

Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

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- 1 (a) Sketch the graph of $y = |2x + 1|$.

[1]

- (b) Solve the inequality $3x + 5 < |2x + 1|$.

[3]

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- 2 On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z| \leq 3$, $\operatorname{Re} z \geq -2$ and $\frac{1}{4}\pi \leq \arg z \leq \pi$. [4]

- 3 Solve the equation $2^{3x-1} = 5(3^{-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

[illegible]

- 4** Solve the equation $\tan(x + 45^\circ) = 2 \cot x$ for $0^\circ < x < 180^\circ$. [5]

[illegible]

- 5** The complex numbers u and w are defined by $u = 2e^{\frac{1}{4}\pi i}$ and $w = 3e^{\frac{1}{3}\pi i}$.

- (a) Find $\frac{u^2}{w}$, giving your answer in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$. Give the exact values of r and θ . [3]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (b) State the least positive integer n such that both $\operatorname{Im} w^n = 0$ and $\operatorname{Re} w^n > 0$. [1]

6 (a) Prove the identity $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4 \theta$.

[4]

[illegible]

[3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 7** The equation of a curve is $y = \frac{x}{\cos^2 x}$, for $0 \leq x < \frac{1}{2}\pi$. At the point where $x = a$, the tangent to the curve has gradient equal to 12.

(a) Show that $a = \cos^{-1} \left(\sqrt[3]{\frac{\cos a + 2a \sin a}{12}} \right)$. [3]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- (b) Verify by calculation that a lies between 0.9 and 1.

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- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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- 8 In a certain chemical reaction the amount, x grams, of a substance is increasing. The differential equation satisfied by x and t , the time in seconds since the reaction began, is

$$\frac{dx}{dt} = kxe^{-0.1t},$$

where k is a positive constant. It is given that $x = 20$ at the start of the reaction.

- (a) Solve the differential equation, obtaining a relation between x , t and k . [5]

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- (b) Given that $x = 40$ when $t = 10$, find the value of k and find the value approached by x as t becomes large. [3]

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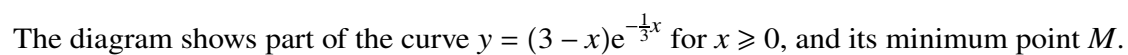
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- [5]

[illegible]

- (b) Find the area of the shaded region bounded by the curve and the axes, giving your answer in terms of e . [5]

This image shows a full page of a handwriting practice worksheet. It consists of approximately 20 horizontal rows. Each row is defined by two parallel dotted lines, creating a series of uniform gaps for letter height. The entire page is otherwise blank, with no margins, text, or other markings.

10 Let $f(x) = \frac{2x^2 + 7x + 8}{(1+x)(2+x)^2}$.

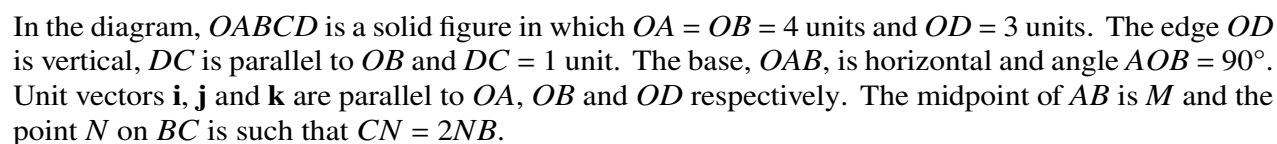
(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

[illegible]



- [illegible]

- (b) Calculate the angle in degrees between the directions of $\overrightarrow{MD}$ and $\overrightarrow{ON}$. [3]

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- (c) Show that the length of the perpendicular from M to ON is $\sqrt{\frac{22}{5}}$. [4]

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[illegible]

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2022

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Any blank pages are indicated.

- 1 Solve the equation $2^{3x-1} = 5(3^{1-x})$. Give your answer in the form $\frac{\ln a}{\ln b}$ where a and b are integers. [4]

[illegible]

- 2 The polynomial $2x^3 - x^2 + a$, where a is a constant, is denoted by $p(x)$. It is given that $(2x + 3)$ is a factor of $p(x)$.

(a) Find the value of a .

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(b) When a has this value, solve the inequality $p(x) < 0$.

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- 3** The equation of a curve is $y = \sin x \sin 2x$. The curve has a stationary point in the interval $0 < x < \frac{1}{2}\pi$.
Find the x -coordinate of this point, giving your answer correct to 3 significant figures. [6]

[illegible]

- 4 (a) Express $4 \cos x - \sin x$ in the form $R \cos(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

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- (b) Hence solve the equation $4 \cos 2x - \sin 2x = 3$ for $0^\circ < x < 180^\circ$. [5]

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- 5 (a) Solve the equation $z^2 - 6iz - 12 = 0$, giving the answers in the form $x + iy$, where x and y are real and exact. [3]

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- (b) On a sketch of an Argand diagram with origin O , show points A and B representing the roots of the equation in part (a). [1]

(c) Find the exact modulus and argument of each root.

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(d) Hence show that the triangle OAB is equilateral.

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6 Relative to the origin O , the points A , B and C have position vectors given by

$$\overrightarrow{OA} = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 5 \\ 3 \\ -2 \end{pmatrix}.$$

(a) Using a scalar product, find the cosine of angle BAC .

