



Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
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- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **16** pages.



- 1 Solve the equation $4|5^x - 1| = 5^x$, giving your answers correct to 3 decimal places. [4]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 (a) Express $5 \sin x - 3 \cos x$ in the form $R \sin(x - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$. Give the exact value of R and give α correct to 2 decimal places. [3]

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- (b) Hence state the greatest and least possible values of $(5 \sin x - 3 \cos x)^2$. [2]

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3 The curve with equation $y = xe^{1-2x}$ has one stationary point.

(a) Find the coordinates of this point.

[4]

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(b) Determine whether the stationary point is a maximum or a minimum.

[2]

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- 4** Using the substitution $u = \sqrt{x}$, find the exact value of

$$\int_3^{\infty} \frac{1}{(x+1)\sqrt{x}} \, dx.$$

[6]

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 5 (a) Show that the equation

$$\cot 2\theta + \cot \theta = 2$$

can be expressed as a quadratic equation in $\tan \theta$.

[3]

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- (b) Hence solve the equation $\cot 2\theta + \cot \theta = 2$, for $0 < \theta < \pi$, giving your answers correct to 3 decimal places.

[3]

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- 6** When $(a + bx)\sqrt{1 + 4x}$, where a and b are constants, is expanded in ascending powers of x , the coefficients of x and x^2 are 3 and -6 respectively.

Find the values of a and b .

[6]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

7 (a) Given that $y = \ln(\ln x)$, show that

$$\frac{dy}{dx} = \frac{1}{x \ln x}. \quad [1]$$

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The variables x and t satisfy the differential equation

$$x \ln x + t \frac{dx}{dt} = 0.$$

It is given that $x = e$ when $t = 2$.

(b) Solve the differential equation obtaining an expression for x in terms of t , simplifying your answer. [7]

[illegible]

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8 The constant a is such that $\int_1^a \frac{\ln x}{\sqrt{x}} \, dx = 6$.

(a) Show that $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$. [5]

[$\exp(x)$ is an alternative notation for e^x .]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- (b) Verify by calculation that a lies between 9 and 11.

[2]

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- (c) Use an iterative formula based on the equation in part (a) to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

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- 9 Two lines l and m have equations $\mathbf{r} = 3\mathbf{i} + 2\mathbf{j} + 5\mathbf{k} + s(4\mathbf{i} - \mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = \mathbf{i} - \mathbf{j} - 2\mathbf{k} + t(-\mathbf{i} + 2\mathbf{j} + 2\mathbf{k})$ respectively.

(a) Show that l and m are perpendicular. [2]

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(b) Show that l and m intersect and state the position vector of the point of intersection. [5]

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- (c) Show that the length of the perpendicular from the origin to the line m is $\frac{1}{3}\sqrt{5}$. [4]

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- 10** The complex number $1 + 2i$ is denoted by u . The polynomial $2x^3 + ax^2 + 4x + b$, where a and b are real constants, is denoted by $p(x)$. It is given that u is a root of the equation $p(x) = 0$.

(a) Find the values of a and b .

[4]

[illegible]

(b) State a second complex root of this equation.

[1]

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- (c) Find the real factors of $p(x)$. [2]

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- (d) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - u| \leq \sqrt{5}$ and $\arg z \leq \frac{1}{4}\pi$. [4]

- (ii) Find the least value of $\operatorname{Im} z$ for points in the shaded region. Give your answer in an exact form. [1]

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[illegible]

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2021

1 hour 50 minutes

You must answer on the question paper.

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INSTRUCTIONS

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INFORMATION

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This document has **20** pages. Any blank pages are indicated.

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- 1** Find the value of x for which $3(2^{1-x}) = 7^x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [4]

[illegible]

- 2** Solve the inequality $|3x - a| > 2|x + 2a|$, where a is a positive constant. [4]

[illegible]

- 3 (a)** Given the complex numbers $u = a + ib$ and $w = c + id$, where a , b , c and d are real, prove that $(u + w)^* = u^* + w^*$. [2]

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- (b)** Solve the equation $(z + 2 + i)^* + (2 + i)z = 0$, giving your answer in the form $x + iy$ where x and y are real. [4]

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4 Express $\frac{4x^2 - 13x + 13}{(2x - 1)(x - 3)}$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 5 (a) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - 2i| \leq 1$ and $\text{Im } z \geq 2$. [4]

- (b) Find the greatest value of $\arg z$ for points in the shaded region, giving your answer in degrees. [3]

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- 6 (a)** Using the expansions of $\sin(3x + 2x)$ and $\sin(3x - 2x)$, show that

$$\frac{1}{2}(\sin 5x + \sin x) \equiv \sin 3x \cos 2x. \quad [3]$$

[illegible]

[Turn over

- 7 The variables x and y satisfy the differential equation

$$e^{2x} \frac{dy}{dx} = 4xy^2,$$

and it is given that $y = 1$ when $x = 0$.

Solve the differential equation, obtaining an expression for y in terms of x .

[7]

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8 (a) By first expanding $(\cos^2 \theta + \sin^2 \theta)^2$, show that

$$\cos^4 \theta + \sin^4 \theta \equiv 1 - \frac{1}{2} \sin^2 2\theta. \quad [3]$$

[illegible]

for $0^\circ < \theta < 180^\circ$.

