6.46 = \ln a + 2.2 \ln b$ leading to $\ln a = 1.4$, $\ln b = 2.3$.
	Obtain value $a = 4.06$	A1	
	Obtain value $b = 9.97$	A1	SC B1 for $a = e^{1.4}$ and $b = e^{2.3}$.
	Alternative Method for Question 3		
	$e^{3.7} = ab^1$ and $e^{6.46} = ab^{2.2}$	B1	
	Divide to obtain $e^{2.76} = b^{1.2}$ and state or imply $2.76 = 1.2 \ln b$	M1	
	Obtain value $a = 4.06$	A1	
	Obtain value $b = 9.97$	A1	
		4	

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Question	Answer	Marks	Guidance
4(a)	Multiply numerator and denominator by $a + 5i$	M1	OE
	Use $i^2 = -1$	M1	At least once.
	Obtain answer $\frac{3a-10}{a^2+25} + \frac{2a+15}{a^2+25}i$	A1	
	Alternative Method for Question 4(a)		
	Multiply $x + iy$ by $a - 5i$ and use $i^2 = -1$	M1	
	Compare real and imaginary parts	M1	$3 = ax + 5y$, $2 = ay - 5x$.
	Obtain answer $\frac{3a-10}{a^2+25} + \frac{2a+15}{a^2+25}i$	A1	
4(b)		3	
	State or imply $\text{Im}(\mathbf{a}) \div \text{Re}(\mathbf{a}) = 1$	M1	Or $\text{Im}(\mathbf{a}) = \text{Re}(\mathbf{a})$ or equivalent for <i>their</i> u .
	Obtain answer $a = 25$	A1	
		2	

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Question	Answer	Marks	Guidance
5(a)	Use correct trig formulae and obtain an equation in $\sin x$ and $\cos x$	*M1	Allow one sign error.
	Obtain a correct equation in any form	A1	e.g. $2 \cos x \sin \frac{\pi}{6} = -2 \sin x \sin \frac{\pi}{3}$.
	Substitute exact trig ratios and obtain an expression for $\tan x$	DM1	Allow one sign error.
	Obtain answer $\tan x = -\frac{1}{\sqrt{3}}$	A1	Or exact equivalent.
		4	
5(b)	Obtain answer, e.g. $x = \frac{5\pi}{6}$	B1	
	Obtain second answer, e.g. $x = \frac{11\pi}{6}$ and no others in the interval	B1FT	FT first answer + π (provided $0 \leq \text{first answer} \leq \pi$). Or FT first answer – π (provided $\pi \leq \text{first answer} \leq 2\pi$). Ignore any answers outside interval.
		2	

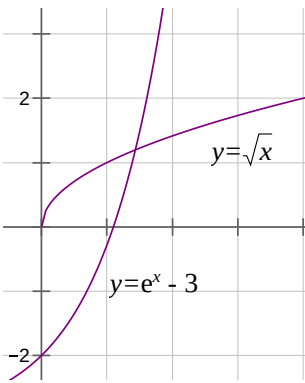
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Question	Answer	Marks	Guidance
6(a)	State correct derivative of x or y with respect to t	B1	$\frac{dx}{dt} = \frac{1}{2}t^{-\frac{1}{2}}, \quad \frac{dy}{dt} = \frac{1}{t}.$
	Use $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$	M1	Use correct chain rule.
	Obtain answer $\frac{dy}{dx} = \frac{2}{\sqrt{t}}$	A1	Or simplified equivalent e.g. $2t^{-\frac{1}{2}}$ or $\frac{2\sqrt{t}}{t}.$
		3	
6(b)	State or imply <i>their</i> $\frac{dy}{dx} = \frac{1}{2}$	M1	
	Obtain $\sqrt{t} = 4$	A1	Or equivalent.
	Obtain answer (7, ln 16)	A1	Or exact equivalent. Can state the two components separately.
		3	

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Question	Answer	Marks	Guidance
7	Separate variables correctly	B1	$\int \tan \theta d\theta = \int \frac{x}{x^2 + 3} dx$. Condone missing integral signs or missing dx, dθ. Can be implied by later work.
	Obtain term $-\ln(\cos \theta)$	B1	Or equivalent e.g. $\ln(\sec \theta)$.
	Obtain term of the form $a \ln(x^2 + 3)$	M1	
	Obtain term $\frac{1}{2} \ln(x^2 + 3)$	A1	
	Use $x = 1, \theta = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a \ln(x^2 + 3)$ and $b \ln(\cos \theta)$	M1	If they have rearranged then the constant must be of the correct form.
	Obtain correct answer in any form	A1	$\frac{1}{2} \ln(x^2 + 3) = -\ln \cos \theta + \ln 2$.
	Obtain final answer $x^2 = \frac{4}{\cos^2 \theta} - 3$	A1	Or equivalent e.g. $x^2 = 4 \sec^2 x - 3$. lns removed.
		7	

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Question	Answer	Marks	Guidance
8(a)	Sketch a relevant graph, e.g. $y = e^x - 3$ $y = e^x - 3$ Should cut vertical axis at $(0, -2)$ and have increasing gradient.	B1	 Ignore anything outside 1st and 4th quadrants. For second B1 need to mark intersection with a dot, a cross, or say root at point of intersection, or equivalent.
	Sketch a second relevant graph, e.g. $y = \sqrt{x}$ and justify the given statement $y = \sqrt{x}$ should start at $(0, 0)$ and have reducing grading	B1	
		2	
8(b)	Calculate the values of a relevant expression or pair of expressions at $x = 1$ and $x = 2$	M1	
	Complete the argument correctly with correct calculated values	A1	e.g. $1 > -0.28..$, $1.41 < 4.39..$ $1.28 > 0$, $-2.98 < 0$.
		2	
8(c)	State $x = \ln(3 + \sqrt{x})$ and rearrange to the given equation $\sqrt{x} = e^x - 3$	B1	Or rearrange $\sqrt{x} = e^x - 3$ to $x = \ln(3 + \sqrt{x})$ and state iterative formula of $x_{n+1} = \ln(3 + \sqrt{x_n})$. AG
		1	

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Question	Answer	Marks	Guidance
8(d)	Use the iterative process correctly at least once	M1	
	Obtain final answer 1.43	A1	
	Show sufficient iterations to at least 4 d.p. to justify 1.43 to 2 d.p. or show there is a sign change in the interval (1.425, 1.435) Condone recovery and small differences in the final figure in the iteration	A1	e.g. 1, 1.3864, 1.4297, 1.4341, ... 1.5, 1.4210, 1.4332, 1.4344, 1.4345, ... 2, 1.4848, 1.4395, 1.4350, 1.4346, 1.4345, ...
		3	

Question	Answer	Marks	Guidance
9(a)	Use the correct product rule	*M1	Condone error in chain rule.
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = -\frac{x^2}{2}e^{-\frac{x^2}{4}} + e^{-\frac{x^2}{4}}$.
	Equate derivative to zero and solve for x	DM1	
	Obtain answer $\left(\sqrt{2}, \sqrt{2}e^{-\frac{1}{2}}\right)$	A1	Or exact equivalent. Can state the components separately.
		4	

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Question	Answer	Marks	Guidance
9(b)	State or imply $dx = \frac{1}{2}u^{-\frac{1}{2}} du$	B1	Or equivalent e.g. $du = 2x dx$. Alternative substitution: $u = -\frac{1}{4}x^2$.
	Substitute for x and dx	M1	
	Obtain correct integral $\frac{1}{2} \int e^{-\frac{1}{4}u} du$	A1	OE
	Use correct limits in an integral of the form $ae^{-\frac{1}{4}u}$ or $ae^{-\frac{1}{4}x^2}$	M1	$u = 9$ and $u = 0$ or $x = 3$ and $x = 0$.
	Obtain answer $2 - 2e^{-\frac{9}{4}}$	A1	Or exact equivalent.
	Alternative Method for Question 9(b)		
	$\int x e^{-\frac{1}{4}x^2} dx = a e^{-\frac{1}{4}x^2}$	M1	Recognition used.
	a negative	A1	
	$a = -2$	A1	
	Use correct limits in an integral of the form $ae^{-\frac{1}{4}x^2}$	M1	$x = 3$ and $x = 0$.
	Obtain answer $2 - 2e^{-\frac{9}{4}}$	A1	Or exact equivalent.
		5	

