



# Cambridge International A Level

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**MATHEMATICS**

**9709/31**

Paper 3 Pure Mathematics 3

**October/November 2021**

**MARK SCHEME**

Maximum Mark: 75

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| <p><b>Published</b></p> |
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This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

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This document consists of **16** printed pages.

**PUBLISHED****Generic Marking Principles**

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**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

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Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

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- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

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| Mathematics Specific Marking Principles |                                                                                                                                                                                                                                               |
|-----------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1                                       | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.                                     |
| 2                                       | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.                                                            |
| 3                                       | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.                                                                                                               |
| 4                                       | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).                                                                                            |
| 5                                       | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6                                       | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.                                                                                              |

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer                                                                                                         | Marks     | Guidance |
|----------|----------------------------------------------------------------------------------------------------------------|-----------|----------|
| 1        | State or imply non-modular equation $4^2(5^x - 1)^2 = (5^x)^2$ or pair of equations $4(5^x - 1) = \pm 5^x$     | <b>M1</b> |          |
|          | Obtain $5^x = \frac{4}{3}$ and $5^x = \frac{4}{5}$ (or $5^{x+1} = 4$ )                                         | <b>A1</b> |          |
|          | Use correct method for solving an equation of the form $5^x = a$ , or $5^{x+1} = b$ where $a > 0$ , or $b > 0$ | <b>M1</b> |          |
|          | Obtain answers $x = 0.179$ and $x = -0.139$                                                                    | <b>A1</b> |          |
|          | <b>Alternative method for question 1</b>                                                                       |           |          |
|          | Obtain $5^x = \frac{4}{3}$ by solving an equation                                                              | <b>B1</b> |          |
|          | Obtain $5^x = \frac{4}{5}$ (or $5^{x+1} = 4$ ) by solving an equation                                          | <b>B1</b> |          |
|          | Use correct method for solving an equation of the form $5^x = a$ , or $5^{x+1} = b$ where $a > 0$ , or $b > 0$ | <b>M1</b> |          |
|          | Obtain answers $x = 0.179$ and $x = -0.139$                                                                    | <b>A1</b> |          |
|          |                                                                                                                | <b>4</b>  |          |

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| Question | Answer                             | Marks        | Guidance                                                                                                    |
|----------|------------------------------------|--------------|-------------------------------------------------------------------------------------------------------------|
| 2(a)     | State $R = \sqrt{34}$              | <b>B1</b>    |                                                                                                             |
|          | Use trig formulae to find $\alpha$ | <b>M1</b>    | $\tan \alpha = \frac{3}{5}$ or $\sin \alpha = \frac{3}{\sqrt{34}}$ or $\cos \alpha = \frac{5}{\sqrt{34}}$ . |
|          | Obtain $\alpha = 0.54$             | <b>A1</b>    | 30.96° scores <b>M1A0</b> .                                                                                 |
|          |                                    | <b>3</b>     |                                                                                                             |
| 2(b)     | State greatest value 34            | <b>B1 FT</b> | <i>Their <math>R^2</math> .</i>                                                                             |
|          | State least value 0                | <b>B1</b>    |                                                                                                             |
|          |                                    | <b>2</b>     |                                                                                                             |

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| Question | Answer                                                                | Marks     | Guidance                                                |
|----------|-----------------------------------------------------------------------|-----------|---------------------------------------------------------|
| 3(a)     | Use correct product rule                                              | <b>M1</b> |                                                         |
|          | Obtain correct derivative in any form                                 | <b>A1</b> | $\frac{dy}{dx} = e^{1-2x} - 2xe^{1-2x}$                 |
|          | Equate derivative to zero and solve for $x$                           | <b>M1</b> |                                                         |
|          | Obtain $x = \frac{1}{2}$ and $y = \frac{1}{2}$                        | <b>A1</b> |                                                         |
|          |                                                                       | <b>4</b>  |                                                         |
| 3(b)     | Use a correct method for determining the nature of a stationary point | <b>M1</b> | e.g. $\frac{d^2y}{dx^2} = -2e^{1-2x} - 2(1-2x)e^{1-2x}$ |
|          | Show that it is a maximum point                                       | <b>A1</b> |                                                         |
|          |                                                                       | <b>2</b>  |                                                         |

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| Question | Answer                                                                            | Marks     | Guidance                                                                       |
|----------|-----------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------|
| 4        | State that $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ or $du = \frac{1}{2\sqrt{x}} dx$ | <b>B1</b> |                                                                                |
|          | Substitute throughout for $x$ and $dx$                                            | <b>M1</b> |                                                                                |
|          | Obtain a correct integral with integrand $\frac{2}{u^2 + 1}$                      | <b>A1</b> |                                                                                |
|          | Integrate and obtain term of the form $k \tan^{-1} u$                             | <b>M1</b> | $(2 \tan^{-1} u)$                                                              |
|          | Use limits $\sqrt{3}$ and $\infty$ for $u$ or equivalent and evaluate trig.       | <b>A1</b> | e.g. $2\left(\frac{\pi}{2} - \frac{\pi}{3}\right)$ Must be working in radians. |
|          | Obtain answer $\frac{1}{3}\pi$                                                    | <b>A1</b> | Or equivalent single term.                                                     |
|          |                                                                                   | <b>6</b>  |                                                                                |

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| Question | Answer                                                                   | Marks     | Guidance                                                                                          |
|----------|--------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------------|
| 5(a)     | Use correct trig formulae and express equation in terms of $\tan \theta$ | <b>M1</b> |                                                                                                   |
|          | Obtain a correct equation in $\tan \theta$ in any form                   | <b>A1</b> | e.g. $\frac{1 - \tan^2 \theta}{2 \tan \theta} + \frac{1}{\tan \theta} = 2$                        |
|          | Reduce to $\tan^2 \theta + 4 \tan \theta - 3 = 0$ , or 3-term equivalent | <b>A1</b> |                                                                                                   |
|          |                                                                          | <b>3</b>  |                                                                                                   |
| 5(b)     | Solve a 3-term quadratic for $\tan \theta$ and calculate $\theta$        | <b>M1</b> | $(\tan \theta = -2 \pm \sqrt{7})$                                                                 |
|          | Obtain answer, e.g. 0.573                                                | <b>A1</b> | Must be 3 d.p.                                                                                    |
|          | Obtain second answer, e.g. 1.783 and no other                            | <b>A1</b> | Ignore answers outside the given interval. Treat answers in degrees as a misread. (32.9°, 102.1°) |
|          |                                                                          | <b>3</b>  |                                                                                                   |