[4]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

9 The equation of a curve is $ye^{2x} - y^2e^x = 2$.

(a) Show that $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$. [4]

[illegible]

- (b) Find the exact coordinates of the point on the curve where the tangent is parallel to the y-axis. [4]

[illegible]

- 10 With respect to the origin O , the position vectors of the points A and B are given by $\overrightarrow{OA} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\overrightarrow{OB} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix}$.

(a) Find a vector equation for the line l through A and B . [3]

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(b) The point C lies on l and is such that $\overrightarrow{AC} = 3\overrightarrow{AB}$.

Find the position vector of C . [2]

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- (c) Find the possible position vectors of the point P on l such that $OP = \sqrt{14}$. [5]

[illegible]

11 The equation of a curve is $y = \sqrt{\tan x}$, for $0 \leq x < \frac{1}{2}\pi$.

- (a) Express $\frac{dy}{dx}$ in terms of $\tan x$, and verify that $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$. [4]

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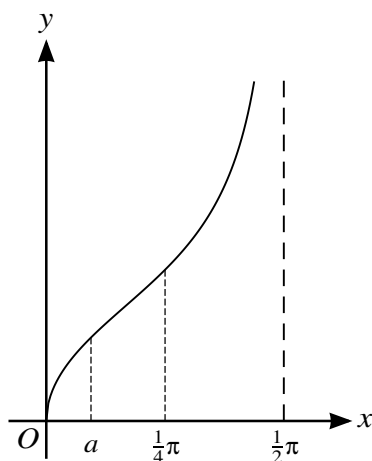
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The value of $\frac{dy}{dx}$ is also 1 at another point on the curve where $x = a$, as shown in the diagram.



- (b) Show that $t^3 + t^2 + 3t - 1 = 0$, where $t = \tan a$. [4]

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(c) Use the iterative formula

$$a_{n+1} = \tan^{-1} \left(\frac{1}{3} (1 - \tan^2 a_n - \tan^3 a_n) \right)$$

to determine a correct to 2 decimal places, giving the result of each iteration to 4 decimal places. [3]

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This document has **20** pages. Any blank pages are indicated.

- 1** Find the quotient and remainder when $2x^4 + 1$ is divided by $x^2 - x + 2$. [3]

[illegible]

- 2 (a) Sketch the graph of $y = |2x - 3|$.

[1]

- (b) Solve the inequality $|2x - 3| < 3x + 2$.

[3]

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- 3** Solve the equation $4^{x-2} = 4^x - 4^2$, giving your answer correct to 3 decimal places. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

4 Find the exact value of $\int_{\frac{1}{3}\pi}^{\pi} x \sin \frac{1}{2}x \, dx$.

[5]

[illegible]

5 Solve the equation $\sin \theta = 3 \cos 2\theta + 2$, for $0^\circ \leq \theta \leq 360^\circ$.

[5]

[illegible]

- 6 (a)** By first expanding $\cos(x - 60^\circ)$, show that the expression

$$2 \cos(x - 60^\circ) + \cos x$$

can be written in the form $R \cos(x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give the exact value of R and the value of α correct to 2 decimal places. [5]

[illegible]

- (b)** Hence find the value of x in the interval $0^\circ < x < 360^\circ$ for which $2 \cos(x - 60^\circ) + \cos x$ takes its least possible value. [2]

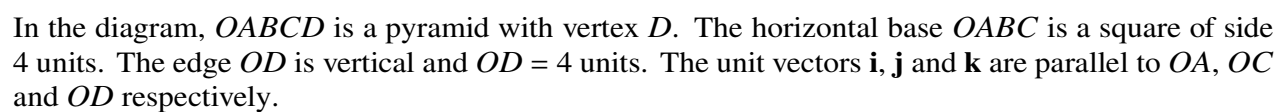
[illegible]

7 The equation of a curve is $\ln(x + y) = x - 2y$.

(a) Show that $\frac{dy}{dx} = \frac{x+y-1}{2(x+y)+1}$. [4]

[illegible]

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(a) Find a vector equation for the line through M and N . [5]

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[4]

[illegible]

9 Let $f(x) = \frac{1}{(9-x)\sqrt{x}}$.

- (a) Find the x -coordinate of the stationary point of the curve with equation $y = f(x)$. [4]

[illegible]

[Turn over

- 10** A large plantation of area 20 km^2 is becoming infected with a plant disease. At time t years the area infected is $x \text{ km}^2$ and the rate of increase of x is proportional to the ratio of the area infected to the area not yet infected.

When $t = 0$, $x = 1$ and $\frac{dx}{dt} = 1$.

- (a)** Show that x and t satisfy the differential equation

$$\frac{dx}{dt} = \frac{19x}{20 - x}. \quad [2]$$

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- (b)** Solve the differential equation and show that when $t = 1$ the value of x satisfies the equation $x = e^{0.9+0.05x}$. [5]

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- (c) Use an iterative formula based on the equation in part (b), with an initial value of 2, to determine x correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- (d) Calculate the value of t at which the entire plantation becomes infected. [1]

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11 The complex number $-\sqrt{3} + i$ is denoted by u .

- (a) Express u in the form $re^{i\theta}$, where $r > 0$ and $-\pi < \theta \leq \pi$, giving the exact values of r and θ . [2]

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- (b) Hence show that u^6 is real and state its value. [2]

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- (c) (i) On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $0 \leq \arg(z - u) \leq \frac{1}{4}\pi$ and $\operatorname{Re} z \leq 2$. [4]

- (ii) Find the greatest value of $|z|$ for points in the shaded region. Give your answer correct to 3 significant figures. [2]

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[illegible]

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