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Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{1-2x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$	B1	
	Use a correct method for finding a coefficient	M1	$A(2+x)^2 + B(1-2x)(2+x) + C(1-2x)$ $= 24x + 13.$
	Obtain one of $A = 4$, $B = 2$ and $C = -7$	A1	If errors in equating still allow A marks for A and C.
	Obtain a second value	A1	
	Obtain the third value	A1	Mark the form $\frac{A}{1-2x} + \frac{Dx+E}{(2+x)^2}$, where $A = 4$, $D = 2$ and $E = -3$, B1 M1 A1 A1 A1 as above. If there are extra term in partial fractions, that is 4 unknowns A , B , D and E then B0 unless recover at end, e.g. by setting $B = 0$. If B set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+ F$, mark as above but need $F = 0$ to recover B1.
		5	

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Question	Answer	Marks	Guidance
10(b)	Use a correct method to find the first two terms of the expansion of $(1-2x)^{-1}$, $(2+x)^{-1}$, $(2+x)^{-2}$, $\left(1+\frac{x}{2}\right)^{-1}$ or $\left(1+\frac{x}{2}\right)^{-2}$	M1	Symbolic coefficients are not sufficient for the M1.
	Obtain correct un-simplified expansions up to the term in x^2 of each partial fraction	A1 FT	$A\left(1+(-1)(-2x)+\frac{(-1)(-2)}{2}(-2x)^2+..\right)$ $A = 4.$
		A1 FT	$\frac{B}{2}\left(1+(-1)\left(\frac{x}{2}\right)+\frac{(-1)(-2)}{2}\left(\frac{x}{2}\right)^2+...\right)$ $B = 2.$
		A1 FT	$\frac{C}{4}\left(1+(-2)\left(\frac{x}{2}\right)+\frac{(-2)(-3)}{2}\left(\frac{x}{2}\right)^2+...\right)$ $C = -7$ $= 4(1+2x+4x^2) + 2/2(1-x/2+x^2/4) - 7/4(1-x+3x^2/4)$ $= (4+1-7/4) + (8-1/2+7/4)x + (16+1/4-21/16)x^2$ The FT is on A, B, C.

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Question	Answer	Marks	Guidance
	Obtain final answer $\frac{13}{4} + \frac{37}{4}x + \frac{239}{16}x^2$	A1	OE $(Dx + E)/4 [1 + (-2)(x/2) + (-2)(-3)(x/2)^2/2 \dots]$ $D = 2 \quad E = -3$ The FT is on A, D, E. Maclaurin's Series $f(0) = 13/4 \quad B1 \quad f'(0) = 37/4 \quad B1 \quad f''(0) = 239/8 \quad B1.$ $\frac{13}{4} + \frac{37}{4}x + \frac{239}{8}x^2/2$ or equivalent M1 A1. If $1 + \frac{37}{4}x + \frac{239}{8}x^2/2$ then M0 A0 unless <i>their</i> $f(0)$ actually is 1. For the A, D, E form of fractions, give M1 A1FT A1FT for the expanded partial fractions, then, if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer. If final answer has been multiplied throughout (e.g. by 16) then A0 at the end
		5	
10(c)	$ x < \frac{1}{2}$	B1	OE
		1	

Question	Answer	Marks	Guidance
11(a)	Obtain $3\mathbf{i} + 2\mathbf{j} + \mathbf{k}$	B1	Accept coordinates in place of position vector.
		1	

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Question	Answer	Marks	Guidance
11(b)	$\overrightarrow{AM}$ or $\overrightarrow{AP}$ correct soi	B1	$\overrightarrow{AM} = 2\mathbf{j} + \mathbf{k}$, or $\overrightarrow{AP} = -2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$.
	Carry out correct process for evaluating the scalar product of $\overrightarrow{AM}$ and $\overrightarrow{AP}$	M1	or $\overrightarrow{MA}$ and $\overrightarrow{PA}$: $0 + 2 + 2$.
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and obtain the inverse cosine of the result	M1	For their vectors. $\theta = \cos^{-1}\left(\frac{4}{3\sqrt{5}}\right)$.
	Obtain answer 53.4° or 0.932°	A1	
		4	
11(c)	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on the line passing through O and M ,	B1 FT	e.g. $\overrightarrow{PQ} = -(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$. Follow <i>their M</i> .
	Calculate the scalar product of $\overrightarrow{PQ}$ and a direction vector for the line passing through O and M and equate to zero	*M1	
	Solve and obtain correct solution e.g. $\mu = -\frac{1}{2}$	A1	
	Carry out method to calculate PQ	DM1	$\sqrt{.5^2 + 0 + 1.5^2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	A1	Or exact equivalent.
	Alternative Method 1 for Question 11(c)		
	Find $\overrightarrow{PQ}$ (or $\overrightarrow{QP}$) for a general point Q on the line passing through O and M ,	B1 FT	e.g. $\overrightarrow{PQ} = -(\mathbf{i} + \mathbf{j} + 2\mathbf{k}) + \mu(3\mathbf{i} + 2\mathbf{j} + \mathbf{k})$. Follow <i>their M</i> .
	Use a correct method to express PQ^2 (or PQ) in terms of μ	*M1	
	Obtain a correct equation in any form	A1	e.g. $PQ^2 = (1 + 3\mu)^2 + (1 + 2\mu)^2 + (2 + \mu)^2$

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Question	Answer	Marks	Guidance
11(c)	Carry out a complete method for finding its minimum	DM1	e.g. $6(1+3\mu) + 4(1+2\mu) + 2(2+\mu) = 0$, $\mu = -\frac{1}{2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	A1	Or exact equivalent.
	Alternative Method 2 for Question 11(c)		
	Calling (0, 0, 0) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
	Use a scalar product to find the projection of $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) on the line passing through O and M	M1	
	Obtain correct answer $\frac{7}{\sqrt{14}}$	A1	OE
	Use Pythagoras to find the perpendicular	M1	$d = \sqrt{AP^2 - AQ^2} = \sqrt{1+1+2^2 - \left(\frac{7}{\sqrt{14}}\right)^2}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	A1	Or exact equivalent.
	Alternative Method 3 for Question 11(c)		
	Calling (0, 0, 0) A, state $\overrightarrow{PA}$ (or $\overrightarrow{AP}$) in component form, e.g. $\mathbf{i} + \mathbf{j} + 2\mathbf{k}$	B1	
	Calculate the vector product of $\overrightarrow{PA}$ and a direction vector for the line passing through O and M	M1	
	Obtain correct answer, e.g. $3\mathbf{i} - 5\mathbf{j} + \mathbf{k}$	A1	

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Question	Answer	Marks	Guidance
11(c)	Divide modulus of the product by that of the direction vector	M1	e.g. $\frac{\sqrt{3^2 + 5^2 + 1^2}}{\sqrt{3^2 + 2^2 + 1^2}}$.
	Obtain answer $\frac{\sqrt{10}}{2}$	A1	Or exact equivalent.
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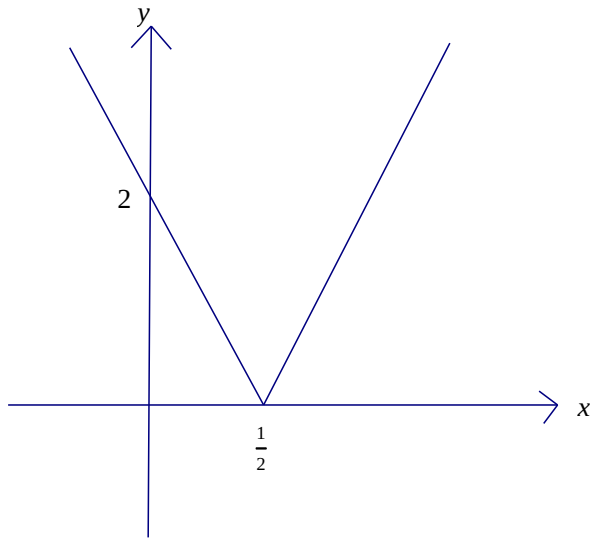
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1(a)		B1	Show a recognizable sketch graph of $y = 4x - 2$. Roughly symmetrical. Should extend into the second quadrant. Ignore $y = 4x - 2$ below the axis if intention is clear e.g. dashed or the required lines are clearly bolder. Some indication of scale on both axes – accept dashes. Must go beyond (0, 2) and (1, 2). Ignore any attempt to sketch $y = 1 + 3x$.
		1	