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| Question | Answer                                                                                                                                          | Marks     | Guidance                                   |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------|
| 6        | State or imply $1 + 2x$ as first terms of the expansion of $\sqrt{1+4x}$                                                                        | <b>B1</b> | Allow for correct unsimplified expression. |
|          | State or imply $-2x^2$ as third term of the expansion of $\sqrt{1+4x}$                                                                          | <b>B1</b> | Allow for correct unsimplified expression. |
|          | Form an expression for the coefficient of $x$ or coefficient of $x^2$ in the expansion of $(a + bx)\sqrt{1+4x}$ and equate to given coefficient | <b>M1</b> | All relevant terms considered.             |
|          | Obtain $2a + b = 3$ , or equivalent                                                                                                             | <b>A1</b> | One correct equation.                      |
|          | Obtain $-2a + 2b = -6$ or equivalent                                                                                                            | <b>A1</b> | Second correct equation.                   |
|          | Obtain answer $a = 2$ and $b = -1$                                                                                                              | <b>A1</b> |                                            |
|          |                                                                                                                                                 | <b>6</b>  |                                            |

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| Question | Answer                                                                                           | Marks     | Guidance                                                |
|----------|--------------------------------------------------------------------------------------------------|-----------|---------------------------------------------------------|
| 7(a)     | Show sufficient working to justify the given answer                                              | <b>B1</b> |                                                         |
|          |                                                                                                  | <b>1</b>  |                                                         |
| 7(b)     | Correct separation of variables                                                                  | <b>B1</b> | e.g. $-\int \frac{1}{t} dt = \int \frac{1}{x \ln x} dx$ |
|          | Obtain term $\ln(\ln x)$                                                                         | <b>B1</b> |                                                         |
|          | Obtain term $-\ln t$                                                                             | <b>B1</b> |                                                         |
|          | Evaluate a constant or use $x = e$ and $t = 2$ as limits in an expression involving $\ln(\ln x)$ | <b>M1</b> |                                                         |
|          | Obtain correct solution in any form, e.g. $\ln(\ln x) = -\ln t + \ln 2$                          | <b>A1</b> |                                                         |
|          | Use log laws to enable removal of logarithms                                                     | <b>M1</b> |                                                         |
|          | Obtain answer $x = e^{\frac{2}{t}}$ , or simplified equivalent                                   | <b>A1</b> |                                                         |
|          |                                                                                                  | <b>7</b>  |                                                         |
| 7(c)     | State that $x$ tends to 1 coming from $x = e^{\frac{k}{t}}$                                      | <b>B1</b> |                                                         |
|          |                                                                                                  | <b>1</b>  |                                                         |

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| Question | Answer                                                                                                                     | Marks      | Guidance                                                                                                                              |
|----------|----------------------------------------------------------------------------------------------------------------------------|------------|---------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | Commence integration and reach $a\sqrt{x} \ln x + b \int \sqrt{x} \cdot \frac{1}{x} dx$ , or equivalent                    | <b>*M1</b> |                                                                                                                                       |
|          | Obtain $2\sqrt{x} \ln x - \int 2\sqrt{x} \cdot \frac{1}{x} dx$ , or equivalent                                             | <b>A1</b>  |                                                                                                                                       |
|          | Obtain integral $2\sqrt{x} \ln x - 4\sqrt{x}$ , or equivalent                                                              | <b>A1</b>  |                                                                                                                                       |
|          | Substitute limits and equate result to 6                                                                                   | <b>DM1</b> |                                                                                                                                       |
|          | Rearrange and obtain $a = \exp\left(\frac{1}{\sqrt{a}} + 2\right)$                                                         | <b>A1</b>  | Obtain <b>given answer</b> from full and correct working.                                                                             |
|          |                                                                                                                            | <b>5</b>   |                                                                                                                                       |
| 8(b)     | Calculate the values of a relevant expression or pair of expressions at $a = 9$ and $a = 11$                               | <b>M1</b>  | e.g. $\begin{cases} 9 < 10.31 \\ 11 > 9.99 \end{cases}$ or $1.31 > 0, -1.01 < 0$                                                      |
|          | Complete the argument correctly with correct values                                                                        | <b>A1</b>  |                                                                                                                                       |
|          |                                                                                                                            | <b>2</b>   |                                                                                                                                       |
| 8(c)     | Use the iterative process $a_{n+1} = \exp\left(\frac{1}{\sqrt{a_n}} + 2\right)$ correctly at least once                    | <b>M1</b>  |                                                                                                                                       |
|          | Obtain answer 10.12                                                                                                        | <b>A1</b>  |                                                                                                                                       |
|          | Show sufficient iterations to 4dp to justify 10.12 to 2dp, or show there is a sign change in the interval (10.115, 10.125) | <b>A1</b>  | e.g. 10, 10.1374, 10.1156, 10.1190, ...,<br>9, 10.3123, 10.0886, 10.1233, 10.1178, ...,<br>11, 9.9893, 10.1391, 10.1153, 10.1191, ... |
|          |                                                                                                                            | <b>3</b>   |                                                                                                                                       |

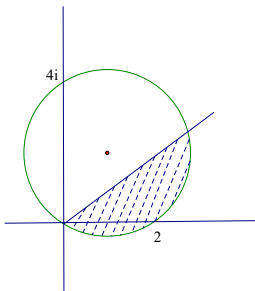
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| Question | Answer                                                                                                               | Marks     | Guidance                                                                          |
|----------|----------------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------------|
| 9(a)     | Use correct method to evaluate the scalar product of relevant vectors                                                | <b>M1</b> | $(-4 - 2 + 6)$                                                                    |
|          | Obtain answer zero and deduce the given statement                                                                    | <b>A1</b> | Need a conclusion or a statement in advance that the scalar product will be zero. |
|          |                                                                                                                      | <b>2</b>  |                                                                                   |
| 9(b)     | Express general point of $l$ or $m$ in component form, e.g. $(3 + 4s, 2 - s, 5 + 3s)$ or $(1 - t, -1 + 2t, -2 + 2t)$ | <b>B1</b> |                                                                                   |
|          | Equate at least two pairs of components and solve for $s$ or for $t$                                                 | <b>M1</b> |                                                                                   |
|          | Obtain correct answer $s = -1$ and $t = 2$                                                                           | <b>A1</b> |                                                                                   |
|          | Verify that all three equations are satisfied                                                                        | <b>A1</b> |                                                                                   |
|          | State position vector of the intersection $-\mathbf{i} + 3\mathbf{j} + 2\mathbf{k}$ , or equivalent                  | <b>A1</b> | Can come from 1 correct value and no contradictory statement.                     |
|          |                                                                                                                      | <b>5</b>  |                                                                                   |

