



Cambridge International A Level

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2022

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **17** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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Mathematics Specific Marking Principles	
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2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

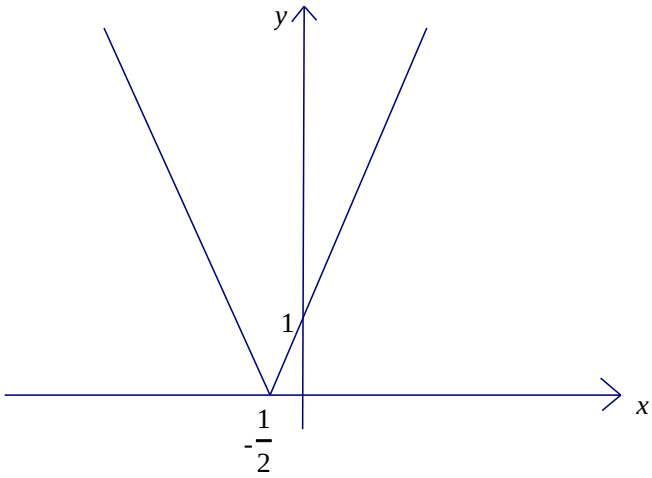
Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
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WWW	Without Wrong Working
AWRT	Answer Which Rounds To

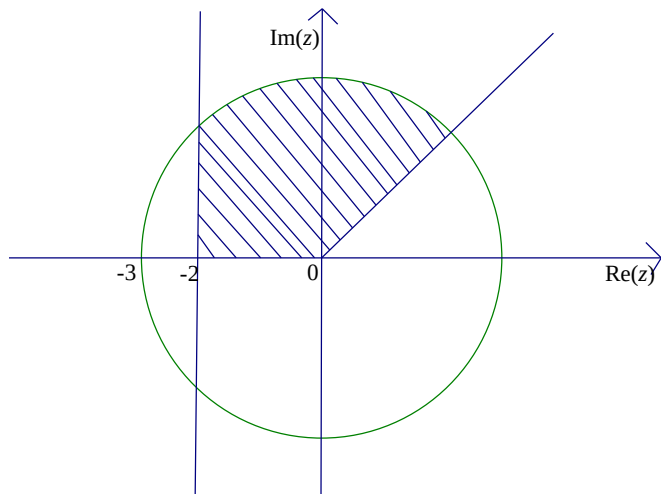
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Question	Answer	Marks	Guidance
1(a)	Show a recognisable sketch graph of $y = 2x + 1 $	B1	 Ignore $y = 3x + 5$ if also drawn on the sketch.
		1	

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Question	Answer	Marks	Guidance
1(b)	Find x-coordinate of intersection with $y = 3x + 5$	M1	
	Obtain $x = -\frac{6}{5}$	A1	
	State final answer $x < -\frac{6}{5}$ only	A1	Do not condone $\leq$ for $<$ in the final answer.
	Alternative method 1 for question 1(b)		
	Solve the linear inequality $3x + 5 < -(2x + 1)$, or corresponding equation	M1	Must solve the relevant equation.
	Obtain critical value $x = -\frac{6}{5}$	A1	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	A1	
	Alternative method 2 for question 1(b)		
	Solve the quadratic inequality $(3x + 5)^2 < (2x + 1)^2$, or corresponding equation	M1	$5x^2 + 26x + 24 < 0$
	Obtain critical value $x = -\frac{6}{5}$	A1	Ignore -4 if seen.
	State final answer $x < -\frac{6}{5}$ only	A1	
		3	

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Question	Answer	Marks	Guidance
2	Show a circle with radius 3 and centre the origin	B1	 For the vertical line and the circle, allow the B1 marks if all you see is the relevant part.
	Show the line $x = -2$	B1	
	Show the correct half line for $\frac{\pi}{4}$	B1	
	Shade the correct region	B1	
		4	

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Question	Answer	Marks	Guidance
3	Use law of logarithm of a product or power	M1	One correct application of a log law.
	Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2 = \ln 5 - x\ln 3$	A1	
	Solve for x	M1	As far as $x = \dots$ with only minor slips in processing.
	Obtain answer $x = \frac{\ln 10}{\ln 24}$	A1	
	Alternative method for question 3		
	Use laws of indices to split at least one exponential term	M1	e.g. $\frac{2^{3x}}{2}$ or an arrangement with 8^x
	Obtain $24^x = 10$	A1	OE
	Solve for x	M1	
	Obtain answer $x = \frac{\ln 10}{\ln 24}$	A1	
		4	

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Question	Answer	Marks	Guidance
4	Use correct $\tan(A+B)$ formula and obtain an equation in $\tan x$ or an equation in $\cos x$ and $\sin x$	M1	e.g. $\frac{\tan x + \tan 45^\circ}{1 - \tan x \tan 45^\circ} = \frac{2}{\tan x}$ Allow if 2 in denominator or $\frac{\sin x \cos 45^\circ + \cos x \sin 45^\circ}{\cos x \cos 45^\circ - \sin x \sin 45^\circ} = \frac{2 \cos x}{\sin x}$.
	Obtain correct 3 term equation $\tan^2 x + 3 \tan x - 2 = 0$, or equivalent	A1	or $3 \sin x \cos x = 2 \cos^2 x - \sin^2 x$
	Solve a 3-term quadratic in $\tan x$ and obtain a value for x	M1	
	Obtain answer, e.g. 29.3°	A1	29.316...
	Obtain second answer, e.g. 105.7° and no other	A1	105.583.... Ignore answers outside the given interval. Treat answers in radians as a misread.
		5	