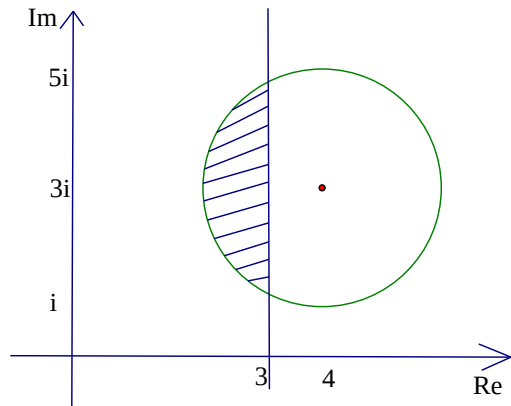
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Question	Answer	Marks	Guidance
1(b)	Obtain critical value $x = 3$	B1	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Solve the linear equation $1 + 3x = 2 - 4x$	M1	Or corresponding linear inequality.
	Obtain critical value $\frac{1}{7}$	A1	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x < \frac{1}{7}$ [or] $x > 3$	A1	Or equivalent. Allow with a comma, or nothing between. Strict inequalities only. Exact values. A0 for $\frac{1}{7} > x > 3$ A0 for $x < \frac{1}{7}$ and $x > 3$.
	Alternative method for question 1(b)		
	Solve the quadratic inequality $(4x - 2)^2 > (1 + 3x)^2$, or corresponding quadratic equation	M1	e.g. $7x^2 - 22x + 3 = 0$. Available if they start with the correct equation / inequality, have a correct method for squaring (i.e. not $(a + b)^2 = a^2 + b^2$) and a correct method for solving. Need to obtain at least one critical value.
	Obtain critical value $x = 3$	A1	Allow incorrect inequality. Allow if later rejected. Allow $\frac{21}{7}$.
	Obtain critical value $\frac{1}{7}$	A1	Allow 0.143 or better. Allow incorrect inequality. Allow if later rejected.
	Obtain final answer $x < \frac{1}{7}$ [or] $x > 3$	A1	Or equivalent. Strict inequalities only. Allow with a comma, or nothing between. Exact values. A0 for $\frac{1}{7} > x > 3$ A0 for $x < \frac{1}{7}$ and $x > 3$.
		4	

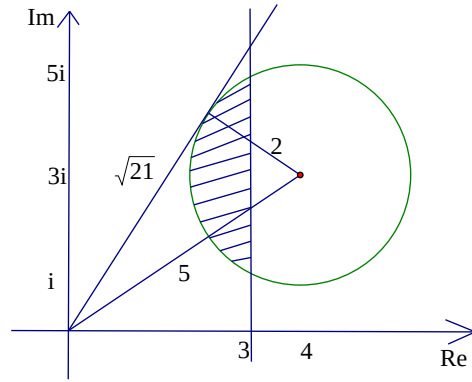
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Question	Answer	Marks	Guidance
2	Obtain $\frac{dx}{dt} = \frac{2}{t} \ln t$	B1	Any equivalent form.
	Obtain $\frac{dy}{dt} = -2te^{2-t^2}$	B1	Any equivalent form.
	$\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$ and substitute $t = e$	M1	Correct use of chain rule for $\frac{dy}{dx} \left(\frac{-2e^2 e^{2-e^2}}{2 \ln e} \right)$. Condone an error between correct combination of the derivatives and attempt to substitute e.
	Obtain $-e^{4-e^2}$	A1	ISW Accept $-0.0337(405..)$. Accept $-e^4 e^{-e^2}$, $\frac{-e^4}{e^{e^2}}$ and $-e^2 e^{2-e^2}$. Allow M1A1 for a correct decimal answer following B1B1 seen.
		4	

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Question	Answer	Marks	Guidance
4(a)	Show a circle with centre $4 + 3i$. Accept a curved shape with correct point roughly in the middle.	B1	 Need some indication of scale e.g. label the centre, mark key points on the axes or dashes on the axes. Condone dotted lines in place of solid lines Condone correct shaded shape but not an entire circle
	Show a circle with radius 2 and centre not at the origin. The shape should be consistent with their scales	B1	
	Show correct vertical line. Enough to meet correct circle twice or complete line for any other circle.	B1	
	Shade the correct region on a correct diagram Any other shading must be accompanied by words to explain which region is required	B1	
		4	

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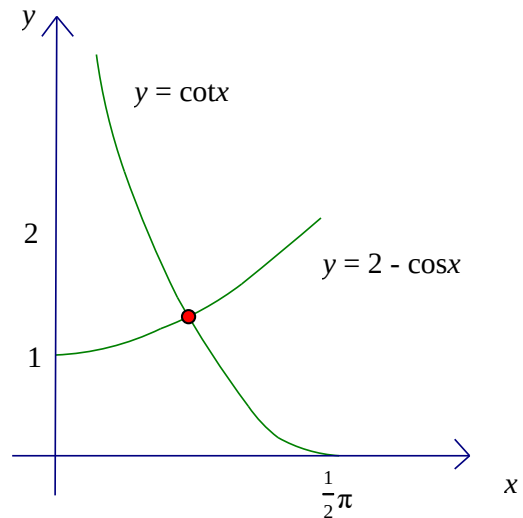
Question	Answer	Marks	Guidance
4(b)	Carry out a complete method for finding the greatest value of $\arg(z)$ e.g $\tan^{-1}\frac{3}{4} + \sin^{-1}\frac{2}{5}$ (0.6435 + 0.4115)	M1	
	Obtain answer 1.06 (accept 1.055 or 1.056) radians or 60.45° (accept 60.4° or 60.5°)	A1	
	Alternative method for question 4(b)		
	Tangent to circle passing through origin has equation $y = mx$. The equation $(x - 4)^2 + (y - 3)^2 = 4$ will have one root. Hence $(1 + m^2)x^2 - (8 + 6m)x + 21 = 0$, discriminant $= 0 = 48m^2 - 96m + 20$ and $m = \frac{6 \pm \sqrt{21}}{6}$ with the larger value needed to give greatest $\arg(z)$. Required angle is $\tan^{-1} m$.	M1	Complete method for finding the greatest value of $\arg(z)$.
	Obtain answer 1.06 radians or 60.45°	A1	Accept 1.055 or 1.056 radians. Accept 60.4° or 60.5°.
		2	

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Question	Answer	Marks	Guidance
5	Split fraction to obtain $1 + \frac{x-4}{x^2+4}$	B1	
	Attempt integration and obtain $p \ln(x^2+4)$ or $q \tan^{-1}\left(\frac{x}{2}\right)$ from correct working	M1	Allow for $p \ln(x^2+4)$ from $\int \frac{x}{x^2+4} dx$ but only if a correct method for splitting has been used.
	Obtain $\frac{1}{2} \ln(x^2+4)$	A1 FT	Follow through is on their coefficients in the partial fraction. Allow from $\frac{x^2}{x^2+4} + \frac{x}{x^2+4}$ even if the split of the fraction is not complete. If $1 - \frac{4}{x^2+4} + \frac{x}{x^2+4}$ later seen or implied, award the B1. Only available from a correct split, not from an approach using parts that is incomplete.
	Obtain $-2 \tan^{-1}\left(\frac{x}{2}\right)$	A1 FT	Only available from a correct split, not from an approach using parts that is incomplete.
	Correct use of correct limits 0 and 6 in an expression involving $p \ln(x^2+4)$, $q \tan^{-1}\left(\frac{x}{2}\right)$ and no incorrect terms.	M1	p and q should be constants. The x term is not required at this stage.
	Obtain $6 + \frac{1}{2} \ln 10 - 2 \tan^{-1} 3$	A1	ISW Or three term equivalent. (Must combine the $\ln$ terms.) Accept with $\frac{1}{2} \ln 10 $.