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| Question | Answer                                                                                                                                                                                                                                                                                          | Marks      | Guidance                                                                                                        |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------|-----------------------------------------------------------------------------------------------------------------|
| 9(c)     | Taking a general point $P$ on $m$ , form an equation in $t$ by <i>either</i> equating a relevant scalar product to zero, <i>or</i> equating the derivative of $ \overline{OP} $ to zero, <i>or</i> taking a specific point $Q$ on $m$ , e.g. $(1, -1, -2)$ , using Pythagoras in triangle $OPQ$ | <b>*M1</b> | e.g. $\begin{pmatrix} 1-t \\ -1+2t \\ -2+2t \end{pmatrix} \cdot \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = 0$ |
|          | Obtain $t = \frac{7}{9}$                                                                                                                                                                                                                                                                        | <b>A1</b>  |                                                                                                                 |
|          | Carry out correct method to find $OP$                                                                                                                                                                                                                                                           | <b>DM1</b> |                                                                                                                 |
|          | Obtain $\frac{\sqrt{5}}{3}$                                                                                                                                                                                                                                                                     | <b>A1</b>  | Obtain the <b>given answer</b> from full and correct working.                                                   |
|          | <b>Alternative method for question 9(c)</b>                                                                                                                                                                                                                                                     |            |                                                                                                                 |
|          | Take a specific point $Q$ on $m$ , e.g. $(-1, 3, 2)$ and use a scalar product to find $QN$ , the projection of $OQ$ on $m$                                                                                                                                                                      | <b>*M1</b> |                                                                                                                 |
|          | Obtain $QN = \frac{11}{3}$ , or equivalent                                                                                                                                                                                                                                                      | <b>A1</b>  |                                                                                                                 |
|          | Use Pythagoras to obtain $ON$                                                                                                                                                                                                                                                                   | <b>DM1</b> |                                                                                                                 |
|          | Obtain the given answer correctly                                                                                                                                                                                                                                                               | <b>A1</b>  |                                                                                                                 |
|          |                                                                                                                                                                                                                                                                                                 | <b>4</b>   |                                                                                                                 |

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| Question  | Answer                                                                          | Marks     | Guidance                                                                             |
|-----------|---------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------|
| 10(a)     | Substitute $1 + 2i$ in the polynomial and attempt expansions of $x^2$ and $x^3$ | <b>M1</b> | $u^2 = -3 + 4i$ , $u^3 = -11 - 2i$<br>Full substitution but need not simplify.       |
|           | Equate real and/or imaginary parts to zero                                      | <b>M1</b> | $-18 - 3a + b = 0$ , $4 + 4a = 0$                                                    |
|           | Obtain $a = -1$                                                                 | <b>A1</b> |                                                                                      |
|           | Obtain $b = 15$                                                                 | <b>A1</b> |                                                                                      |
|           |                                                                                 | <b>4</b>  |                                                                                      |
| 10(b)     | State second root $1 - 2i$                                                      | <b>B1</b> |                                                                                      |
|           |                                                                                 | <b>1</b>  |                                                                                      |
| 10(c)     | State the quadratic factor $x^2 - 2x + 5$                                       | <b>B1</b> |                                                                                      |
|           | State the linear factor $2x + 3$                                                | <b>B1</b> |                                                                                      |
|           |                                                                                 | <b>2</b>  |                                                                                      |
| 10(d)(i)  | Show a circle with centre $1 + 2i$                                              | <b>B1</b> |  |
|           | Show circle passing through the origin                                          | <b>B1</b> |                                                                                      |
|           | Show the half line $y = x$ in the first quadrant (accept chord of circle)       | <b>B1</b> |                                                                                      |
|           | Shade the correct region on a correct diagram                                   | <b>B1</b> |                                                                                      |
|           |                                                                                 | <b>4</b>  |                                                                                      |
| 10(d)(ii) | State answer $2 - \sqrt{5}$                                                     | <b>B1</b> |                                                                                      |
|           |                                                                                 | <b>1</b>  |                                                                                      |



# Cambridge International A Level

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**MATHEMATICS**

**9709/32**

Paper 3 Pure Mathematics 3

**October/November 2021**

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer                                                     | Marks      | Guidance                                                                                                        |
|----------|------------------------------------------------------------|------------|-----------------------------------------------------------------------------------------------------------------|
| 1        | Use law of the logarithm of a product, a quotient or power | <b>*M1</b> | e.g. $\ln(7^x) = x \ln 7$                                                                                       |
|          | Obtain a correct linear equation in any form               | <b>A1</b>  | e.g. $\ln 3 + (1-x) \ln 2 = x \ln 7$                                                                            |
|          | Solve a linear equation for $x$                            | <b>DM1</b> |                                                                                                                 |
|          | Obtain answer $x = \frac{\ln 6}{\ln 14}$                   | <b>A1</b>  | Maximum 3 out of 4 available if final answer not in required form e.g. 0.67...<br>ISW once correct answer seen. |
|          | <b>Alternative method for Question 1</b>                   |            |                                                                                                                 |
|          | $2^{1-x} = 2 \times 2^{-x}$                                | <b>*M1</b> | OE                                                                                                              |
|          | $6 = 2^x 7^x [= 14^x]$                                     | <b>A1</b>  |                                                                                                                 |
|          | Use law of the logarithm of a power to solve for $x$       | <b>DM1</b> | Must be a linear power. Allow $x = \ln_{14}(6)$ .                                                               |
|          | Obtain answer $x = \frac{\ln 6}{\ln 14}$                   | <b>A1</b>  | ISW once correct answer seen.                                                                                   |
|          |                                                            | <b>4</b>   |                                                                                                                 |

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| Question | Answer                                                                                                                                                       | Marks     | Guidance                                                                                                                             |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------|
| 2        | State or imply non-modular inequality $(3x - a)^2 > 2^2(x + 2a)^2$ , or corresponding quadratic equation, or pair of linear equations or linear inequalities | <b>B1</b> | Need $2^2$ seen or implied.                                                                                                          |
|          | Make reasonable attempt to solve a 3-term quadratic, or solve two linear equations for $x$ in terms of $a$                                                   | <b>M1</b> | $(5x^2 - 22ax - 15a^2 = 0)$                                                                                                          |
|          | Obtain critical values $x = 5a$ and $x = -\frac{3}{5}a$ and no others                                                                                        | <b>A1</b> | OE<br>Accept incorrect inequalities with correct critical values.<br>Must state 2 values i.e. $\frac{a \pm b}{c}$ is not sufficient. |
|          | State final answer $x > 5a, x < -\frac{3}{5}a$                                                                                                               | <b>A1</b> | Do not condone $\geq$ for $>$ or $\leq$ for $<$ in the final answer.<br>$5a < x < -\frac{3}{5}a$ is <b>A0</b> , 'and' is <b>A0</b> . |
|          | <b>Alternative method for Question 2</b>                                                                                                                     |           |                                                                                                                                      |
|          | Obtain critical value $x = 5a$ from a graphical method, or by solving a linear equation or linear inequality                                                 | <b>B1</b> |                                                                                                                                      |
|          | Obtain critical value $x = -\frac{3}{5}a$ similarly                                                                                                          | <b>B2</b> | Maximum 2 marks if more than 2 critical values.                                                                                      |
|          | State final answer $x > 5a, x < -\frac{3}{5}a$                                                                                                               | <b>B1</b> | Do not condone $\geq$ for $>$ or $\leq$ for $<$ in the final answer.<br>$5a < x < -\frac{3}{5}a$ is <b>B0</b> , 'and' is <b>B0</b> . |
|          |                                                                                                                                                              | <b>4</b>  |                                                                                                                                      |