Question	Answer	Marks	Guidance
5(a)	State or imply $u^2 = 4e^{\frac{1}{2}\pi i}$	B1	
	Obtain answer $v = \frac{4}{3}e^{\frac{1}{6}\pi i}$	B1 + B1	For the modulus and the argument.
		3	
5(b)	State $n = 6$	B1	
		1	

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Question	Answer	Marks	Guidance
6(a)	Express $\cos 4\theta$ in terms of $\cos 2\theta$ and/or $\sin 2\theta$	B1	
	Express $\cos 2\theta$ in terms of $\cos \theta$ and/or $\sin \theta$	B1	Anywhere
	Expand to obtain a correct expression in terms of $\cos \theta$	B1	e.g. $2(2\cos^2 \theta - 1)^2 - 1 + 4(2\cos^2 \theta - 1) + 3$
	Reduce correctly to $\cos 4\theta + 4\cos 2\theta + 3 \equiv 8\cos^4 \theta$	B1	AG
		4	
6(b)	Use the identity and carry out method to calculate a root	M1	$8\cos^4 \theta - 3 = 4$
	Obtain answer, e.g. 14.7°	A1	
	Obtain second answer, e.g. 165.3° , and no other in the given interval	A1 FT	Ignore answers outside the given interval. Treat answers in radians as a misread.
		3	

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Question	Answer	Marks	Guidance
7(a)	Use correct product or quotient rule	M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = \frac{\cos^2 x + 2x \sin x \cos x}{\cos^4 x}$ or $\frac{dy}{dx} = \sec^2 x + 2x \sec^2 x \tan x$
	Equate derivative at $x = a$ to 12 and obtain $a = \cos^{-1} \left(\sqrt[3]{\frac{\cos a + 2a \sin a}{12}} \right)$	A1	AG
		3	
7(b)	Evaluate a relevant expression or pair of expressions at $a = 0.9$ and $a = 1$	M1	Must be calculated in radians.
	Complete the argument correctly with correct calculated values	A1	e.g. $\cos 0.9 = 0.622 > 0.553$ or $0.9 < 0.985$ or $0.0846 > 0$ $\cos 1 = 0.540 < 0.570$ or $1 > 0.964$ or $-0.0358 < 0$ or could be looking at values of the gradient 8.46 & 14.1
		2	
7(c)	Use the process $a_{n+1} = \cos^{-1} \left(\sqrt[3]{\frac{\cos a_n + 2a_n \sin a_n}{12}} \right)$ correctly at least once	M1	Must be working in radians.
	Obtain final answer 0.97	A1	
	Show sufficient iterations to 4 d.p. to justify 0.97 to 2 d.p., or show there is a sign change in the interval (0.965, 0.975)	A1	e.g. 0.95, 0.9743, 0.9694, 0.9704
		3	

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Question	Answer	Marks	Guidance
8(a)	Separate variables correctly	B1	$\int \frac{1}{x} dx = \int ke^{-0.1t} dt$
	Obtain term $\ln x$	B1	
	Obtain term $-10ke^{-0.1t}$	B1	Not from $\int xe^{-0.1t} dt$
	Use $x = 20, t = 0$ to evaluate a constant or as limits in a solution containing terms $a \ln x, be^{-0.1t}$ where $ab \neq 0$	M1	
	Obtain $\ln x = 10k(1 - e^{-0.1t}) + \ln 20$	A1	or equivalent ISW
		5	
8(b)	Use $x = 40, t = 10$ to find k or $10k$	M1	Available for their function of the correct structure even if they found no constant in (a) .
	Obtain $10k = 1.09654$	A1	or equivalent e.g. $10k = \frac{\ln 2}{1 - e^{-1}}$
	State that x tends to 59.9	A1	Need a number, not an expression for that value 3 sf or better 59.87595.....
		3	

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Question	Answer	Marks	Guidance
9(a)	Use correct product or quotient rule	*M1	
	Obtain correct derivative in any form	A1	e.g. $\frac{dy}{dx} = -e^{-\frac{x}{3}} - \frac{1}{3}(3-x)e^{-\frac{x}{3}}$
	Equate their derivative to zero and solve for x	DM1	
	Obtain $x = 6$	A1	
	Obtain $y = -3e^{-2}$	A1	Or exact equivalent.
		5	
9(b)	Commence integration and reach $a(3-x)e^{\frac{1}{3}x} + b\int e^{\frac{1}{3}x} dx$, where $ab \neq 0$	*M1	
	Obtain $-3(3-x)e^{\frac{1}{3}x} - 3\int e^{\frac{1}{3}x} dx$, or equivalent	A1	
	Complete integration and obtain $3xe^{\frac{1}{3}x}$, or equivalent	A1	$-3e^{-\frac{x}{3}}(3-x) + 9e^{-\frac{x}{3}}$
	Substitute limits $x = 0$ and $x = 3$, having integrated twice	DM1	
	Obtain answer $\frac{9}{e}$, or exact equivalent	A1	
		5	

