
MATHEMATICS

9709/72

Paper 7

October/November 2019

MARK SCHEME

Maximum Mark: 50

Published

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This document consists of **11** printed pages.



Cambridge Assessment
International Education

Generic Marking Principles

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GENERIC MARKING PRINCIPLE 1:

Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:

Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:

Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:

Rules must be applied consistently e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:

Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:

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The following notes are intended to aid interpretation of mark schemes in general, but individual mark schemes may include marks awarded for specific reasons outside the scope of these notes.

Types of mark

- M** Method mark, awarded for a valid method applied to the problem. Method marks are not lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. Correct application of a formula without the formula being quoted obviously earns the M mark and in some cases an M mark can be implied from a correct answer.
- A** Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated method mark is earned (or implied).
- B** Mark for a correct result or statement independent of method marks.
- DM or DB** When a part of a question has two or more “method” steps, the M marks are generally independent unless the scheme specifically says otherwise; and similarly, when there are several B marks allocated. The notation DM or DB is used to indicate that a particular M or B mark is dependent on an earlier M or B (asterisked) mark in the scheme. When two or more steps are run together by the candidate, the earlier marks are implied and full credit is given.
- FT** Implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A or B marks are given for correct work only.

Abbreviations

AEF/OE	Any Equivalent Form (of answer is equally acceptable) / Or Equivalent
AG	Answer Given on the question paper (so extra checking is needed to ensure that the detailed working leading to the result is valid)
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ISW	Ignore Subsequent Working
SOI	Seen Or Implied
SC	Special Case (detailing the mark to be given for a specific wrong solution, or a case where some standard marking practice is to be varied in the light of a particular circumstance)
WWW	Without Wrong Working
AWRT	Answer Which Rounds To

Question	Answer	Marks	Guidance
1(i)	Binomial	B1	
	$n = 500$ and $p = \frac{1}{150}$ or 0.00667	B1	Or $B\left(500, \frac{1}{150}\right)$ for B1B1
		2	
1(ii)	Poisson	B1	
	n large and mean = $\frac{10}{3}$ or 3.3 or better, which is < 5	B1	Accept $n > 50$
		2	
1(iii)	$1 - e^{-\frac{10}{3}} \times \left(1 + \frac{10}{3} + \frac{\left(\frac{10}{3}\right)^2}{2}\right)$	M1	1-P($X = 0, 1, 2$)
	$= 1 - 0.353$	A1	Correct expression with $\lambda = 3.3$ or better
	$= 0.647$ (3 sf)	A1	SC Use of Binomial scores B1 for 0.648. Use of Normal scores B1 for 0.67(0) to 0.677
		3	

Question	Answer	Marks	Guidance
2(i)(a)	Assume standard deviation for the region is 7.1	B1	Or standard deviation is same as for whole population OE
	$\frac{63.2 - 65.2}{\frac{7.1}{\sqrt{n}}} = -2.182$	M1	Attempt to find correct equation (accept +2.182)
	$n = \{-2.182 \times 7.1 \div (-2)\}^2$	A1	Any correct expression for n or $\sqrt{n}$. SOI
	$n = 60$	A1	CWO. Must be an integer
		4	
2(i)(b)	H ₀ : population mean (or μ) = 65.2 H ₁ : population mean (or μ) < 65.2	B1	Not just 'mean'
	$2.182 > 1.751$	M1	Or valid area comparison.
	There is evidence that animals are shorter in this region	A1	CWO. No contradictions
		3	
2(ii)	Population unknown or population not given as normal	B1	Allow population not normal. Accept distribution of X unknown.
		1	

Question	Answer	Marks	Guidance
3(i)	$\text{est}(\mu) = \frac{25110}{50} (= 502.2)$	B1	
	$\text{est}(\sigma^2) = \frac{50}{49} \left(\frac{12610300}{50} - \frac{25110^2}{50} \right) \left(= \frac{50}{49} \times \frac{58}{50} = 1.1836 \right)$	M1	OE
	1.18 (3 sf) or $\frac{58}{49}$	A1	Accept SD = 1.0879
	$z = 2.054$ or 2.055	B1	
	$502.2 \pm z \times \frac{\sqrt{1.1836}}{\sqrt{50}}$	M1	Must be of correct form.
	501.9 to 502.5 (1dp)	A1	CWO. Must be in interval. SC accept use of biased variance (1.16) for M1 A1
		6	
3(ii)	More confident or z would be greater, Hence wider.	B1	OE Reason needed
		1	