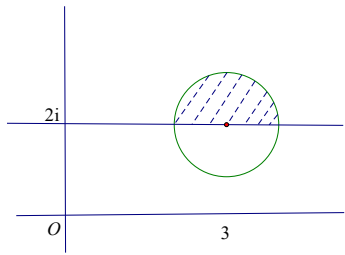
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| Question | Answer                                                                        | Marks     | Guidance                                                                  |
|----------|-------------------------------------------------------------------------------|-----------|---------------------------------------------------------------------------|
| 3(a)     | Substitute for $u$ and $w$ and state correct conjugate of one side            | <b>B1</b> |                                                                           |
|          | Express the other side without conjugates and confirm $(u + w)^* = u^* + w^*$ | <b>B1</b> | <b>Given answer.</b> Needs explicit reference to conjugate of both sides. |
|          |                                                                               | <b>2</b>  |                                                                           |
| 3(b)     | Substitute and remove conjugates to obtain a correct equation in $x$ and $y$  | <b>B1</b> | e.g. $x + 2 - (y + 1)i + (2 + i)(x + iy) = 0$                             |
|          | Use $i^2 = -1$ and equate real and imaginary parts to zero                    | <b>M1</b> |                                                                           |
|          | Obtain two correct equations in $x$ and $y$                                   | <b>A1</b> | e.g. $3x - y + 2 = 0$ and $x + y - 1 = 0$ . Allow $xi + yi - i = 0$ .     |
|          | Solve and obtain answer $z = -\frac{1}{4} + \frac{5}{4}i$                     | <b>A1</b> | Allow for real and imaginary parts stated separately.                     |
|          |                                                                               | <b>4</b>  |                                                                           |

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| Question | Answer                                                       | Marks     | Guidance                                                                                    |
|----------|--------------------------------------------------------------|-----------|---------------------------------------------------------------------------------------------|
| 4        | State or imply the form $A + \frac{B}{2x-1} + \frac{C}{x-3}$ | <b>B1</b> | $\frac{Dx+E}{2x-1} + \frac{F}{x-3}$ and $\frac{P}{2x-1} + \frac{Qx+R}{x-3}$ are also valid. |
|          | Use a correct method for finding a constant                  | <b>M1</b> |                                                                                             |
|          | Obtain one of $A = 2$ , $B = -3$ and $C = 2$                 | <b>A1</b> | Allow maximum <b>M1A1</b> for one or more ‘correct’ values after <b>B0</b> .                |
|          | Obtain a second value                                        | <b>A1</b> |                                                                                             |
|          | Obtain the third value                                       | <b>A1</b> |                                                                                             |
|          | <b>Alternative method for Question 4</b>                     |           |                                                                                             |
|          | Divide numerator by denominator                              | <b>M1</b> |                                                                                             |
|          | Obtain $2 \left[ + \frac{Px+Q}{(2x-1)(x-3)} \right]$         | <b>A1</b> | $\left[ 2 + \frac{x+7}{(2x-1)(x-3)} \right]$                                                |
|          | State or imply the form $\frac{D}{2x-1} + \frac{E}{x-3}$     | <b>B1</b> |                                                                                             |
|          | Obtain one of $D = -3$ and $E = 2$                           | <b>A1</b> |                                                                                             |
|          | Obtain a second value                                        | <b>A1</b> |                                                                                             |
|          |                                                              | <b>5</b>  |                                                                                             |

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| Question | Answer                                                                                                     | Marks     | Guidance                                                                                                                                                                                                                                             |
|----------|------------------------------------------------------------------------------------------------------------|-----------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 5(a)     | Show circle with centre $3 + 2i$                                                                           | <b>B1</b> |                                                                                                                                                                   |
|          | Show circle with radius 1. Must match <i>their</i> scales: if scales not identical should have an ellipse. | <b>B1</b> |                                                                                                                                                                                                                                                      |
|          | Show line $y = 2$ in at least the diameter of a circle in the first quadrant                               | <b>B1</b> |                                                                                                                                                                                                                                                      |
|          | Shade the correct region in a correct diagram                                                              | <b>B1</b> |                                                                                                                                                                                                                                                      |
|          |                                                                                                            | <b>4</b>  |                                                                                                                                                                                                                                                      |
| 5(b)     | Identify the correct point                                                                                 | <b>B1</b> |                                                                                                                                                                                                                                                      |
|          | Carry out a correct method for finding the argument                                                        | <b>M1</b> | e.g. $\arg x = \tan^{-1} \frac{2}{3} + \sin^{-1} \frac{1}{\sqrt{13}}$<br>Exact working required.                                                                                                                                                     |
|          | Obtain answer $49.8^\circ$                                                                                 | <b>A1</b> | Or better. 0.869 radians scores <b>B1M1A0</b> .                                                                                                                                                                                                      |
|          |                                                                                                            | <b>3</b>  | <b>Special Case 1: B1M0</b> for $45^\circ$ if they have shaded the wrong half of the circle.<br><b>Special Case 2:</b> 3 out of 3 available if they identify the correct point on the correct circle and it is consistent with <i>their</i> shading. |

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| Question | Answer                                                                        | Marks     | Guidance                                                                                                                 |
|----------|-------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------|
| 6(a)     | State correct expansion of $\sin(3x + 2x)$ or $\sin(3x - 2x)$                 | <b>B1</b> |                                                                                                                          |
|          | Substitute expansions in $\frac{1}{2}(\sin 5x + \sin x)$ , or equivalent      | <b>M1</b> |                                                                                                                          |
|          | Simplify and obtain $\frac{1}{2}(\sin 5x + \sin x) = \sin 3x \cos 2x$         | <b>A1</b> | Obtain the <b>given identity</b> correctly.                                                                              |
|          |                                                                               | <b>3</b>  |                                                                                                                          |
| 6(b)     | Obtain integral $-\frac{1}{10}\cos 5x - \frac{1}{2}\cos x$ , or equivalent    | <b>B1</b> |                                                                                                                          |
|          | Substitute limits correctly in an expression of the form $p\cos 5x + q\cos x$ | <b>M1</b> | Correct limits and subtracted the right way around.<br>Do not need values of trig functions for M1.<br>Maximum one slip. |
|          | Obtain $\frac{1}{5}(3 - \sqrt{2})$                                            | <b>A1</b> | Substitute values and obtain the <b>given answer</b> following full, correct and exact working.                          |
|          |                                                                               | <b>3</b>  |                                                                                                                          |