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Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{1+x} + \frac{B}{2+x} + \frac{C}{(2+x)^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 3$, $B = -1$ and $C = -2$	A1	SR after B0 can score M1A1 for one correct value
	Obtain a second value	A1	
	Obtain the third value	A1	$\frac{A}{1+x} + \frac{Dx+E}{(2+x)^2}$, where $A = 3$, $D = -1$ and $E = -4$, is awarded B1 M1 A1 A1 A1 as above.
		5	
10(b)	Use a correct method to find the first two terms of the expansion of $(1+x)^{-1}$, $(2+x)^{-1}$, $\left(1+\frac{1}{2}x\right)^{-1}$, $(2+x)^{-2}$ or $\left(1+\frac{1}{2}x\right)^{-2}$	M1	For the A, D, E form of fractions, award M1 A1FT A1FT for the expanded partial fractions, then if $D \neq 0$, M1 for multiplying out fully, and A1 for the final answer.
	Obtain correct unsimplified expansions up to the term in x^2 of each partial fraction	A3 FT	$3(1 - x + x^2 \dots)$ $-\frac{1}{2}\left(1 - \frac{x}{2} + \frac{x^2}{4} \dots\right)$ $-\frac{2}{4}\left(1 - x + \frac{3}{4}x^2\right)$
	Obtain final answer $2 - \frac{9}{4}x + \frac{5}{2}x^2$	A1	
		5	

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Question	Answer	Marks	Guidance
11(a)	State $\overrightarrow{OM} = 2\mathbf{i} + 2\mathbf{j}$ or equivalent	B1	Can be implied by $\overrightarrow{MB} = -2\mathbf{i} + 2\mathbf{j}$ or $\overrightarrow{MA} = 2\mathbf{i} - 2\mathbf{j}$.
	Obtain $\overrightarrow{MD} = -2\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	e.g. $\overrightarrow{OC} + \frac{2}{3}\overrightarrow{CB}$
	Obtain answer $3\mathbf{j} + \mathbf{k}$	A1	
		4	
11(b)	Use the correct process for evaluating the scalar product of $\overrightarrow{MD}$ and $\overrightarrow{ON}$	M1	
	Using the correct process for the moduli, divide the scalar product by the product of the moduli and reach the inverse cosine of the result	M1	$\cos^{-1}\left(\frac{-6+3}{\sqrt{10}\sqrt{17}}\right)$
	Obtain final answer 103.3°	A1	
		3	

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Question	Answer	Marks	Guidance
11(c)	Taking a general point P of ON to have position vector $\lambda(3\mathbf{j} + \mathbf{k})$, form an equation in λ by <i>either</i> equating the scalar product of $\overrightarrow{ON}$ and $\overrightarrow{MP}$ to zero, <i>or</i> applying Pythagoras to triangle OMP , <i>or</i> equating the derivative of $ \overrightarrow{MP} $ to zero	M1	e.g. $\begin{pmatrix} -2 \\ -2+3\lambda \\ \lambda \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} = 0$
	Solve and obtain $\lambda = \frac{3}{5}$	A1	
	Substitute for λ and calculate MP	M1	$\overrightarrow{MP} = -2\mathbf{i} - \frac{1}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$
	Obtain $\sqrt{\frac{22}{5}}$	A1	AG
	Alternative method for question 11(c)		
	Use a scalar product to find the projection OQ of OM on OM	M1	
	Obtain $OQ = \frac{6}{\sqrt{10}}$	A1	
	Use Pythagoras in triangle OMQ to find MQ	M1	
	Obtain $\sqrt{\frac{22}{5}}$	A1	AG
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Question	Answer	Marks	Guidance
1	Use law of the logarithm of a power or product	M1	Ignoring the 3 or the 5 is not a misread.
	Obtain a correct linear equation in any form, e.g. $(3x-1)\ln 2 = \ln 5 + (1-x)\ln 3$	A1	Condone invisible brackets if they are used correctly later.
	Solve for x	M1	Get as far as $x = \dots$ Condone minor slips in the processing e.g. sign errors and losing a term that had been there, but award M0 for a fundamental error e.g. $3x\ln 2 + x\ln 3 = 3x\ln 6$ or ignoring the 3 or the 5 completely. Condone working in decimals.
	Obtain final answer $x = \frac{\ln 30}{\ln 24}$	A1	Do not ISW
	Alternative method for question 1		
	Use laws of indices to split at least one exponential term	M1	e.g. $\frac{2^{3x}}{2}$ or an arrangement with 8^x and/or 3^x .
	Obtain $24^x = 30$	A1	Or equivalent e.g. $3^x 8^x = 30$ not for $3^x 2^{3x} = 30$ (need two factors with the same index).
	Solve for x	M1	Get as far as $x = \dots$
	Obtain final answer $x = \frac{\ln 30}{\ln 24}$	A1	Do not ISW
		4	