[4]

[illegible]

(b) Hence find the area of triangle ABC . Give your answer in a simplified exact form. [4]

- 7 The variables x and θ satisfy the differential equation

$$x \sin^2 \theta \frac{dx}{d\theta} = \tan^2 \theta - 2 \cot \theta,$$

for $0 < \theta < \frac{1}{2}\pi$ and $x > 0$. It is given that $x = 2$ when $\theta = \frac{1}{4}\pi$.

- (a) Show that $\frac{d}{d\theta}(\cot^2 \theta) = -\frac{2 \cot \theta}{\sin^2 \theta}$.

(You may assume without proof that the derivative of $\cot \theta$ with respect to θ is $-\operatorname{cosec}^2 \theta$.) [1]

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- (b) Solve the differential equation and find the value of x when $\theta = \frac{1}{6}\pi$. [7]

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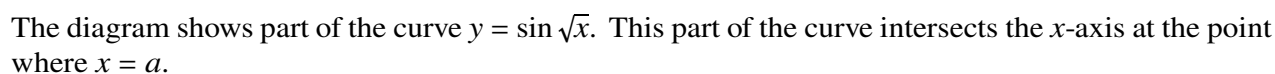
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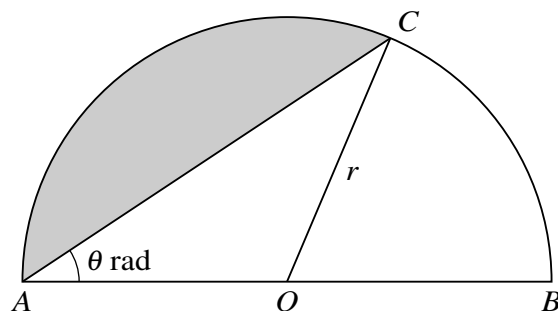
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- [illegible]



The diagram shows a semicircle with diameter AB , centre O and radius r . The shaded region is the minor segment on the chord AC and its area is one third of the area of the semicircle. The angle CAB is θ radians.

- (a) Show that $\theta = \frac{1}{3}(\pi - 1.5 \sin 2\theta)$. [4]

[illegible]

- (b) Verify by calculation that $0.5 < \theta < 0.7$.

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- (c) Use an iterative formula based on the equation in part (a) to determine θ correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

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10 Let $f(x) = \frac{4 - x + x^2}{(1 + x)(2 + x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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- (b) Find the exact value of $\int_0^4 f(x) \, dx$. Give your answer as a single logarithm. [5]

[illegible]

[illegible]

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- 1 Solve the equation $\ln(2x - 1) = 2\ln(x + 1) - \ln x$. Give your answer correct to 3 decimal places. [4]

[illegible]

- 2 Expand $\sqrt{\frac{1+2x}{1-2x}}$ in ascending powers of x , up to and including the term in x^2 , simplifying the coefficients. [5]

[illegible]

3 Find the exact value of $\int_0^{\frac{1}{4}\pi} x \sec^2 x \, dx$.

[5]

[illegible]

- 4 The parametric equations of a curve are

$$x = 2t - \tan t, \quad y = \ln(\sin 2t),$$

for $0 < t < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot t$.

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- 5 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z + 2| \leq 2$ and $\text{Im } z \geq 1$. [4]

- (b) Find the greatest value of $\arg z$ for points in the shaded region. [2]

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- 6** Solve the quadratic equation $(1 - 3i)z^2 - (2 + i)z + i = 0$, giving your answers in the form $x + iy$, where x and y are real. [6]

This image shows a full page of a worksheet designed for handwriting practice. It features approximately 20 evenly spaced horizontal dotted lines across the entire width of the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- 7 (a) Show that the equation $\sqrt{5} \sec x + \tan x = 4$ can be expressed as $R \cos(x + \alpha) = \sqrt{5}$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [4]

[illegible]

[Turn over

- 8** The curve with equation $y = \frac{x^3}{e^x - 1}$ has a stationary point at $x = p$, where $p > 0$.

(a) Show that $p = 3(1 - e^{-p})$.

[3]

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- (b) Verify by calculation that p lies between 2.5 and 3.

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- (c) Use an iterative formula based on the equation in part (a) to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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- 9 With respect to the origin O , the position vectors of the points A , B and C are given by

$$\overrightarrow{OA} = \begin{pmatrix} 0 \\ 5 \\ 2 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OC} = \begin{pmatrix} 4 \\ -3 \\ -2 \end{pmatrix}.$$

The midpoint of AC is M and the point N lies on BC , between B and C , and is such that $BN = 2NC$.

- (a) Find the position vectors of M and N . [3]

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- (b) Find a vector equation for the line through M and N . [2]

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- (c) Find the position vector of the point Q where the line through M and N intersects the line through A and B . [4]

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- 10** A gardener is filling an ornamental pool with water, using a hose that delivers 30 litres of water per minute. Initially the pool is empty. At time t minutes after filling begins the volume of water in the pool is V litres. The pool has a small leak and loses water at a rate of $0.01V$ litres per minute.

The differential equation satisfied by V and t is of the form $\frac{dV}{dt} = a - bV$.

- (a) Write down the values of the constants a and b . [1]

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- (b) Solve the differential equation and find the value of t when $V = 1000$. [6]

[illegible]

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- (c) Obtain an expression for V in terms of t and hence state what happens to V as t becomes large. [2]

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11 Let $f(x) = \frac{5 - x + 6x^2}{(3 - x)(1 + 3x^2)}$.

(a) Express $f(x)$ in partial fractions.

[5]

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- (b) Find the exact value of $\int_0^1 f(x) \, dx$, simplifying your answer. [5]

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