

Cambridge International AS & A Level

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MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 75.
- The number of marks for each question or part question is shown in brackets [].

This document has **20** pages. Blank pages are indicated.

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1 Solve the inequality $2 - 5x > 2|x - 3|$.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. [4]

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$.

[5]

[illegible]

4 Solve the equation

$$\log_{10}(2x+1) = 2\log_{10}(x+1) - 1.$$

Give your answers correct to 3 decimal places.

[6]

[illegible]

- 5 (a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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- (b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1} \left(\frac{1}{e^{-\frac{1}{2}x_n} + 1} \right),$$

with initial value $x_1 = 2$, converges to one of these roots.

Use the formula to determine this root correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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- 6 (a)** Express $\sqrt{6} \cos \theta + 3 \sin \theta$ in the form $R \cos(\theta - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 2 decimal places. [3]

[illegible]

- (b)** Hence solve the equation $\sqrt{6} \cos \frac{1}{3}x + 3 \sin \frac{1}{3}x = 2.5$, for $0^\circ < x < 360^\circ$. [4]

[illegible]

- 7 (a) Verify that $-1 + \sqrt{5}i$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

[illegible]

(b) Find the other roots of this equation.

[4]

[illegible]

- 8 The coordinates (x, y) of a general point of a curve satisfy the differential equation

$$x \frac{dy}{dx} = (1 - 2x^2)y,$$

for $x > 0$. It is given that $y = 1$ when $x = 1$.

Solve the differential equation, obtaining an expression for y in terms of x .

[6]

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9 Let $f(x) = \frac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

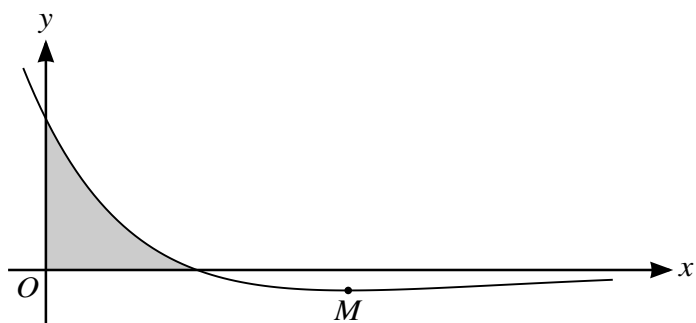
(a) Express $f(x)$ in partial fractions.

[5]

[illegible]

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 . [5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.



(a) Find the exact coordinates of M .

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dashed lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e . [5]

[illegible]

- 11 Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.

- (a) Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

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MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3

October/November 2020

1 hour 50 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

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- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

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1 Solve the equation

$$\ln(1 + e^{-3x}) = 2.$$

Give the answer correct to 3 decimal places.

[3]

[illegible]

- 2 (a)** Expand $\sqrt[3]{1+6x}$ in ascending powers of x , up to and including the term in x^3 , simplifying the coefficients. [4]

[illegible]

- (b) State the set of values of x for which the expansion is valid. [1]

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3 The variables x and y satisfy the relation $2^y = 3^{1-2x}$.

- (a) By taking logarithms, show that the graph of y against x is a straight line. State the exact value of the gradient of this line. [3]

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- (b) Find the exact x -coordinate of the point of intersection of this line with the line $y = 3x$. Give your answer in the form $\frac{\ln a}{\ln b}$, where a and b are integers. [2]

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- 4 (a)** Show that the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$ can be written in the form

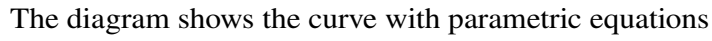
$$\tan^2 \theta + 3\sqrt{3} \tan \theta - 2 = 0. \quad [3]$$

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(b) Hence solve the equation $\tan(\theta + 60^\circ) = 2 \cot \theta$, for $0^\circ < \theta < 180^\circ$.

[3]

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for $-\frac{1}{2}\pi < \theta < \frac{1}{2}\pi$.

- [illegible]

The gradient of the curve has its maximum value at the point P .

(b) Find the exact value of the x -coordinate of P . [4]

[illegible]

- 6** The complex number u is defined by

$$u = \frac{7+i}{1-i}.$$

- (a)** Express u in the form $x + iy$, where x and y are real. [3]

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- (b)** Show on a sketch of an Argand diagram the points A , B and C representing u , $7 + i$ and $1 - i$ respectively. [2]

$$\tan^{-1}\left(\frac{4}{3}\right) = \tan^{-1}\left(\frac{1}{7}\right) + \frac{1}{4}\pi. \quad [3]$$

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7 The variables x and t satisfy the differential equation

$$e^{3t} \frac{dx}{dt} = \cos^2 2x,$$

for $t \geq 0$. It is given that $x = 0$ when $t = 0$.

(a) Solve the differential equation and obtain an expression for x in terms of t . [7]

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(b) State what happens to the value of x when t tends to infinity. [1]

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- 8 With respect to the origin O , the position vectors of the points A , B , C and D are given by

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ 1 \\ 5 \end{pmatrix}, \quad \overrightarrow{OB} = \begin{pmatrix} 4 \\ -1 \\ 1 \end{pmatrix}, \quad \overrightarrow{OC} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} \quad \text{and} \quad \overrightarrow{OD} = \begin{pmatrix} 3 \\ 2 \\ 3 \end{pmatrix}.$$

- (a) Show that $AB = 2CD$.