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Question	Answer	Marks	Guidance
2(a)	Substitute $x = -\frac{3}{2}$ and equate result to zero	M1	Or divide by $2x + 3$ and set constant remainder equal to zero. Or state $(2x^3 - x^2 + a) = (2x + 3)(x^2 + px + q)$, compare coefficients and solve for p or q .
	Obtain $a = 9$	A1	
		2	

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Question	Answer	Marks	Guidance
2(b)	Commence division by $(2x+3)$ reaching a partial quotient $x^2 + kx$	*M1	The M1 is earned if inspection reaches an unknown factor: $x^2 + Bx + C$ and an equation in B and/or C , or an unknown factor $Ax^2 + Bx + 3$ and an equation in A and/or B .
	Obtain factorisation $(2x+3)(x^2 - 2x + 3)$	A1	Allow if the correct quotient seen. Correct factors seen in (a) and quoted or used here scores M1A1.
	Show that $x^2 - 2x + 3$ is always positive, or $2x^3 - x^2 + 9$ only intersects the x -axis once	DM1	Must use their quadratic factor. SC If M0, allow B1 if state $x < -\frac{3}{2}$ and no error seen
	State final answer $x < -\frac{3}{2}$ from correct work	A1	
		4	

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Question	Answer	Marks	Guidance
3	Use correct product rule on given expression	*M1	
	Obtain correct derivative in any form	A1	e.g. $\cos x \sin 2x + 2 \sin x \cos 2x$
	Use correct double angle formulae to express derivative in terms of $\sin x$ and $\cos x$	*M1	
	Equate derivative to zero and obtain an equation in one trig variable	DM1	dependent on the 2 previous M Marks.
	Obtain $3\sin^2 x = 2$, $3\cos^2 x = 1$ or $\tan^2 x = 2$	A1	OE
	Solve and obtain $x = 0.955$	A1	3 sf only. Final answer in degrees is A0. Ignore any attempt to find the corresponding value of y .
	Alternative method for the first three marks		
	Use correct double angle formula to obtain $y = 2 \cos x - 2 \cos^3 x$	*M1	or $y = 2 \sin^2 x \cos x$
	Use chain rule and / or product rule	*M1	
	Obtain derivative $y' = -2 \sin x + 6 \sin x \cos^2 x$	A1	$y' = -2 \sin^3 x + 4 \sin x \cos^2 x$
	Alternative method for the second and third M marks		
	Equate derivative to zero and obtain an equation in $\tan x$ and $\tan 2x$	*M1	
	Use correct double angle formula to obtain an equation in $\tan x$	DM1	
		6	

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Question	Answer	Marks	Guidance
4(a)	State $R = \sqrt{17}$	B1	Allow if working from an incorrect expansion but not from decimals.
	Use correct trig formulae to find α (Correct expansion and correct expression for trig ratio for α)	M1	NB: $\cos \alpha = 4$ and $\sin \alpha = 1$ scores M0A0. M0 for incorrect expansion of $\cos(x - \alpha)$ M1 for correct expression for trig ratio for α and no errors seen.
	Obtain $\alpha = 14.04^\circ$	A1	2 d.p. required Allow M1A1 for correct answer with no working shown. Correct answer from incorrect working (e.g. $\tan^{-1}\left(-\frac{1}{4}\right)$) is awarded M0A0. $180^\circ - \tan^{-1}\left(-\frac{1}{4}\right)$ is awarded M1
		3	
4(b)	Evaluate $\cos^{-1}\left(\frac{3}{\sqrt{17}}\right)$ to at least 1 d.p. (43.31 38...°)	B1 FT	FT <i>their R</i> . Accept awrt 43.3° or awrt 316.7° Can be implied by subsequent working.
	Use correct method to find a value of x in the interval	M1	Must be working with $2x$ and <i>their</i> α .
	Obtain answer, e.g. 14.6°	A1	Accept overspecified answers but they need to be correct. (14.6388.... and 151.3249...).
	Use a correct method to find a second answer in the interval	M1	Must be working with $2x$, <i>their</i> α and $360^\circ - \text{their } 43.3$.
	Obtain second answer in the interval, e.g. 151.3° , and no other in the interval	A1	Ignore answers outside the given interval. Treat answers in radians (0.255... and 2.64...) as a misread.
		5	

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Question	Answer	Marks	Guidance
5(a)	Use quadratic formula, or completing the square $((z - 3i)^2 - 3 = 0)$ and use $i^2 = -1$ to find a root	M1	
	Obtain a root, e.g. $\sqrt{3} + 3i$	A1	Or exact 2 term equivalent e.g. $\frac{6i}{2} + \frac{\sqrt{12}}{2}$ ISW.
	Obtain the other root, e.g. $-\sqrt{3} + 3i$	A1	Or exact 2 term equivalent ISW.
		3	
5(b)	Show points representing the roots correctly	B1 FT	2 roots consistent with <i>their (a)</i> and with no errors seen on the diagram. B0 if they only have one root or more than 2 roots Must match their scale and $1 < \sqrt{3} < 2$ Linear scales seen or implied. Need some indication of scale (numbers or dashes). Scales along an axis must be approximately consistent but scales may be different on the 2 axes.
		1	