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Question	Answer	Marks	Guidance
5	Alternative method for question 5		
	Use the substitution $x = 2 \tan \theta$ to obtain $\int 2 \tan^2 \theta + \tan \theta \, d\theta$	B1	
	Attempt integration and obtain $p \tan \theta$ or $r \ln(\sec \theta)$ from correct working	M1	
	Obtain $2 \tan \theta (-2\theta)$ and	A1 FT	Follow through on <i>their</i> coefficients after the substitution.
	Obtain $\ln \sec \theta$	A1 FT	Follow through on <i>their</i> coefficients after the substitution.
	Use correct limits 0 and $\tan^{-1} 3$ in an expression involving $u \tan \theta$, $v \ln \sec \theta$ and no incorrect terms	M1	u and v should be constants. The θ term is not required at this stage.
	Obtain $6 + \ln \left \sec(\tan^{-1} 3) \right - 2 \tan^{-1} 3$	A1	ISW Or three term equivalent. Not required to simplify $\ln \left \sec(\tan^{-1} 3) \right $.
		6	

Question	Answer	Marks	Guidance
6(a)	Sketch a relevant graph. e.g. $y = \cot x$: x intercept should be correct. Not touching the y -axis. No incorrect curvature. Ignore anything outside $0 < x \leq \frac{1}{2}\pi$.	B1	 2nd B1 requires a mark at the point of intersection or a suitable comment for the justification.
	Sketch a second relevant graph and justify the given statement e.g. $y = 2 - \cos x$: Condone if looks almost straight, but not if drawn with a ruler and not incorrect curvature. Correct y intercept. Needs to be drawn for $0 < x \leq \frac{1}{2}\pi$. Ignore outside this.	B1	
		2	
6(b)	Calculate the value of a relevant expression or values of a pair of expressions at $x = 0.6$ and $x = 0.8$. Must be working in radians. Values correct to at least 2 significant figures. Need all relevant values but only one (pair) needs to be correct to award M1. Complete set of values for their expression. If not comparing with 0 or 1 then the pairing must be clear, not just embedded values.	M1	e.g. $1.17 < 1.46$, $1.30 > 0.971$, $-0.29 < 0$, $0.33 > 0$. $-0.20 < 0$, $0.342 > 0$ from $\tan x(2 - \cos x) - 1 = 0$. $0.80 < 1$, $1.34 > 1$ from $\tan x(2 - \cos x) = 1$. $0.146 > 0$, $-0.105 < 0$ from $x - \tan^{-1}\left(\frac{1}{2 - \cos x}\right)$.
	Complete the argument correctly with correct calculated values (awrt 2 s.f.). Clear comparison for their expression. Allow work on a smaller interval.	A1	Accept truncated values. If comparing with 0 can either indicate different signs or a negative product.
		2	

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Question	Answer	Marks	Guidance
6(c)	Use the iterative process correctly at least once. Must be working in radians	M1	
	Obtain final answer 0.68	A1	Must be a clear conclusion.
	Show sufficient iterations to at least 4 d.p. to justify 0.68 to 2 d.p. or show there is a sign change in the interval (0.675, 0.685). Allow recovery. Allow truncation. Allow small differences in the 4 th s.f.	A1	e.g. 0.7, 0.6806, 0.6855, 0.6843, 0.6846 0.6, 0.7053, 0.6792, 0.6858, 0.6842, 0.6846 0.8, 0.6545, 0.6920, 0.6826, 0.6850, 0.6844, 0.6845 .
		3	

Question	Answer	Marks	Guidance
7(a)	Use correct expansion for $\cos(2\theta + \theta)$	*M1	
	Use correct double angle formulae to express $\cos 3\theta$ in terms of $\cos \theta$ and $\sin \theta$	DM1	
	Show sufficient working to confirm $\cos 3\theta \equiv 4\cos^3 \theta - 3\cos \theta$	A1	AG
		3	

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Question	Answer	Marks	Guidance
7(b)	Use the identity and correct double angle formula to obtain an equation in $\cos \theta$ only. Must come from using all three terms in the given equation.	*M1	e.g. $4 \cos^3 \theta - 3 \cos \theta + \cos \theta (2 \cos^2 \theta - 1) = \cos^2 \theta$ $6 \cos^3 \theta - \cos^2 \theta - 4 \cos \theta = 0$ or $6 \cos^2 \theta - \cos \theta - 4 = 0$.
	Obtain $\theta = 90^\circ$	B1	Allow if $\cos \theta$ obtained correctly as a factor of <i>their</i> expression (even if there is an error in the quadratic factor). Can follow M0.
	Solve a 3-term quadratic in $\cos \theta$ to obtain a value of θ	DM1	
	Obtain one value e.g. 25.3°	A1	Accept awrt 25.3° .
	Obtain a second value e.g. 137.5° and no extras in range	A1	Accept awrt 137.5° . Ignore values outside the range. Mark solutions in radians as a misread (0.442, 1.57, 2.40).
		5	

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Question	Answer	Marks	Guidance
8(a)	Multiply both sides by $a + 2i$ and attempt expansion of right-hand side	*M1	
	Use of $i^2 = -1$ seen at least once (or implied)	DM1	e.g. $2 + 3ai = \lambda(2a + 2) + \lambda i(-a + 4)$
	Compare real and imaginary parts to obtain an equation in a only $[2 = \lambda(2a + 2), \quad 3a = \lambda(-a + 4)]$	M1	e.g. $\frac{3a}{2} = \frac{-a + 4}{2a + 2}$. Any equivalent form.
	Obtain $3a^2 + 4a - 4 = 0$ from correct working	A1	AG
	Alternative method for question 8(a)		
	Multiply top and bottom of the left-hand side by $a - 2i$ and attempt both expansions	*M1	Do not need the right-hand side at this stage.
	Use of $i^2 = -1$ seen at least once or implied	DM1	e.g. $[\lambda(2 - i) =] \frac{8a + i(3a^2 - 4)}{a^2 + 4}$.
	Compare real and imaginary parts to obtain an equation in a only	M1	e.g. $8a = -2(3a^2 - 4)$. Any equivalent form.
	Obtain $3a^2 + 4a - 4 = 0$ from correct working	A1	AG
		4	
8(b)	Solve given quadratic to obtain a value of a and use this to form an equation in λ only (based on an equation seen in <i>their</i> working in (a) or (b))	M1	Can be implied by relevant working seen or a correct value for λ seen.
	Obtain $a = -2, \lambda = -1$ or $a = \frac{2}{3}, \lambda = \frac{3}{5}$	A1	Allow $\frac{6}{10}$ and 0.6.
	Obtain second correct pair of values	A1	
		3	

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Question	Answer	Marks	Guidance
9(a)	Use correct product rule	*M1	As far as $p \cos x \cos 2x + q \sin x \sin 2x$ or full working (u , v , du/dx , dv/dx) shown.
	Obtain $\frac{dy}{dx} = \cos x \cos 2x - 2 \sin x \sin 2x$	A1	OE
	Equate derivative to zero and use correct double angle formulae	DM1	Allow if only have one double angle in their derivative.
	Obtain $\cos x(1 - 6\sin^2 x) = 0$ or equivalent	A1	e.g. $\cos x(6\cos^2 x - 5) = 0$, $5\tan^2 x = 1$. Simplified but not necessarily factorised - like terms must be collected.
	Obtain $a = 0.42$	A1	Only. Accept $x = 0.42$.
	Alternative method for question 9(a)		
	Use correct double angle formula	*M1	
	Obtain $\sin x - 2\sin^3 x$ or equivalent	A1	
	Use correct chain rule or product rule to differentiate and equate the derivative to zero	DM1	
	Obtain $\cos x(1 - 6\sin^2 x) = 0$	A1	OE
	Obtain $a = 0.42$	A1	Only. Accept $x = 0.42$.
		5	

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Question	Answer	Marks	Guidance
9(b)	Use double angle formula and obtain $p \cos^3 x + q \cos x$ correctly	*M1	e.g. from $\int 2 \cos^2 x \sin x - \sin x \, dx$.
	Obtain $\pm \left(-\frac{2}{3} \cos^3 x + \cos x \right)$	A1	Correct for <i>their</i> integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm \left(-\frac{2}{3} \left[\left(\frac{-1}{\sqrt{2}} \right)^3 - \left(\frac{1}{\sqrt{2}} \right)^3 \right] - \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right).$
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.
	Alternative method 1 for question 9(b)		
	Use integration by parts twice and obtain $r \cos x \cos 2x + s \sin x \sin 2x$	*M1	Seen, not just implied.
	Obtain $\frac{1}{3} \cos x \cos 2x + \frac{2}{3} \sin x \sin 2x$	A1	Accept $\pm$ (correct for <i>their</i> integral).
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\pm \frac{1}{3} \left(0 + 2 \times \frac{1}{\sqrt{2}} \times -1 - 0 - 2 \times \frac{1}{\sqrt{2}} \times 1 \right).$
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or simplified exact equivalent. Final answer must be positive.