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| Question | Answer                                                                                                                                                        | Marks     | Guidance                                    |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|---------------------------------------------|
| 7        | Separate variables correctly                                                                                                                                  | <b>B1</b> | $\int \frac{1}{y^2} dy = \int 4xe^{-2x} dx$ |
|          | $\int \frac{1}{y^2} dy = -\frac{1}{y}$                                                                                                                        | <b>B1</b> | OE                                          |
|          | Commence the other integration and reach $axe^{-2x} + b \int e^{-2x} dx$                                                                                      | <b>M1</b> |                                             |
|          | Obtain $-2xe^{-2x} + 2 \int e^{-2x} dx$ or $-\frac{1}{2}xe^{-2x} + \frac{1}{2} \int e^{-2x} dx$                                                               | <b>A1</b> | SOI (might have taken out factor of 4)      |
|          | Complete integration and obtain $-2xe^{-2x} - e^{-2x}$                                                                                                        | <b>A1</b> |                                             |
|          | Evaluate a constant or use $x = 0$ and $y = 1$ as limits in a solution containing terms of the form $\frac{p}{y}$ , $qxe^{-2x}$ , $re^{-2x}$ , or equivalent. | <b>M1</b> |                                             |
|          | Obtain $y = \frac{e^{2x}}{2x+1}$ , or equivalent expression for $y$                                                                                           | <b>A1</b> | ISW                                         |
|          |                                                                                                                                                               | <b>7</b>  |                                             |

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| Question | Answer                                                                                                 | Marks        | Guidance                                                                                                                                                                                                              |
|----------|--------------------------------------------------------------------------------------------------------|--------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 8(a)     | Expand the square and equate to 1                                                                      | <b>B1</b>    |                                                                                                                                                                                                                       |
|          | Use correct double angle formula                                                                       | <b>M1</b>    | Need to see $\frac{4}{2}$ or $\sin 2\theta = 2 \sin \theta \cos \theta$ stated.                                                                                                                                       |
|          | Obtain $\cos^4 \theta + \sin^4 \theta = 1 - \frac{1}{2} \sin^2 2\theta$                                | <b>A1</b>    | Obtain the <b>given result</b> correctly.                                                                                                                                                                             |
|          |                                                                                                        | <b>3</b>     |                                                                                                                                                                                                                       |
| 8(b)     | Use the identity and carry out a method for finding a root                                             | <b>M1</b>    | $\left(1 - \frac{1}{2} \sin^2 2\theta = \frac{5}{9}\right)$                                                                                                                                                           |
|          | Obtain answer $35.3^\circ$                                                                             | <b>A1</b>    | Must be correct if overspecified: 35.264...                                                                                                                                                                           |
|          | Obtain a second answer, e.g. $54.7^\circ$                                                              | <b>A1 FT</b> | [e.g. $90^\circ - \text{their } 35.3^\circ$ ]<br>Do not FT if mixing degrees and radians.                                                                                                                             |
|          | Obtain the remaining answers, e.g. $144.7^\circ$ and $125.3^\circ$ and no others in the given interval | <b>A1 FT</b> | [e.g. $180^\circ - \dots$ and $180^\circ - \dots$ ]<br>Ignore answers outside the given interval.<br>Treat answers in radians as a misread.<br>(0.615, 0.955, 2.19, 2.53)<br>Do not FT if mixing degrees and radians. |
|          |                                                                                                        | <b>4</b>     |                                                                                                                                                                                                                       |

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| Question | Answer                                                                                                                  | Marks     | Guidance                                                                                                                                |
|----------|-------------------------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------------------------------------------------------------------|
| 9(a)     | State correct derivative of $ye^{2x}$ with respect to $x$                                                               | <b>B1</b> | $2ye^{2x} + e^{2x} \frac{dy}{dx}$                                                                                                       |
|          | State correct derivative of $y^2e^x$ with respect to $x$                                                                | <b>B1</b> | $2ye^x \frac{dy}{dx} + y^2e^x$                                                                                                          |
|          | Equate attempted derivative of the LHS to zero and solve for $\frac{dy}{dx}$                                            | <b>M1</b> |                                                                                                                                         |
|          | Obtain $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$                                                                   | <b>A1</b> | Obtain the <b>given answer</b> correctly.<br>Condone multiplication by $\frac{-1}{-1}$ and cancelling of $e^x$ without comment.         |
|          | <b>Alternative method for Question 9(a)</b>                                                                             |           |                                                                                                                                         |
|          | Rearrange as $y = \frac{2}{e^{2x} - ye^x} \Rightarrow \frac{d}{dx}(e^{2x} - ye^x) = 2e^{2x} - ye^x - e^x \frac{dy}{dx}$ | <b>B1</b> | Other rearrangements are possible e.g.<br>$y = 2e^{-2x} + y^2e^{-x} \quad \frac{d}{dx}(y^2e^{-x}) = 2ye^{-x} \frac{dy}{dx} - y^2e^{-x}$ |
|          | $\frac{dy}{dx} = -\frac{2}{(e^{2x} - ye^x)^2} \times \left( 2e^{2x} - ye^x - e^x \frac{dy}{dx} \right)$                 | <b>B1</b> | $\Rightarrow \frac{dy}{dx} = -4e^{-x} + 2ye^{-x} \frac{dy}{dx} - y^2e^{-x}$                                                             |
|          | Solve for $\frac{dy}{dx}$                                                                                               | <b>M1</b> |                                                                                                                                         |
|          | Obtain $\frac{dy}{dx} = \frac{2ye^x - y^2}{2y - e^x}$                                                                   | <b>A1</b> | Obtain the <b>given answer</b> correctly.                                                                                               |
|          |                                                                                                                         | <b>4</b>  |                                                                                                                                         |

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| Question | Answer                                                                                      | Marks      | Guidance                          |
|----------|---------------------------------------------------------------------------------------------|------------|-----------------------------------|
| 9(b)     | Equate denominator to zero and substitute for $y$ or for $e^x$ in the equation of the curve | <b>*M1</b> |                                   |
|          | Obtain equation of the form $ae^{3x} = b$ or $cy^3 = d$                                     | <b>DM1</b> | $(e^{3x} = 8, \quad y^3 = 1)$ SOI |
|          | Obtain $x = \ln 2$                                                                          | <b>A1</b>  | Accept $\frac{1}{3} \ln 8$ ISW    |
|          | Obtain $y = 1$                                                                              | <b>A1</b>  |                                   |
|          |                                                                                             | <b>4</b>   |                                   |

| Question | Answer                                                                                                                                           | Marks     | Guidance                                                                                                                                                                             |
|----------|--------------------------------------------------------------------------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(a)    | Obtain direction vector $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ , or equivalent                                                                 | <b>B1</b> | Accept answers as column vectors throughout.                                                                                                                                         |
|          | Use a correct method to form a vector equation                                                                                                   | <b>M1</b> |                                                                                                                                                                                      |
|          | State answer $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - \mathbf{k} + \lambda(-\mathbf{i} + \mathbf{j} + 2\mathbf{k})$ , or equivalent correct form | <b>A1</b> | e.g. $\mathbf{r} = \begin{pmatrix} 0 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}$ Allow $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ for $\mathbf{r}$ . |
|          |                                                                                                                                                  | <b>3</b>  |                                                                                                                                                                                      |
| 10(b)    | Use a correct method to find the position vector of $C$                                                                                          | <b>M1</b> | e.g. $\mathbf{OC} = \mathbf{OA} + \mathbf{AC} = \begin{pmatrix} 1-3 \\ 2+3 \\ -1+6 \end{pmatrix}$                                                                                    |
|          | Obtain answer $-2\mathbf{i} + 5\mathbf{j} + 5\mathbf{k}$ , or equivalent                                                                         | <b>A1</b> | Accept as coordinates.                                                                                                                                                               |
|          |                                                                                                                                                  | <b>2</b>  |                                                                                                                                                                                      |