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Question	Answer	Marks	Guidance
5(c)	State modulus of either root is $2\sqrt{3}$, or simplified exact equivalent	B1 FT	ISW if converted to decimal . Ignore modulus of second root if seen. Follow their root(s) not on either axis (from (a) or (b)).
	Find the argument of one of their roots – get as far as $\tan^{-1}(\dots)$	M1	SOI but must be correct for their root.
	Obtain correct arguments $\frac{1}{3}\pi$ and $\frac{2}{3}\pi$, or simplified exact equivalents	A1	Must obtain values. Allow degrees.
		3	
5(d)	Give a complete justification that the correct triangle is equilateral	B1	Check <i>their</i> diagram in (b) . Possible justifications: 3 equal sides, or all angles equal to $\frac{\pi}{3}$, or isosceles and an angle of $\frac{\pi}{3}$.
		1	

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Question	Answer	Marks	Guidance
6(a)	State or imply $\overrightarrow{AB}$ or $\overrightarrow{AC}$ correctly in component form	B1	$(\overrightarrow{AB} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}, \overrightarrow{AC} = 4\mathbf{i} - 3\mathbf{k}).$
	Using the correct process with relevant vectors to evaluate the scalar product $\overrightarrow{AB} \cdot \overrightarrow{AC}$,	M1	or $\overrightarrow{BA} \cdot \overrightarrow{CA}$ ($8 - 3 = 5$). M0 for $\overrightarrow{AB} \cdot \overrightarrow{CA}$.
	Using the correct process for the moduli, divide <i>their</i> scalar product by the product of <i>their</i> moduli to obtain $\cos \theta$ or θ	M1	$\left(\frac{5}{\sqrt{9}\sqrt{25}} \right)$ Independent of the first M1.
	Obtain answer $\frac{1}{3}$	A1	ISW. Need to see a value for $\cos \theta$. Accept $\frac{5}{15}$ or 0.333 ($\cos^{-1} \frac{1}{3}$ alone is not sufficient)
		4	

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Question	Answer	Marks	Guidance
6(b)	Use correct method to find an exact value for the sine of angle BAC from <i>their</i> (a)	M1	$\left(\sqrt{1-\frac{1}{9}}\right)$
	Obtain answer $\frac{2}{3}\sqrt{2}$, or equivalent	A1	
	Use correct area formula to find the area of triangle ABC with <i>their</i> versions of relevant vectors	M1	$\left(\frac{1}{2}\sqrt{9}\sqrt{25} \times \text{their } \sin \theta\right)$ or $\frac{1}{2}\sqrt{9}\sqrt{25} \times \sin\left(\cos^{-1}\frac{1}{3}\right)$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	A1	Only ISW
	Alternative method 1 for question 6(b)		
	Use correct method to find the perpendicular distance from A to BC (or B to AC or C to AB)	M1	$\begin{pmatrix} 2+2\lambda \\ -2+2\lambda \\ 1-4\lambda \end{pmatrix} \cdot \begin{pmatrix} 2 \\ 2 \\ -4 \end{pmatrix} = 0 \Rightarrow \lambda = \frac{1}{6}$
	Obtain $\frac{1}{3}\sqrt{75}$	A1	$\left(\left \frac{7}{3}\mathbf{i} - \frac{5}{3}\mathbf{j} + \frac{1}{3}\mathbf{k}\right \right)$
	Use correct area formula to find the area of triangle ABC	M1	$\left(\frac{1}{2} \times \text{their } \sqrt{24} \times \text{their } \frac{1}{3}\sqrt{75}\right)$ The length they use for <i>their</i> base must be found correctly.
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	A1	

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Question	Answer	Marks	Guidance
6(b)	Alternative method 2 for question 6(b)		
	Correct method to find the semi-perimeter	M1	
	Obtain $4 + \sqrt{6}$	A1	
	Correct application of Hero's (Heron's) formula	M1	$\sqrt{(4 + \sqrt{6})(1 + \sqrt{6})(-1 + \sqrt{6})(4 - \sqrt{6})}$
	Obtain answer $5\sqrt{2}$ or $\sqrt{50}$	A1	
		4	

Question	Answer	Marks	Guidance
7(a)	Show sufficient working to justify the given statement	B1	e.g. see $2 \cot \theta \times -\operatorname{cosec}^2 \theta$ in the working or express in terms of $\sin \theta$ and $\cos \theta$ and use quotient rule to obtain the given result. Solution must have θ present throughout and must reach the given answer.
		1	