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Question	Answer	Marks	Guidance
	Alternative method 2 for question 9(b)		
	Use factor formula and integrate to obtain $g \cos 3x + h \cos x$	*M1	$\int \frac{1}{2}(\sin 3x - \sin x) dx$.
	Obtain $\pm(-\frac{1}{6} \cos 3x + \frac{1}{2} \cos x)$	A1	Correct for <i>their</i> integral.
	Correct use of limits $\frac{1}{4}\pi$ and $\frac{3}{4}\pi$ (or use double the integral from $\frac{1}{4}\pi$ to $\frac{1}{2}\pi$)	DM1	OE $\mp \frac{1}{\sqrt{2}}(\frac{1}{6} + \frac{1}{2} + \frac{1}{6} + \frac{1}{2})$.
	Obtain $\frac{2\sqrt{2}}{3}$	A1	Or exact equivalent. Final answer must be positive.
		4	

Question	Answer	Marks	Guidance
10(a)	Use the correct process to calculate the scalar product of the direction vectors	M1	$(-2 + 4 + 2c)$.
	Divide the scalar product by the product of the moduli and equate the result to $\cos 60^\circ$	M1	Or equivalent e.g. $2 + 2c = \sqrt{6}\sqrt{20 + c^2} \cos 60^\circ$. Allow for the correct process using 60° but the wrong vectors.
	Obtain correct equation in c	A1	e.g. $\frac{2 + 2c}{\sqrt{6}\sqrt{20 + c^2}} = \frac{1}{2}$ or $10c^2 + 32c - 104 = 0$.
	Obtain $c = 2$	A1	Only.
		4	

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Question	Answer	Marks	Guidance
10(b)	Calling $(6, -3, 6)$ A , find $\overrightarrow{AP}$ for a general point P on l	B1	e.g. $\begin{pmatrix} -3+\lambda \\ 1+\lambda \\ -5+2\lambda \end{pmatrix}$.
	Equate the scalar product of <i>their</i> $\overrightarrow{AP}$ and a direction vector for l to zero and obtain an equation in λ	*M1	e.g. $(-3+\lambda)+(1+\lambda)+(-10+4\lambda)=0$.
	Solve and obtain $\lambda=2$	A1	
	Carry out a method to calculate $ \overrightarrow{AP} $	DM1	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$.
	Obtain $\sqrt{11}$ from correct working	A1	AG
	Alternative method 1 for question 10(b)		
	Calling $(6, -3, 6)$ A , find $\overrightarrow{AP}$ for a general point P on l	B1	e.g. $\begin{pmatrix} -3+\lambda \\ 1+\lambda \\ -5+2\lambda \end{pmatrix}$
	Differentiate the modulus of $\overrightarrow{AP}$ or the square of the modulus and equate the derivative to zero	*M1	e.g. $2(-3+\lambda)+2(1+\lambda)+4(-5+2\lambda)=0$
	Solve and obtain $\lambda=2$	A1	
	Carry out a method to calculate $ \overrightarrow{AP} $	DM1	e.g. $(-1)^2+3^2+(-1)^2$ or $1^2+3^2+1^2$
	Obtain $\sqrt{11}$ from correct working	A1	AG

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Question	Answer	Marks	Guidance
	Alternative method 2 for question 10(b)		
	Vector from $(6, -3, 6)$ to $(3, -2, 1)$ is $-3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$	B1	The method works for vector from $(6, -3, 6)$ to any point on l .
	Use scalar product to find the angle between <i>their</i> vector and the direction of l	M1	
	Obtain $\cos \theta = \frac{3-1+10}{\sqrt{35}\sqrt{6}} \left(= \sqrt{\frac{24}{35}} \right)$ or $\sin \theta = \sqrt{\frac{11}{35}}$	A1	
	Correct use of trig to find the projection of their vector on the normal to l	M1	$\sqrt{35} \sin \theta = \sqrt{35} \times \sqrt{\frac{11}{35}}$.
	Obtain $\sqrt{11}$ from correct working	A1	AG
	Alternative method 3 for question 10(c)		
	Vector from $(6, -3, 6)$ to $(3, -2, 1)$ is $-3\mathbf{i} + \mathbf{j} - 5\mathbf{k}$	B1	
	Find the vector product of <i>their</i> vector and the direction of l	M1	
	Obtain $\mathbf{i}(2+5) - \mathbf{j}(-6+5) + \mathbf{k}(-3-1) (= 7\mathbf{i} + \mathbf{j} - 4\mathbf{k})$	A1	
	Correct use of trig to find the perpendicular distance	M1	$\frac{ \text{their vector product} }{ \text{direction vector} }$.
	Distance $= \frac{\sqrt{66}}{\sqrt{6}} = \sqrt{11}$	A1	
		5	

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Question	Answer	Marks	Guidance
11(a)	Correct separation of variables.	B1	$\int \frac{1}{y^2 + y} dy = \int -\frac{1}{x^2} dx$. Condone missing integral signs or missing dx, dy, but not both.
	Obtain $\frac{1}{x}$	B1	
	Express $\frac{1}{y^2 + y}$ in partial fractions or express the denominator of the fraction as a difference of two squares	*M1	Allow for the correct split of $\frac{\pm 1}{(y^2 \pm y)}$.
	Obtain $\frac{1}{y} - \frac{1}{y+1}$ or $\frac{1}{(y + \frac{1}{2})^2 - (\frac{1}{2})^2}$	A1	Allow if coefficients for the partial fractions are correct but followed by an error.
	Obtain $\ln y - \ln(y+1)$	A1	Or equivalent, dependent on where they left the minus sign.
	Use $x=1, y=1$ to find constant of integration or as limits in a definite integral in an expression containing terms of the form $\frac{p}{x}, q \ln y$ and $r \ln(1+y)$	DM1	$\ln \frac{1}{2} = 1 + C$ If they rearrange the equation before finding the constant of integration then the constant must be of the correct form.
	Correct equation in x and y	A1	$\ln \frac{y}{1+y} = \frac{1}{x} - 1 + \ln \frac{1}{2}$.
	Obtain $y = \frac{e^{\frac{1}{x}-1}}{2 - e^{\frac{1}{x}-1}}$	A1	Or equivalent e.g. $y = \frac{1}{2e^{\frac{1}{x}-1} - 1}$, $y = \frac{1}{e^{1-\frac{1}{x}+\ln 2} - 1}$. Accept with decimal value for e^{-1} .
		8	

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Question	Answer	Marks	Guidance
11(b)	State that y approaches $\frac{1}{2e-1}$	B1 FT	Or exact equivalent. Condone $y = \frac{1}{2e-1}$. FT on an expression in $e^{\frac{1}{x}}$.
		1	



Cambridge International A Level

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2023

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2023 series for most Cambridge IGCSE, Cambridge International A and AS Level components, and some Cambridge O Level components.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

Mathematics Specific Marking Principles

- 1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
- 2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
- 3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
- 4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
- 5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.
- 6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

Mark Scheme Notes

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no “follow through” from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	State or imply non-modular inequality $-0.5 < 2^{x+1} - 2 < 0.5$, can be in two separate statements, or $(2^{x+1} - 2)^2 < 0.5^2$ or corresponding pair of linear equations $0.5 = 2^{x+1} - 2$ and $-0.5 = 2^{x+1} - 2$ or quadratic equation $(2^{x+1} - 2)^2 = 0.5^2$	B1	$-0.25 < 2^x - 1 < 0.25$, can be in two separate statements, or $(2^x - 1)^2 < 0.25^2$ or corresponding pair of linear equations $0.25 = 2^x - 1$ and $-0.25 = 2^x - 1$ or quadratic equation $(2^x - 1)^2 = 0.25^2$. Incorrect inequality mark recoverable by correct final answer or $x < 0.32$ and $x > -0.42$.
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	M1	Reach $(x + 1)\ln 2 = \ln a$ or equivalent, do not need to reach $x = \dots$
	Obtain critical values $x = 0.322$ and -0.415 or awrt $x = 0.32$ and -0.42 or exact equivalents	A1	e.g. $\frac{\ln 2.5}{\ln 2} - 1$ and $\frac{\ln 1.5}{\ln 2} - 1$.
	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	A1	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
	Alternative method for Question 1		
	Use correct method for solving an equation or inequality of the form $2^{x+1} = a$ or $2^x = b$ where $a, b > 0$	M1	May see $2^{x+1} = 1.5$ and $2^{x+1} = 2.5$. Reach $(x + 1)\ln 2 = \ln a$ or equivalent, don't need to reach $x = \dots$
	Obtain one critical value, e.g. 0.322 or awrt $x = 0.32$ or exact equivalent	A1	e.g. $\frac{\ln 2.5}{\ln 2} - 1$.
	Obtain the other critical value e.g. -0.415 or awrt $x = -0.42$ or exact equivalent	A1	e.g. $\frac{\ln 1.5}{\ln 2} - 1$.