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| Question | Answer                                                                                                                                                | Marks        | Guidance                                                                                                                                                                               |
|----------|-------------------------------------------------------------------------------------------------------------------------------------------------------|--------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 10(c)    | State $\overline{OP}$ in component form                                                                                                               | <b>B1 FT</b> |                                                                                                                                                                                        |
|          | Form an equation in $\lambda$ by equating the modulus of $OP$ to $\sqrt{14}$ , or equivalent                                                          | <b>M1</b>    |                                                                                                                                                                                        |
|          | Simplify and obtain $3\lambda^2 - \lambda - 4 = 0$ , or equivalent                                                                                    | <b>A1</b>    | $3\lambda^2 + \lambda - 4 = 0$ if using $\mathbf{i} - \mathbf{j} - 2\mathbf{k}$ in (a).<br>$3\mu^2 + 5\mu - 2 = 0$ if using $-\mathbf{i} + \mathbf{j} + 2\mathbf{k}$ in (a) and $OB$ . |
|          | Solve a 3-term quadratic and find a position vector                                                                                                   | <b>M1</b>    | $\left(\lambda = -1, \frac{4}{3} \text{ or } \lambda = 1, -\frac{4}{3} \text{ or } \mu = \frac{1}{3}, -2 \text{ or } \mu = -\frac{1}{3}, 2\right)$                                     |
|          | Obtain answers $2\mathbf{i} + \mathbf{j} - 3\mathbf{k}$ and $-\frac{1}{3}\mathbf{i} + \frac{10}{3}\mathbf{j} + \frac{5}{3}\mathbf{k}$ , or equivalent | <b>A1</b>    | Accept as coordinates.                                                                                                                                                                 |
|          |                                                                                                                                                       | <b>5</b>     |                                                                                                                                                                                        |

| Question | Answer                                                                           | Marks     | Guidance                                                                                         |
|----------|----------------------------------------------------------------------------------|-----------|--------------------------------------------------------------------------------------------------|
| 11(a)    | Use chain rule                                                                   | <b>M1</b> | Allow if not starting with the correct index.                                                    |
|          | Obtain correct derivative in any form                                            | <b>A1</b> | e.g. $\frac{dy}{dx} = \frac{\sec^2 x}{2\sqrt{\tan x}}$                                           |
|          | Use correct Pythagoras to obtain correct derivative in terms of $\tan x$         | <b>A1</b> | e.g. $\frac{dy}{dx} = \frac{1 + \tan^2 x}{2\sqrt{\tan x}}$                                       |
|          | Use a correct derivative to obtain $\frac{dy}{dx} = 1$ when $x = \frac{1}{4}\pi$ | <b>B1</b> | Confirm the <b>given statement</b> from correct work.<br>Should see at least $\frac{2}{2} = 1$ . |
|          |                                                                                  | <b>4</b>  |                                                                                                  |

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| Question | Answer                                                                                                           | Marks      | Guidance                                                                                                                                        |
|----------|------------------------------------------------------------------------------------------------------------------|------------|-------------------------------------------------------------------------------------------------------------------------------------------------|
| 11(b)    | Equate answer to part (a) to 1 and obtain a quartic equation in $t$ or $\tan x$                                  | <b>*M1</b> | At least as far as $(1 + \tan^2 x)^2 = 4 \tan x$ .                                                                                              |
|          | Obtain correct answer, i.e. $t^4 + 2t^2 - 4t + 1 = 0$                                                            | <b>A1</b>  | Or equivalent horizontal form.                                                                                                                  |
|          | Commence division by $t - 1$                                                                                     | <b>DM1</b> | As far as $t^3 + t^2 + \dots$ by long division or inspection.<br>Allow verification by multiplying given answer by $t - 1$ .                    |
|          | Obtain the given answer                                                                                          | <b>A1</b>  |                                                                                                                                                 |
|          |                                                                                                                  | <b>4</b>   |                                                                                                                                                 |
| 11(c)    | Use the iterative process correctly with the given formula at least once                                         | <b>M1</b>  | Obtain one value and use that to obtain the next.<br>Must be working in radians.                                                                |
|          | Obtain final answer $a = 0.29$                                                                                   | <b>A1</b>  |                                                                                                                                                 |
|          | Show sufficient iterations to 4 d.p. to justify 0.29 to 2 d.p., or show there is a sign change in (0.285, 0.295) | <b>A1</b>  | e.g. 0.3, 0.2854, 0.2894, 0.2883, .....<br>0.4, 0.2436, 0.2984, 0.2841, 0.2883, 0.2871, ...<br>0.5, 0.1776, 0.3103, 0.2805, 0.2893, 0.2868, ... |
|          |                                                                                                                  | <b>3</b>   |                                                                                                                                                 |



## Cambridge International AS & A Level

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**MATHEMATICS**

**9709/33**

Paper 3 Pure Mathematics 3

**October/November 2021**

**MARK SCHEME**

Maximum Mark: 75

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|                         |
|-------------------------|
| <p><b>Published</b></p> |
|-------------------------|

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the October/November 2021 series for most Cambridge IGCSE™, Cambridge International A and AS Level components and some Cambridge O Level components.

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This document consists of **14** printed pages.

**PUBLISHED****Generic Marking Principles**

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptors for a question. Each question paper and mark scheme will also comply with these marking principles.

**GENERIC MARKING PRINCIPLE 1:**

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

**GENERIC MARKING PRINCIPLE 2:**

Marks awarded are always **whole marks** (not half marks, or other fractions).

**GENERIC MARKING PRINCIPLE 3:**

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

**GENERIC MARKING PRINCIPLE 4:**

Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

**GENERIC MARKING PRINCIPLE 5:**

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

**GENERIC MARKING PRINCIPLE 6:**

Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

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| Mathematics Specific Marking Principles |                                                                                                                                                                                                                                               |
|-----------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 1                                       | Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.                                     |
| 2                                       | Unless specified in the question, answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.                                                            |
| 3                                       | Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.                                                                                                               |
| 4                                       | Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).                                                                                            |
| 5                                       | Where a candidate has misread a number in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 mark for the misread. |
| 6                                       | Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.                                                                                              |

**PUBLISHED****Mark Scheme Notes**

The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

**Types of mark**

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more ‘method’ steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.
- A or B marks are given for correct work only (not for results obtained from incorrect working) unless follow through is allowed (see abbreviation FT above).
  - For a numerical answer, allow the A or B mark if the answer is correct to 3 significant figures or would be correct to 3 significant figures if rounded (1 decimal place for angles in degrees).
  - The total number of marks available for each question is shown at the bottom of the Marks column.
  - Wrong or missing units in an answer should not result in loss of marks unless the guidance indicates otherwise.
  - Square brackets [ ] around text or numbers show extra information not needed for the mark to be awarded.