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Question	Answer	Marks	Guidance
7(b)	Separate variables correctly Check for relevant working in (a)	B1	$\int x dx = \int \frac{\tan^2 \theta}{\sin^2 \theta} - \frac{2 \cot \theta}{\sin^2 \theta} d\theta$ Condone incorrect notation e.g. missing dx. Need either the integral sign or the dx, dθ.
	Obtain term $\frac{1}{2}x^2$	B1	
	Obtain terms $\tan \theta + \cot^2 \theta$	B1 + B1	Alternative: $\int \frac{2 \cot \theta}{\sin^2 \theta} d\theta = \int \frac{2 \cos \theta}{\sin^3 \theta} d\theta = -\frac{1}{\sin^2 \theta} (+C)$
	Form an equation for the constant of integration, or use limits $x = 2$, $\theta = \frac{1}{4}\pi$, in a solution with at least two correctly obtained terms of the form ax^2 , $b \tan \theta$ and $c \cot^2 \theta$, where $abc \neq 0$	M1	Need to have 3 terms. Constant of correct form.
	State correct solution in any form, e.g. $\frac{1}{2}x^2 = \tan \theta + \cot^2 \theta$	A1	or $\frac{1}{2}x^2 = \tan \theta + \operatorname{cosec}^2 \theta - 1$ If everything else is correct, allow a correct final answer to imply this A1.
	Substitute $\theta = \frac{1}{6}\pi$ and obtain answer $x = 2.67$	A1	2.6748... $\sqrt{\frac{18+2\sqrt{3}}{3}}$ If see a correctly rounded value ISW.
		7	

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Question	Answer	Marks	Guidance
8(a)	State $(a =) \pi^2$	B1	Allow 32400, 180^2 . Accept $(x =) \pi^2$.
		1	
8(b)	State or imply $dx = 2u \, du$ or equivalent	B1	e.g. $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ Incorrect statements e.g. $du = \frac{1}{2\sqrt{x}}$ is B0.
	Substitute for x and dx throughout the integral	M1	
	Obtain $\int 2u \sin u \, du$	A1	Allow with missing du .
	Commence integration of $\int k u \sin u \, du$ by parts and reach $\mp k u \cos u \pm \int k \cos u \, du$	*M1	
	Obtain integral $-ku \cos u + k \sin u$	A1	
	Substitute limits $u = 0$ and $u = \sqrt{\text{their } a}$, $a \neq 0$, a in radians or $x = 0$ and $\text{their } a$ in the complete integral	DM1	$-2\pi \cos \pi + 2 \sin \pi (+0 - 2 \sin 0)$ Need limits stated but condone if zeros not shown in substitution.
	Obtain answer 2π	A1	
		7	

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Question	Answer	Marks	Guidance
9(a)	State or imply angle $AOC = \pi - 2\theta$	B1	Might be seen on the printed diagram.
	Use correct formulae for the area of a sector and triangle, or of a segment, and find the area of the shaded region	M1	$\frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2 \sin(\pi - 2\theta)$ or $\frac{1}{2}\pi r^2 - \left[\frac{1}{2}r^2(2\theta) + \frac{1}{2}r^2 \sin(\pi - 2\theta) \right]$ M0 if subtraction the wrong way round.
	Equate to $\frac{1}{6}\pi r^2$ and obtain a correct equation in any form	A1	e.g. $\frac{1}{6}\pi r^2 = \frac{1}{2}r^2(\pi - 2\theta) - \frac{1}{2}r^2 \sin(\pi - 2\theta)$.
	Obtain $\theta = \frac{1}{3}(\pi - 1.5\sin 2\theta)$ correctly	A1	AG Condone if state / imply $\sin(\pi - 2\theta) = \sin 2\theta$.
		4	
9(b)	Evaluate a relevant expression or pair of expressions at $\theta = 0.5$ and $\theta = 0.7$	M1	Allow work on a smaller interval. Need to evaluate for both limits, with at least one correct. When using $x = f(x)$ embedded values are not sufficient e.g. $f(0.5) \dots$ is accepted but $\frac{1}{3}(\pi - 1.5\sin 2 \times 0.5) = \dots$ is not.
	Complete the argument correctly with correct calculated values	A1	e.g. $0.5 < 0.626$, $0.7 > 0.554$ or $0.126 > 0$, $-0.146 < 0$ If using pairs then the pairing must be clear. Need to see the inequalities or an appropriate comment. Need to see values calculated to at least 2 sf.
		2	

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Question	Answer	Marks	Guidance
9(c)	Use the iterative process $\theta_{n+1} = \frac{1}{3}(\pi - 1.5 \sin 2\theta_n)$ correctly at least once	M1	i.e obtain one value and use that value to obtain a second value. Must be working in radians.
	Obtain final answer 0.586	A1	
	Show sufficient iterations to 5 d.p. to justify 0.586 to 3 d.p., or show there is a sign change in the interval (0.5855, 0.5865).	A1	0.5, 0.62646, 0.57225, 0.59195, 0.58416, 0.58715, e.g. 0.58599, 0.58644 0.6, 0.58118, 0.58833, 0.58553, 0.58661, 0.58619, 0.58636 0.7, 0.55447, 0.59958, 0.58133, 0.58827, 0.58556, 0.58661, 0.58620, 0.58636 Allow working to more than 5 dp, but not less.
		3	