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Question	Answer	Marks	Guidance
1	State final answer $-0.415 < x < 0.322$ or $(-0.415, 0.322)$	A1	Need 3 significant figures. Need combined result, not $x < 0.32$ and $x > -0.42$. Must be strict inequalities. No working, 0/4.
		4	

Question	Answer	Marks	Guidance
2	Show a circle centre (2, 0)	B1	
	Show the relevant part of a circle with radius 1	B1 FT	FT centre not at the origin even if centre at $1 - 2i$. Must clearly go through (1, 0) or (3, 0) (oe for FT mark).
	Show the point representing $1 - 2i$	B1	Can be implied by correct perpendicular bisector
	Show the perpendicular bisector of the line joining $1 - 2i$ and the origin. Perpendicular to OP by eye and at midpoint of OP by eye sufficient. Must reach midpoint of OP and if extended will cut BE .	B1 FT	FT on the position of $1 - 2i$.

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Question	Answer	Marks	Guidance
2	Shade the correct region. Dependent on all previous marks, except in case 3 below, and the perpendicular must cut axes between CF and BE, but not actually through C or F and not through B or E Scale can be implied by dashes 1 Scale only on y-axis and $2OA = OC$ B1, B1FT, B1, B1FT, B1 2 Scale only on x-axis and $2OB = OE$ B1, B1FT, B1, B1FT, B1 3 No scale on either axis, but $2OA = OC$ then $2OB = OE$ B0, B1FT, B0, B1FT, B1	B1	
		5	

Question	Answer	Marks	Guidance
3	$2(-2)^3 + a(-2)^2 + b(-2) + 6 = -38$ Allow errors $x + 2 \frac{2x^2 + (a-4)x + b - 2a + 8}{2x^3 + ax^2 + bx + 6}$ $\frac{2x^3 + 4x^2}{(a-4)x^2 + bx}$ $\frac{(a-4)x^2 + (2a-8)x}{(b-2a+8)x + 6}$ $\frac{(b-2a+8)x + 2b - 4a + 16}{4a - 2b - 10}$	M1	Substitute $x = -2$ and equate the result to -38 or divide by $x + 2$ to obtain quadratic quotient, and equate constant remainder to -38.

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Question	Answer	Marks	Guidance
3	Obtain a correct evaluated equation, e.g. $-16 + 4a - 2b + 6 = -38$ or $4a - 2b = -28$	A1	
	$2\left(\frac{1}{2}\right)^3 + a\left(\frac{1}{2}\right)^2 + b\left(\frac{1}{2}\right) + 6 = \frac{19}{2}$ Allow errors $2x - 1 \frac{x^2 + \frac{a+1}{2}x + \frac{b}{2} + \frac{a}{4} + \frac{1}{4}}{2x^3 + ax^2 + bx + 6}$ $\frac{2x^3 - x^2}{(a+1)x^2 + bx}$ $\frac{(a+1)x^2 - \left(\frac{a}{2} + \frac{1}{2}\right)x}{\left(b + \frac{a}{2} + \frac{1}{2}\right)x + 6}$ $\frac{\left(b + \frac{a}{2} + \frac{1}{2}\right)x - \left(\frac{b}{2} + \frac{a}{4} + \frac{1}{4}\right)}{6 + \frac{b}{2} + \frac{a}{4} + \frac{1}{4}}$	M1	Substitute $x = \frac{1}{2}$ and equate the result to $\frac{19}{2}$ or divide by $2x - 1$ to obtain quadratic quotient, and equate constant remainder to $\frac{19}{2}$.
	Obtain a correct evaluated equation, e.g. $\frac{1}{4} + \frac{a}{4} + \frac{b}{2} + 6 = \frac{19}{2}$ or $\frac{a}{4} + \frac{b}{2} = \frac{13}{4}$	A1	
	Obtain $a = -3$ and $b = 8$	A1	ISW
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Question	Answer	Marks	Guidance
4	$\frac{2 \pm \sqrt{(-2)^2 - 4(3+i)(3-i)}}{2(3+i)}$	M1	Use quadratic formula to solve for w
	Use $i^2 = -1$ in $(3+i)(3-i)$	M1	
	Obtain one of the answers $w = \frac{2+6i}{6+2i}$ or $w = \frac{2-6i}{6+2i}$	A1	Must be simplified to this form.
	Show intention to multiply numerator and denominator by conjugate of their denominator.	M1	Independent of previous M marks but must be of the same form, e.g. $\frac{a}{b+ci}$.
	Obtain final answers $\frac{3}{5} + \frac{4}{5}i$ and $-i$ Accept $0.6 + 0.8i$ and $0 - i$	A1	SC Both correct final answers from $w = \frac{2+6i}{6+2i}$ and $w = \frac{2-6i}{6+2i}$ seen, no evidence of conjugate, then SC B1 for both. Allow $x = \frac{3}{5}, y = \frac{4}{5}$ or $x = 0, y = -1$. A0 for $\frac{3+4i}{5}$.
	Alternative method for Question 4		
	Multiply the equation by $3-i$	M1	
	Use $i^2 = -1$ in $(3+i)(3-i)$	M1	
	Obtain $10w^2 - 2(3-i)w + (3-i)^2 = 0$ or equivalent	A1	
	Use quadratic formula or factorise to solve for w	M1	

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Question	Answer	Marks	Guidance
4	Obtain final answers $\frac{3}{5} + \frac{4}{5}i$ and $-i$ Accept $0.6 + 0.8i$ and $0 - i$	A1	SC Both correct final answers from $10w^2 - 2(3 - i)w + (3 - i)^2 = 0$ with no working then SC B1 for both. Allow $x = \frac{3}{5}, y = \frac{4}{5}$ or $x = 0, y = -1$. A0 for $\frac{3+4i}{5}$.
Alternative method for Question 4			
	Substitute $w = x + iy$ and form equations for real and imaginary parts	M1	
	Use $i^2 = -1$ in $(x + iy)^2$	M1	
	Obtain $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$	A1	OE
	Form quartic equation in x only or y only using the correct substitution and solve for x or y	M1	Use correct $y = \frac{(3-x)}{10x-3}$ to attempt to form and solve $50x^4 - 60x^3 + 63x^2 - 27x = 0$ $x(5x-3)(10x^2-6x+9) = 0$. Use correct $x = \frac{3(1+y)}{10y+1}$ to attempt to form and solve $100y^4 + 40y^3 - 66y^2 - 14y - 8 = 0$ $(y+1)(5y-4)(20y^2+4y+2) = 0$.
	Obtain final answers $\frac{3}{5} + \frac{4}{5}i$ and $-i$ Accept $0.6 + 0.8i$ and $0 - i$	A1	SC Both correct final answers from $3(x^2 - y^2) - 2xy - 2x + 3 = 0$ and $x^2 - y^2 + 6xy - 2y - 1 = 0$ with no working then SC B1 for both. Allow $x = \frac{3}{5}, y = \frac{4}{5}$ or $x = 0, y = -1$. A0 for $\frac{3+4i}{5}$.
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Question	Answer	Marks	Guidance
5	Use correct product or quotient rule	M1	Need attempt at both derivatives condone errors in chain rule. In quotient rule allow BOD in formula if $\pm 2x$ seen unless clear that incorrect formula has been used. If omit denominator or forget to square or complete reversal of signs then M0 A0 M1 A1 A1 A1.
	Obtain correct derivative in any form, e.g. $\frac{6x(1-x^2)e^{3x^2-1} + 2xe^{3x^2-1}}{(1-x^2)^2}$	A1	If $6x(1-x^2)e^{3x^2-1} + 2xe^{3x^2-1} = 0$ from the start, with no wrong formula seen, award M1A1.
	Equate derivative (or its numerator) to zero and solve for x	M1	$6x - 6x^3 + 2x = 0$ and solve. Allow for just one x value. Allow if from solution of 3 term quadratic equation, but if they get $x = 0$ the x must factorise out
	Obtain the point $(0, e^{-1})$ or exact equivalent	A1	Or for all three x coordinates found 0, $\pm \frac{2\sqrt{3}}{3}$ oe and no extras but if this is the case then one pair of correct coordinates A1 and both other pairs of correct coordinates A1. Accept, e.g. $x = 0, y = e^{-1}$ ISW for last 3 marks.
	Obtain the point $\left(\frac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	A1	Allow $\sqrt[3]{(4/3)}$.
	Obtain the point $\left(-\frac{2\sqrt{3}}{3}, -3e^3\right)$ or exact equivalent	A1	
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Question	Answer	Marks	Guidance
6(a)	Use correct Pythagoras $\cot^2 \theta = \operatorname{cosec}^2 \theta - 1$ or $\cot^2 \theta = 1/\sin^2 \theta - 1$ or $\cot^2 \theta = \cos^2 \theta / \sin^2 \theta$ and then $\cos^2 \theta = 1 - \sin^2 \theta$, together with double angle formula $\cos 2\theta = 1 - 2\sin^2 \theta$, to obtain an equation in $\sin \theta$ or $\sin \theta$ and $\operatorname{cosec}^2 \theta$	M1	If consistent omission of brackets, e.g. $(\sin \theta)^2$ written as $\sin \theta^2$ then SC B1 in place of M1A1.
	Obtain a correct equation in $\sin \theta$ in any form	A1	e.g. $1/\sin^2 \theta - 1 + 2(1 - 2\sin^2 \theta) = 4$ or $\frac{1 - \sin^2}{\sin^2} + 2(1 - 2\sin^2) = 4$. If $\frac{\cos^2}{\sin^2} + 2(1 - 2\sin^2) = 4$ then e.g. $1 - \sin^2 + 2(1 - 2\sin^2)\sin^2 = 4$. (missing $\sin^2$ on right) allow M1A1A0.
	Reduce to the given answer of $4\sin^4 \theta + 3\sin^2 \theta - 1 = 0$ correctly	A1	AG Must follow from a horizontal equation (no denominators). If $s = \sin \theta$ used and defined, allow all marks. If not defined, award M1A1A0.
		3	