**Abbreviations**

|        |                                                                                                                                                                                       |
|--------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| AEF/OE | Any Equivalent Form (of answer is equally acceptable) / Or Equivalent                                                                                                                 |
| AG     | Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)                                                   |
| CAO    | Correct Answer Only (emphasising that no ‘follow through’ from a previous error is allowed)                                                                                           |
| CWO    | Correct Working Only                                                                                                                                                                  |
| ISW    | Ignore Subsequent Working                                                                                                                                                             |
| SOI    | Seen Or Implied                                                                                                                                                                       |
| SC     | Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance) |
| WWW    | Without Wrong Working                                                                                                                                                                 |
| AWRT   | Answer Which Rounds To                                                                                                                                                                |

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| Question | Answer                                                               | Marks     | Guidance |
|----------|----------------------------------------------------------------------|-----------|----------|
| 1        | Commence division and reach partial quotient of the form $2x^2 + kx$ | <b>M1</b> |          |
|          | Obtain quotient $2x^2 + 2x - 2$                                      | <b>A1</b> |          |
|          | Obtain remainder $-6x + 5$                                           | <b>A1</b> |          |
|          |                                                                      | <b>3</b>  |          |

| Question | Answer                                             | Marks     | Guidance |
|----------|----------------------------------------------------|-----------|----------|
| 2(a)     | Show a recognizable sketch graph of $y =  2x - 3 $ | <b>B1</b> |          |
|          |                                                    | <b>1</b>  |          |

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| Question | Answer                                                                               | Marks     | Guidance |
|----------|--------------------------------------------------------------------------------------|-----------|----------|
| 2(b)     | Find $x$ -coordinate of intersection with $y = 3x + 2$                               | <b>M1</b> |          |
|          | Obtain $x = \frac{1}{5}$                                                             | <b>A1</b> |          |
|          | State final answer $x > \frac{1}{5}$ only                                            | <b>A1</b> |          |
|          | <b>Alternative method for Question 2(b)</b>                                          |           |          |
|          | Solve the linear inequality $3 - 2x < 3x + 2$ , or corresponding equation            | <b>M1</b> |          |
|          | Obtain critical value $x = \frac{1}{5}$                                              | <b>A1</b> |          |
|          | State final answer $x > \frac{1}{5}$ only                                            | <b>A1</b> |          |
|          | <b>Alternative method for Question 2(b)</b>                                          |           |          |
|          | Solve the quadratic inequality $(2x - 3)^2 < (3x + 2)^2$ , or corresponding equation | <b>M1</b> |          |
|          | Obtain critical value $x = \frac{1}{5}$                                              | <b>A1</b> |          |
|          | State final answer $x > \frac{1}{5}$ only                                            | <b>A1</b> |          |
|          |                                                                                      | <b>3</b>  |          |

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| Question | Answer                                                                             | Marks     | Guidance |
|----------|------------------------------------------------------------------------------------|-----------|----------|
| 3        | Use laws of indices correctly and solve for $4^x$                                  | <b>M1</b> |          |
|          | Obtain correct solution in any form, e.g. $4^x = \frac{256}{15}$                   | <b>A1</b> |          |
|          | Use a correct method for solving an equation of the form $4^x = a$ , where $a > 0$ | <b>M1</b> |          |
|          | Obtain answer 2.047                                                                | <b>A1</b> |          |
|          |                                                                                    | <b>4</b>  |          |

| Question | Answer                                                                                 | Marks      | Guidance |
|----------|----------------------------------------------------------------------------------------|------------|----------|
| 4        | Commence integration and reach $ax \cos \frac{1}{2}x + b \int \cos \frac{1}{2}x \, dx$ | <b>*M1</b> |          |
|          | Obtain $-2x \cos \frac{1}{2}x + 2 \int \cos \frac{1}{2}x \, dx$                        | <b>A1</b>  | OE       |
|          | Complete integration obtaining $-2x \cos \frac{1}{2}x + 4 \sin \frac{1}{2}x$           | <b>A1</b>  | OE       |
|          | Use limits correctly, having integrated twice                                          | <b>DM1</b> |          |
|          | Obtain answer $2 + \frac{\sqrt{3}}{3} \pi$ , or exact equivalent                       | <b>A1</b>  |          |
|          |                                                                                        | <b>5</b>   |          |

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| Question | Answer                                                                                                  | Marks     | Guidance                                                                    |
|----------|---------------------------------------------------------------------------------------------------------|-----------|-----------------------------------------------------------------------------|
| 5        | Use double angle formula and obtain an equation in $\sin \theta$                                        | <b>M1</b> |                                                                             |
|          | Reduce to $6\sin^2 \theta + \sin \theta - 5 = 0$ , or 3-term equivalent                                 | <b>A1</b> |                                                                             |
|          | Solve a 3-term quadratic in $\sin \theta$ and calculate $\theta$                                        | <b>M1</b> |                                                                             |
|          | Obtain answer, e.g. $56.4^\circ$                                                                        | <b>A1</b> |                                                                             |
|          | Obtain second and third answers, e.g. $123.6^\circ$ and $270^\circ$ and no others in the given interval | <b>A1</b> | Ignore answers outside the interval. Treat answers in radians as a misread. |
|          |                                                                                                         | <b>5</b>  |                                                                             |

| Question | Answer                                                                               | Marks     | Guidance |
|----------|--------------------------------------------------------------------------------------|-----------|----------|
| 6(a)     | Use $\cos(A - B)$ formula and obtain an expression in terms of $\sin x$ and $\cos x$ | <b>M1</b> |          |
|          | Collect terms and reach $2 \cos x + \sqrt{3} \sin x$                                 | <b>A1</b> |          |
|          | State $R = \sqrt{7}$                                                                 | <b>A1</b> |          |
|          | Use trig formula to find $\alpha$                                                    | <b>M1</b> |          |
|          | Obtain $\alpha = 40.89^\circ$                                                        | <b>A1</b> |          |
|          |                                                                                      | <b>5</b>  |          |
| 6(b)     | Use correct method to find $x$                                                       | <b>M1</b> |          |
|          | Obtain answer $x = 220.9^\circ$                                                      | <b>A1</b> |          |
|          |                                                                                      | <b>2</b>  |          |