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Question	Answer	Marks	Guidance
10(a)	State or imply the form $\frac{A}{1+x} + \frac{Bx+C}{2+x^2}$	B1	
	Use a correct method for finding a constant	M1	
	Obtain one of $A = 2$, $B = -1$ and $C = 0$	A1	SC: A maximum of M1A1 is available for obtaining $A = 2$ after scoring B0.
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
10(b)	Integrate and obtain term $2\ln(1+x)$	B1 FT	$A\ln(1+x)$
	Integrate and obtain term of the form $k\ln(2+x^2)$ from an integral of the correct form	*M1	Ignore any separate working relating to $C \neq 0$.
	Obtain term $-\frac{1}{2}\ln(2+x^2)$	A1 FT	$\frac{B}{2}\ln(2+x^2)$
	Substitute limits in an integral containing terms of the form $a\ln(1+x) + b\ln(2+x^2)$, where $ab \neq 0$	DM1	Ignore working relating to $C \neq 0$. $(2\ln 5 - 2\ln 1 - \frac{1}{2}\ln 18 + \frac{1}{2}\ln 2)$ Dependent on the first M1. Must be subtracting the correct way round. Must have an exact substitution.
	Obtain answer $\ln\left(\frac{25}{3}\right)$	A1	ISW Any exact equivalent e.g. $\ln \frac{25\sqrt{2}}{\sqrt{18}}$ with no number in front of the logarithm.
		5	



Cambridge International A Level

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3

October/November 2022

MARK SCHEME

Maximum Mark: 75

Published

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2022 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

This document consists of **15** printed pages.

PUBLISHED**Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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Mathematics Specific Marking Principles	
1	Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.
2	Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.
3	Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.
4	Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).
5	Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread.
6	Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

PUBLISHED**Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
 - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
 - The total number of marks available for each question is shown at the bottom of the Marks column.
 - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
 - Square brackets [] around text or numbers show extra information not needed for the mark to be awarded.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
CAO	Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)
CWO	Correct Working Only
ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

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Question	Answer	Marks	Guidance
1	Use law for the logarithm of a product, quotient or power	M1	
	Remove logarithms and state a correct equation, e.g. $x(2x-1) = (x+1)^2$	A1	
	Solve a 3-term quadratic obtaining at least one root	M1	
	Obtain answer 3.303 only	A1	
		4	

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Question	Answer	Marks	Guidance
2	State a correct unsimplified term in x or x^2 of the expansion of either $(1+2x)^{\frac{1}{2}}$ or $(1-2x)^{-\frac{1}{2}}$	B1	
	State correct unsimplified expansion of $(1+2x)^{\frac{1}{2}}$ up to the term in x^2	B1	
	State correct unsimplified expansion of $(1-2x)^{-\frac{1}{2}}$ up to the term in x^2	B1	
	Obtain sufficient terms of the product of the expansions	M1	
	Obtain final answer $1+2x+2x^2$	A1	
	Alternative method for question 2		
	State that the expression equals $(1+2x)(1-4x^2)^{-\frac{1}{2}}$ and state a term of the expansion	B1	
	State correct unsimplified expansion of $(1-4x^2)^{-\frac{1}{2}}$ up to the term in x^2	B1 + B1	
	Obtain sufficient terms of the product of $(1+2x)$ and the expansion	M1	
	Obtain final answer $1+2x+2x^2$	A1	
		5	

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Question	Answer	Marks	Guidance
3	Commence integration by parts and reach $x \tan x \pm \int \tan x \cdot 1 dx$	*M1	
	Use a correct method to integrate $\tan x$	M1	
	Obtain integral $x \tan x - \ln \sec x$, or equivalent	A1	
	Use limits correctly, having integrated twice	DM1	
	Obtain answer $\frac{1}{4}\pi - \frac{1}{2}\ln 2$, or exact equivalent	A1	
		5	

Question	Answer	Marks	Guidance
4	State or imply $\frac{dx}{dt} = 2 - \sec^2 t$ or $\frac{dy}{dt} = 2 \frac{\cos 2t}{\sin 2t}$	B1	
	Use $\frac{dy}{dx} = \frac{dy}{dt} \div \frac{dx}{dt}$	M1	
	Obtain correct answer in any form	A1	
	Use double angle formula to express derivative in terms of $\cos x$ and $\sin x$	M1	
	Obtain the given answer correctly	A1	AG
		5	

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Question	Answer	Marks	Guidance
5(a)	Show a circle with centre -2	B1	
	Show a circle with radius 2 and centre not the origin	B1	
	Show the line $y = 1$	B1	
	Shade the correct region	B1	
		4	
5(b)	Identify the correct point and carry out a correct method to find the argument	M1	
	Obtain answer $\frac{11}{12}\pi$	A1	2.88 radians or 165° .
		2	