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Question	Answer	Marks	Guidance
6(b)	Solve the given quadratic to obtain a value for θ	M1	$(4\sin^2\theta - 1)(\sin^2\theta + 1) = 0$ and solve for θ . Incorrect sign in solution of quadratic seen, e.g. $(4\sin^2\theta - 1)(\sin^2\theta - 1) = 0$ then M0 A0 A0 but if only see $(4\sin^2\theta - 1) = 0$ and nothing incorrect seen allow 3/3.
	Obtain answer, e.g. $\theta = 30^\circ$	A1	$\pi/6$ award A0
	Obtain three further answers, e.g. $\theta = 150^\circ, 210^\circ$ and 330° and no others in the interval	A1	Ignore any answers outside interval. $5\pi/6 \ 7\pi/6 \ 11\pi/6$ award A1.
		3	

Question	Answer	Marks	Guidance
7(a)	State or imply $2y\frac{dy}{dx}$ as the derivative of y^2	B1	Allow for $3x^2dx + 2ydy$ or $F_x = 3x^2 + 6x$ and $F_y = 2y + 3$.
	Equate derivative of LHS to zero and solve for $\frac{dy}{dx}$	M1	$3x^2 + 2y\frac{dy}{dx} + 6x + 3\frac{dy}{dx} = 0$ or $3x^2dx + 2ydy + 6xdx + 3dy = 0$ or $\frac{dy}{dx} = -\frac{F_x}{F_y}$ need evidence from B1 mark or formula must be seen. Allow errors.
	Obtain the given answer	A1	AG $\frac{dy}{dx} = -\frac{3x^2 + 6x}{2y + 3}$ not $\frac{-3x^2 - 6x}{2y + 3}$. Must factorise with $\frac{dy}{dx}$ e.g. $3x^2 + 6x + \frac{dy}{dx}(2y + 3) = 0$ or $3x^2dx + 6xdx + dy(2y + 3) = 0$.
		3	

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Question	Answer	Marks	Guidance
7(b)	Equate numerator to zero and solve for x	*M1	Allow for just one x value.
	Obtain $x = 0$ and $x = -2$ only	A1	
	Substitute their x , [$x = 0$ or $x = -2$] in curve equation to obtain quadratic equation in y equal to 0	DM1	$y^2 + 3y - 4 = 0$ or $y^2 + 3y = 0$.
	Obtain $y = 1$ and $y = -4$ [when $x = 0$]	A1	
	Obtain $y = 0$ and $y = -3$ [when $x = -2$]	A1	ISW If forget $x = 0$ then max 3/5.
		5	

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Question	Answer	Marks	Guidance
8	Separate variables correctly and reach $a \sec^2 3y$ or be^{-4x}	B1	Condone missing integral signs or dy and dx , but allow if recognisable integrals follow. Not for $1/\cos^2 3y$ and $1/e^{4x}$.
	Obtain term $-\frac{1}{4}e^{-4x}$	B1	Can recover the previous B1 if de^{-4x} seen here.
	Obtain only a term of the form $a \tan 3y$	M1	Can recover the first B1 if $a \tan 3y$ seen here.
	Obtain term $\frac{1}{3} \tan 3y$	A1	
	Use $x = 2, y = 0$ to evaluate a constant or as limits in a solution containing terms of the form $a \tan by$ and $ce^{\pm 4x}$	M1	May see $\tan by$ and $e^{\pm 4x}$ here.
	Obtain correct answer in any form	A1	e.g. $\frac{1}{3} \tan 3y = -\frac{1}{4}e^{-4x} + \frac{1}{4}e^{-8}$ or $\frac{1}{3} \tan 3y = -\frac{1}{4}e^{-4x} + 8.39 \times 10^{-5}$
	Obtain final answer $y = \frac{1}{3} \tan^{-1} \left(\frac{3}{4}e^{-8} - \frac{3}{4}e^{-4x} \right)$	A1	ISW OE e.g. $y = \frac{1}{3} \tan^{-1} \left(2.52 \times 10^{-4} - \frac{3}{4}e^{-4x} \right)$
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Question	Answer	Marks	Guidance
9(a)	State or imply the form $\frac{Ax+B}{2+3x^2} + \frac{C}{2-x}$	B1	If incorrect partial fractions e.g. $A = 0$ or $Ax^2 + B$ then M1, A1 A0 for correct C. Only allow single A1 even if other coefficients correct. B1 recoverable by a correct form end statement.
	Use a correct method for finding a coefficient	M1	e.g. $(Ax+B)(2-x) + C(2+3x^2)$ $= (3C-A)x^2 + (2A-B)x + (2B+2C)$ $= 17x^2 - 7x + 16.$
	Obtain one of $A = -2$, $B = 3$ and $C = 5$	A1	If error present in above still allow A1 for C.
	Obtain a second value	A1	
	Obtain the third value	A1	Extra term in partial fractions, $D/(2+3x^2)$, that is 4 unknowns A , B , C and D then B0 unless recover at end, e.g. by setting B or $D = 0$. If B or D set to any value other than 0 and all coefficients correctly found to their new values then allow all A marks, but still B0 for partial fraction expression unless $B + D$ combined. Hence A1 for each coefficient, but nothing for coefficient set to specific value. Another case of extra term in partial fraction expression, namely $+F$, mark as above but need $F = 0$ to recover B1.
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Question	Answer	Marks	Guidance
9(b)	Use a correct method to find the first two terms of the expansion $(2 - x)^{-1} = 2^{-1} + (-1) 2^{-2}(-x) + [(-1)(-2)2^{-3}(-x)^2/2!]$, $\left(1 + \frac{3x^2}{2}\right)^{-1} = 1 - \frac{3x^2}{2}$ or $\left(1 - \frac{x}{2}\right)^{-1} = 1 - \left(\frac{-x}{2}\right)$	M1	Symbolic coefficients are not sufficient for the M1.
	$\frac{Ax+B}{2} \left[1 + (-1)\frac{3x^2}{2} \dots \right] \quad A = -2 \quad B = 3$ $\frac{C}{2} \left[1 + (-1)\left(\frac{-x}{2}\right) + \frac{(-1)(-2)}{2}\left(\frac{-x}{2}\right)^2 \right]$ $\quad + \frac{(-1)(-2)(-3)}{6}\left(\frac{-x}{2}\right)^3 \dots \quad C = 5$	A1 FT	Obtain correct un-simplified expansions up to the term in x^3 of each partial fraction.
	$= \frac{3-2x}{2} \left(1 - \frac{3x^2}{2}\right) + \frac{5}{2} \left(1 + \frac{x}{2} + \frac{x^2}{4} + \frac{x^3}{8}\right)$ $= \left(\frac{3}{2} + \frac{5}{2}\right) + \left(-1 + \frac{5}{4}\right)x + \left(-\frac{9}{4} + \frac{5}{8}\right)x^2 + \left(\frac{3}{2} + \frac{5}{16}\right)x^3$	A1 FT	Un-simplified $(2 - x)^{-1}$ expanded correctly, error in simplifying before their C is involved in the expression, allow A1FT when their C is introduced. The FT is on A, B, C .
	Multiply expansion of $\left(1 + \frac{3x^2}{2}\right)^{-1}$ (must reach $1 \pm \frac{3x^2}{2}$) by $Ax + B$, where $AB \neq 0$, up to the term in x^3 . Allow if used $Cx + D$ ($Ax + B$ miscopied).	M1	Allow either ± 2 or $\pm 2^{-1}$ outside bracket or missing. Allow one error in actual multiplication to acquire the 4 terms [all terms needed]. Ignore errors in higher powers.