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| Question | Answer                                                                          | Marks      | Guidance |
|----------|---------------------------------------------------------------------------------|------------|----------|
| 7(a)     | Use chain rule to differentiate LHS                                             | <b>*M1</b> |          |
|          | Obtain $\frac{1}{x+y} \left( 1 + \frac{dy}{dx} \right)$                         | <b>A1</b>  |          |
|          | Equate derivative of LHS to $1 - 2 \frac{dy}{dx}$ and solve for $\frac{dy}{dx}$ | <b>DM1</b> |          |
|          | Obtain the given answer correctly                                               | <b>A1</b>  |          |
|          |                                                                                 | <b>4</b>   |          |
| 7(b)     | State $x + y = 1$                                                               | <b>B1</b>  |          |
|          | Obtain or imply $x - 2y = 0$                                                    | <b>B1</b>  |          |
|          | Obtain coordinates $x = \frac{2}{3}$ and $y = \frac{1}{3}$                      | <b>B1</b>  |          |
|          |                                                                                 | <b>3</b>   |          |

| Question | Answer                                                                                                                 | Marks     | Guidance |
|----------|------------------------------------------------------------------------------------------------------------------------|-----------|----------|
| 8(a)     | State $\overrightarrow{OM} = 4\mathbf{i} + 2\mathbf{j}$                                                                | <b>B1</b> |          |
|          | Use a correct method to find $\overrightarrow{ON}$                                                                     | <b>M1</b> |          |
|          | Obtain answer $3\mathbf{j} + \mathbf{k}$                                                                               | <b>A1</b> |          |
|          | Use a correct method to find a line equation for $MN$                                                                  | <b>M1</b> |          |
|          | Obtain answer $\mathbf{r} = 3\mathbf{j} + \mathbf{k} + \lambda(4\mathbf{i} - \mathbf{j} - \mathbf{k})$ , or equivalent | <b>A1</b> |          |
|          |                                                                                                                        | <b>5</b>  |          |

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| Question | Answer                                                                                                                                                                                                                                                  | Marks     | Guidance |
|----------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------|----------|
| 8(b)     | Taking a general point $P$ on $MN$ , form an equation in $\lambda$ by <i>either</i> equating a relevant scalar product to zero <i>or</i> equating the derivative of $\overrightarrow{OP}$ to zero <i>or</i> using Pythagoras in triangle $OPM$ or $OPN$ | <b>M1</b> |          |
|          | Obtain $\lambda = \frac{2}{9}$                                                                                                                                                                                                                          | <b>A1</b> | OE       |
|          | Use correct method to find $OP$                                                                                                                                                                                                                         | <b>M1</b> |          |
|          | Obtain the given answer correctly                                                                                                                                                                                                                       | <b>A1</b> |          |
|          | <b>Alternative method to Question 8(b)</b>                                                                                                                                                                                                              |           |          |
|          | Use a scalar product to find the projection of $OM$ (or $ON$ ) on $MN$                                                                                                                                                                                  | <b>M1</b> |          |
|          | Obtain answer $\frac{14}{\sqrt{18}}$ $\left( \text{or } \frac{4}{\sqrt{18}} \right)$                                                                                                                                                                    | <b>A1</b> |          |
|          | Use Pythagoras to obtain the perpendicular                                                                                                                                                                                                              | <b>M1</b> |          |
|          | Obtain the given answer correctly                                                                                                                                                                                                                       | <b>A1</b> |          |
|          |                                                                                                                                                                                                                                                         | <b>4</b>  |          |

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| Question | Answer                                                                                | Marks     | Guidance |
|----------|---------------------------------------------------------------------------------------|-----------|----------|
| 9(a)     | Use quotient or product rule                                                          | <b>M1</b> |          |
|          | Obtain correct derivative in any form                                                 | <b>A1</b> |          |
|          | Equate derivative to zero and solve for $x$                                           | <b>M1</b> |          |
|          | Obtain answer $x = 3$                                                                 | <b>A1</b> |          |
|          |                                                                                       | <b>4</b>  |          |
| 9(b)     | State $\frac{du}{dx} = \frac{1}{2\sqrt{x}}$ , or $dx = 2\sqrt{x}du$ , or $2u du = dx$ | <b>B1</b> |          |
|          | Substitute and obtain integrand $\frac{2}{9-u^2}$                                     | <b>B1</b> |          |
|          | Use given formula for the integral or integrate relevant partial fractions            | <b>M1</b> |          |
|          | Obtain integral $\frac{1}{3} \ln\left(\frac{3+u}{3-u}\right)$                         | <b>A1</b> | OE       |
|          | Use limits $u = 0$ and $u = 2$ correctly                                              | <b>M1</b> |          |
|          | Obtain the given answer correctly                                                     | <b>A1</b> |          |
|          |                                                                                       | <b>6</b>  |          |

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| Question | Answer                                                                                                                                 | Marks        | Guidance  |
|----------|----------------------------------------------------------------------------------------------------------------------------------------|--------------|-----------|
| 10(a)    | State or imply equation of the form $\frac{dx}{dt} = k \frac{x}{20-x}$                                                                 | <b>M1</b>    |           |
|          | Obtain $k = 19$                                                                                                                        | <b>A1</b>    | <b>AG</b> |
|          |                                                                                                                                        | <b>2</b>     |           |
| 10(b)    | Separate variables and integrate at least one side                                                                                     | <b>M1</b>    |           |
|          | Obtain terms $20 \ln x - x$ and $19t$ , or equivalent                                                                                  | <b>A1 A1</b> |           |
|          | Evaluate a constant or use $t = 0$ and $x = 1$ as limits in a solution containing terms $a \ln x$ and $bt$                             | <b>M1</b>    |           |
|          | Substitute $t = 1$ and rearrange the equation in the given form                                                                        | <b>A1</b>    | <b>AG</b> |
|          |                                                                                                                                        | <b>5</b>     |           |
| 10(c)    | Use $x_{n+1} = e^{0.9+0.05x_n}$ correctly at least once                                                                                | <b>M1</b>    |           |
|          | Obtain final answer $x = 2.83$                                                                                                         | <b>A1</b>    |           |
|          | Show sufficient iterations to 4 decimal places to justify 2.83 to 2 d.p. or show there is a sign change in the interval (2.825, 2.835) | <b>A1</b>    |           |
|          |                                                                                                                                        | <b>3</b>     |           |
| 10(d)    | Set $x = 20$ and obtain answer $t = 2.15$                                                                                              | <b>B1</b>    |           |
|          |                                                                                                                                        | <b>1</b>     |           |

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| Question  | Answer                                                            | Marks     | Guidance |
|-----------|-------------------------------------------------------------------|-----------|----------|
| 11(a)     | State or imply $r = 2$                                            | <b>B1</b> |          |
|           | State or imply $\theta = \frac{5}{6}\pi$                          | <b>B1</b> |          |
|           |                                                                   | <b>2</b>  |          |
| 11(b)     | Use a correct method for finding the modulus or argument of $u^6$ | <b>M1</b> |          |
|           | Show correctly that $u^6$ is real and has value -64               | <b>A1</b> |          |
|           |                                                                   | <b>2</b>  |          |
| 11(c)(i)  | Show half lines from the point representing $-\sqrt{3} + i$       | <b>B1</b> |          |
|           | Show correct half lines                                           | <b>B1</b> |          |
|           | Show the line $x = 2$ in the first quadrant                       | <b>B1</b> |          |
|           | Shade the correct region                                          | <b>B1</b> |          |
|           |                                                                   | <b>4</b>  |          |
| 11(c)(ii) | Carry out a correct method to find the greatest value of $ z $    | <b>M1</b> |          |
|           | Obtain answer 5.14                                                | <b>A1</b> |          |
|           |                                                                   | <b>2</b>  |          |