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Question	Answer	Marks	Guidance
6	Use quadratic formula to solve for z	M1	SC M1: For substitution of $x + iy$ and multiplying out.
	Use $i^2 = -1$ throughout	M1	SC M1: Use $i^2 = -1$ throughout.
	Obtain correct answer in any form	A1	SC A1: For two correct equations $x^2 - y^2 + 6xy - 2x + y = 0$ and $-3(x^2 - y^2) + 2xy - x - 2y + 1 = 0$.
	Multiply numerator and denominator by $(1 + 3i)$, or equivalent	M1	
	Obtain final answer, e.g. $-\frac{1}{2} + \frac{1}{2}i$	A1	
	Obtain second final answer, e.g. $\frac{2}{5} + \frac{1}{5}i$	A1	
		6	

Question	Answer	Marks	Guidance
7(a)	Rearrange and obtain $4\cos x - \sin x = \sqrt{5}$	B1	
	State $R = \sqrt{17}$	B1	
	Use trig formulae to find α	M1	
	Obtain $\alpha = 14.04^\circ$	A1	
		4	

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Question	Answer	Marks	Guidance
7(b)	Evaluate $\cos^{-1} \left(\frac{\sqrt{5}}{\sqrt{17}} \right)$	B1 FT	FT <i>their R.</i>
	Carry out a correct method to find a value of x in the given interval	M1	
	Obtain answer, e.g. 21.6°	A1	
	Obtain a second answer, e.g. 144.4° and no other in the interval	A1	Treat answers in radians as a misread. Ignore answers outside the given interval.
		4	

Question	Answer	Marks	Guidance
8(a)	Use quotient or product rule	M1	
	Obtain correct derivative in any form	A1	
	Equate derivative at $x = p$ to zero and obtain the given equation	A1	
		3	
8(b)	Evaluate a relevant expression or pair of relevant pair of expressions at $p = 2.5$ and $p = 3$	M1	
	Complete the argument with correct calculated values	A1	
		2	

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Question	Answer	Marks	Guidance
8(c)	Use the iterative formula $p_{n+1} = 3(1 - e^{-p_n})$ correctly at least once	M1	
	Obtain final answer $p = 2.82$	A1	
	Show sufficient iterations to 4 d.p. to justify 2.82 to 2 d.p., or show there is a sign change in the interval (2.815, 2.825)	A1	
		3	

Question	Answer	Marks	Guidance
9(a)	State $\overrightarrow{OM} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$	B1	
	Use a correct method to find $\overrightarrow{ON}$	M1	
	Obtain answer $\begin{pmatrix} 3 \\ -2 \\ -1 \end{pmatrix}$	A1	
		3	
9(b)	Carry out a correct method to form a vector equation for MN	M1	
	Obtain a correct equation in any form, e.g. $\mathbf{r} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ -3 \\ -1 \end{pmatrix}$	A1	OE
		2	

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Question	Answer	Marks	Guidance
9(c)	State a correct vector equation for AB in any form, e.g. $\mathbf{r} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 5 \\ 1 \end{pmatrix}$	B1	
	Equate components of AB and MN and solve for λ or for μ	M1	
	Obtain $\lambda = -3$ or $\mu = 2$	A1	
	Obtain position vector $\begin{pmatrix} -1 \\ 10 \\ 3 \end{pmatrix}$, or equivalent, for Q	A1	
		4	

Question	Answer	Marks	Guidance
10(a)	$a = 30$ and $b = 0.01$	B1	
		1	

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Question	Answer	Marks	Guidance
10(b)	Separate variables and integrate one side	M1	
	Obtain terms $-100\ln(30-0.01V)$ and t , or equivalent	A1 FT + A1 FT	FT <i>their a and b.</i>
	Evaluate a constant, or use $t = 0$, $V = 0$ as limits, in a solution containing terms $c \ln(30-0.01V)$ and dt where $cd \neq 0$	M1	
	Obtain solution $100\ln 30 - 100\ln(30-0.01V) = t$, or equivalent	A1	
	Substitute $V = 1000$ and obtain answer $t = 40.5$	A1	
		6	
10(c)	Obtain $V = 3000(1 - e^{-0.01t})$	B1	OE
	State that V approaches 3000	B1	
		2	

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Question	Answer	Marks	Guidance
11(a)	State or imply the form $\frac{A}{3-x} + \frac{Bx+C}{1+3x^2}$	B1	
	Use a correct method to find a constant	M1	
	Obtain one of $A = 2$, $B = 0$ and $C = 1$	A1	
	Obtain a second value	A1	
	Obtain the third value	A1	
		5	
11(b)	Integrate and obtain term $-2\ln(3-x)$	B1 FT	
	Obtain term of the form $b \tan^{-1}(\sqrt{3}x)$	M1	
	Obtain term $\frac{1}{\sqrt{3}} \tan^{-1}(\sqrt{3}x)$	A1 FT	
	Substitute limits correctly in an integral with terms $a \ln(3-x)$ and $b \tan^{-1}(\sqrt{3}x)$, where $ab \neq 0$	M1	
	Obtain answer $2 \ln \frac{3}{2} + \frac{1}{3\sqrt{3}} \pi$, or equivalent	A1	
		5	