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Question	Answer	Marks	Guidance
9(b)	Obtain final answer $4 + \frac{1}{4}x - \frac{13}{8}x^2 + \frac{29}{16}x^3$, or equivalent [If final answer has been multiplied throughout, e.g. by 16 then A0 at the end]	A1	Maclaurin's Series: $f(0) = 4$ B1 $f'(0) = 1/4$ B1. $f''(0) = -13/4$ and $f'''(0) = 87/8$ B1. $4 + \frac{1}{4}x - \frac{\frac{13}{4}x^2}{2} + \frac{\frac{87}{8}x^3}{6}$ or equivalent M1 A1. If $1 + \frac{1}{4}x - \frac{\frac{13}{4}x^2}{2} + \frac{\frac{87}{8}x^3}{6}$ then M0 A0 unless their $f(0)$ actually is 1.
		5	
9(c)	State answer $ x < \sqrt{\frac{2}{3}}$ or $-\sqrt{\frac{2}{3}} < x < \sqrt{\frac{2}{3}}$ clear conclusion required	B1	Or exact equivalent. Strict inequality.
		1	

Question	Answer	Marks	Guidance
10(a)	Use the product rule correctly on $y = x \cos 2x$	M1	$dx/dx \cos 2x + x d/dx(\cos 2x)$ attempted.
	Obtain the correct derivative in any form	A1	e.g. $\cos 2x - 2x \sin 2x$. If $\cos 2x + x - 2\sin 2x$, not recovered, max M1A0A1FTA0 but can recover for full marks by seeing correct substitution.
	Obtain $y = -\frac{\pi}{2}$ and $\frac{dy}{dx} = -1$ when $x = \frac{\pi}{2}$	A1FT	FT their $\frac{dy}{dx}$ with $x = \frac{\pi}{2}$ substituted.
	Obtain answer $x + y = 0$	A1	OE CWO Need to see y and dy/dx at $x = \frac{\pi}{2}$.
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Question	Answer	Marks	Guidance
10(b)	Integrate by parts and reach $ax \sin 2x + b \int \sin 2x dx$	*M1	
	Obtain $\frac{1}{2}x \sin 2x - \frac{1}{2} \int \sin 2x dx$	A1	OE
	Complete integration and obtain $\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x$	A1	OE
	Use limits of $x = 0$ and $x = \frac{\pi}{4}$ in the correct order, having integrated twice to obtain $ax \sin 2x + c \cos 2x$	DM1	If correct, $\frac{1}{2} \left(\frac{\pi}{4} \right) \sin \frac{2\pi}{4} + \frac{1}{4} \cos \frac{2\pi}{4} - \frac{1}{4} \cos 0$ or $\frac{1}{2} \left(\frac{\pi}{4} \right) \sin \frac{2\pi}{4} - \frac{1}{4} \cos 0$. Max one substitution error.
	Obtain answer $\frac{\pi}{8} - \frac{1}{4}$ or exact simplified two term equivalent	A1	ISW Accept $\frac{\pi - 2}{8}$. Accept $\frac{1}{2}x \sin 2x + \frac{1}{4} \cos 2x$ then final answer.
		5	

Question	Answer	Marks	Guidance
11(a)	Use correct process for modulus on direction vector of l , e.g. $\sqrt{(-1)^2 + 1^2 + 2^2}$	M1	SOI Allow -1^2 . Allow $\sqrt{(-\lambda)^2 + \lambda^2 + (2\lambda)^2}$.
	$[\pm] \frac{1}{\sqrt{6}}(-i + j + 2k)$	A1	OE Allow coordinates as row or column, but not row or column with i , j and k included.
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Question	Answer	Marks	Guidance																																
11(b)	Use a correct method to form an equation for line m	M1	Allow even if all signs of point incorrect, namely use $+2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ or $-3\mathbf{i} + \mathbf{j} - \mathbf{k}$.																																
	Obtain $\mathbf{r} = -2\mathbf{i} + 2\mathbf{j} - \mathbf{k} + \mu_1(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$	A1	OE, e.g. $\mathbf{r} = 3\mathbf{i} - \mathbf{j} + \mathbf{k} + \mu_2(-5\mathbf{i} + 3\mathbf{j} - 2\mathbf{k})$ Must have $\mathbf{r} = \dots$																																
		2																																	
11(c)	Justify lines are not parallel	B1	$(-5, 3, -2) \neq d(-1, 1, 2)$ or $(-5, 3, -2)\mathbf{x}(-1, 1, 2) \neq 0$. Can find angle (105° , 74.6° , 1.84° or $1.3(0)^\circ$) instead but if incorrect B0 and A0 at end. Accept direction vectors don't have common factor but not direction vectors are not equal or direction vectors are different or $\mu \neq \lambda$ or scalar product $\neq 0$. Not the line equations are not multiples of each other.																																
	Express l or m in component form e.g. $(-2 - 5\mu_1, 2 + 3\mu_1, -1 - 2\mu_1)$ or $(3 - 5\mu_2, -1 + 3\mu_2, 1 - 2\mu_2)$ or $(1 - \lambda, -2 + \lambda, -3 + 2\lambda)$	B1																																	
	Equate two pairs of components of general points on l and m and solve simultaneously for λ or for μ	M1																																	
	Obtain correct answer for λ or μ , e.g. $\lambda = \frac{11}{2}, \mu_1 = \frac{1}{2}$	A1																																	
	Determine that all three equations are not satisfied and the lines fail to intersect and conclude the lines are skew. Conclusion needs to follow correct working	A1	<table><tr><td>1</td><td>λ</td><td>μ_1</td><td></td><td>2</td><td>λ</td><td>μ_2</td><td></td></tr><tr><td>ij</td><td>11/2</td><td>1/2</td><td>$8 \neq -2$</td><td>ij</td><td>11/2</td><td>3/2</td><td>$8 \neq -2$</td></tr><tr><td>ik</td><td>4/3</td><td>-1/3</td><td>$-2/3 \neq 1$</td><td>ik</td><td>4/3</td><td>2/3</td><td>$-2/3 \neq 1$</td></tr><tr><td>jk</td><td>7/4</td><td>-3/4</td><td>$-3/4 \neq 7/4$</td><td>jk</td><td>7/4</td><td>1/4</td><td>$-3/4 \neq 7/4$</td></tr></table> Dependent on 4 previous marks gained.	1	λ	μ_1		2	λ	μ_2		ij	11/2	1/2	$8 \neq -2$	ij	11/2	3/2	$8 \neq -2$	ik	4/3	-1/3	$-2/3 \neq 1$	ik	4/3	2/3	$-2/3 \neq 1$	jk	7/4	-3/4	$-3/4 \neq 7/4$	jk	7/4	1/4	$-3/4 \neq 7/4$
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		5																																	