[3]

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- (b) Find the angle between the directions of $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

[3]

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9 Let $f(x) = \frac{7x+18}{(3x+2)(x^2+4)}$.

(a) Express $f(x)$ in partial fractions.

[5]

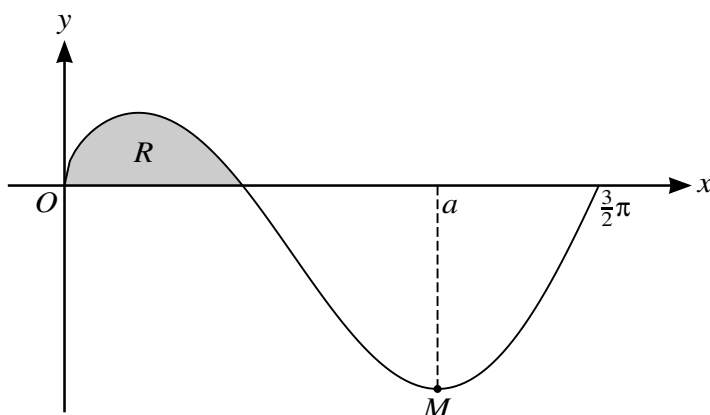
[illegible]

(b) Hence find the exact value of $\int_0^2 f(x) \, dx$.

[6]

[illegible]

10



The diagram shows the curve $y = \sqrt{x} \cos x$, for $0 \leq x \leq \frac{3}{2}\pi$, and its minimum point M , where $x = a$. The shaded region between the curve and the x -axis is denoted by R .

- (a) Show that a satisfies the equation $\tan a = \frac{1}{2a}$. [3]

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- (b) The sequence of values given by the iterative formula $a_{n+1} = \pi + \tan^{-1}\left(\frac{1}{2a_n}\right)$, with initial value $x_1 = 3$, converges to a .

Use this formula to determine a correct to 2 decimal places. Give the result of each iteration to 4 decimal places. [3]

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1 Solve the inequality $2 - 5x > 2|x - 3|$.

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- 2 On a sketch of an Argand diagram, shade the region whose points represent complex numbers z satisfying the inequalities $|z| \geq 2$ and $|z - 1 + i| \leq 1$. [4]

3 The parametric equations of a curve are

$$x = 3 - \cos 2\theta, \quad y = 2\theta + \sin 2\theta,$$

for $0 < \theta < \frac{1}{2}\pi$.

Show that $\frac{dy}{dx} = \cot \theta$.

[5]

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- 5 (a) By sketching a suitable pair of graphs, show that the equation $\operatorname{cosec} x = 1 + e^{-\frac{1}{2}x}$ has exactly two roots in the interval $0 < x < \pi$. [2]

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- (b) The sequence of values given by the iterative formula

$$x_{n+1} = \pi - \sin^{-1} \left(\frac{1}{e^{-\frac{1}{2}x_n} + 1} \right),$$

with initial value $x_1 = 2$, converges to one of these roots.

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[illegible]

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- 7 (a)** Verify that $-1 + \sqrt{5}i$ is a root of the equation $2x^3 + x^2 + 6x - 18 = 0$. [3]

[illegible]

(b) Find the other roots of this equation.

[4]

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9 Let $f(x) = \frac{8 + 5x + 12x^2}{(1 - x)(2 + 3x)^2}$.

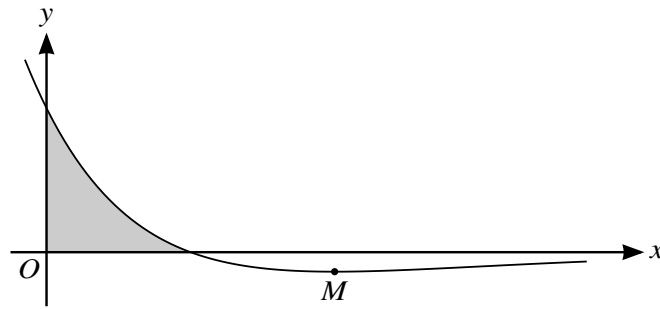
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[5]

[illegible]

- (b)** Hence obtain the expansion of $f(x)$ in ascending powers of x , up to and including the term in x^2 .
[5]

[illegible]



The diagram shows the curve $y = (2 - x)e^{-\frac{1}{2}x}$, and its minimum point M .

- (a) Find the exact coordinates of M . [5]

[illegible]

- (b) Find the area of the shaded region bounded by the curve and the axes. Give your answer in terms of e . [5]

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- 11** Two lines have equations $\mathbf{r} = \mathbf{i} + 2\mathbf{j} + \mathbf{k} + \lambda(a\mathbf{i} + 2\mathbf{j} - \mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$, where a is a constant.
- (a)** Given that the two lines intersect, find the value of a and the position vector of the point of intersection. [5]

[illegible]

- (b)** Given instead that the acute angle between the directions of the two lines is $\cos^{-1}(\frac{1}{6})$, find the two possible values of a . [6]

[illegible]

[illegible]

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