Question	Answer	Marks	Guidance
4(i)	$\frac{1}{2} \times a \times \frac{a}{2} = 1$ or $\frac{1}{2} \int_0^a x dx = 1$ $\frac{a^2}{4} = 1$ OE	M1	Attempt at triangle area or integral $f(x)$ and $= 1$,
	$a = 2$	A1	
		2	
4(ii)	$\frac{1}{2} \int_0^2 x^2 dx$	M1	Attempt integral $xf(x)$
	$= \left[\frac{x^3}{6} \right]_0^2$	M1	Correct integral and limits 0 to their 'a'
	$\left(= \frac{8}{6} \right) = \frac{4}{3}$	A1	AG CWO
		3	

Question	Answer	Marks	Guidance
4(iii)	$P\left(X < \frac{4}{3}\right) = \frac{1}{2} \int_0^{\frac{4}{3}} x dx$	M1	Attempt integral f(x) between correct limits
	$= \frac{4}{9}$	A1	or $\frac{5}{9}$
	$P(E(X) < X < m) = \frac{1}{2} - \frac{4}{9}$	M1	or $\frac{5}{9} - \frac{1}{2}$
	$\frac{1}{18}$	A1	
	Alternative method for question 4(iii)		
	Attempt to find m	M1	
	$m = \sqrt{2}$	A1	
	Integrate f(x) between $\frac{4}{3}$ and ' $\sqrt{2}$ '	M1	
	$\frac{1}{18}$	A1	
		4	

Question	Answer	Marks	Guidance
5(i)	mean = 3250 var. = 61	B1	Or mean = 325 var. = $\frac{6.1}{10}$
	$\frac{3240 - 3250}{\sqrt{61}} (= -1.280)$	M1	Standardise with their values (no mixed methods)
	$\phi(-1.280) = 1 - \phi(1.280)$	M1	Area consistent with their figures
	0.100	A1	Allow 0.1
		4	
5(ii)	$E(D) = 325 - 2 \times 167 = -9$	B1	Accept ± 9
	$\text{Var}(D) = 6.1 + 2^2 \times 5.6 (= 28.5)$	B1	
	$\frac{0 - (-9)}{\sqrt{28.5}} (= 1.686)$	M1	Standardising with <i>their</i> values. Must have a combination attempt on denominator and $\sqrt{}$
	$1 - \phi(1.686)$	M1	Area consistent with their figures
	0.0459	A1	
		5	

Question	Answer	Marks	Guidance
6(i)	H_0 : Pop mean (or λ or μ) is 1.1 H_1 : Pop mean (or λ or μ) is more than 1.1	B1	
	$P(X \geq 4) = 1 - e^{-1.1} \left(1 + 1.1 + \frac{1.1^2}{2} + \frac{1.1^3}{3!} \right)$	M1	Correct expression for either $P(X \geq 4)$ or $P(X \geq 5)$
	0.0257	A1	Correct value of either $P(X \geq 4)$ or $P(X \geq 5)$
	$P(X \geq 5) = 0.0257 - e^{-1.1} \times \frac{1.1^4}{4!} = 0.00544$	B1	B1 for the other value (Note use of $P(X < 4) = 0.9743$ and $P(X < 5) = 0.99456$ can score only if comparison with 0.99 seen)
	$0.00544 < 0.01 < 0.0257$	M1	OE stated (valid comparison)
	There is evidence mean has increased	B1	SC $P(X \geq 6) = 0.000968$ M1A1 Conclusion B1
		6	
6(ii)	Concluding mean has increased when it has not	B1	In context
	'0.00544'	B1FT	FT <i>their</i> $P(X \geq 5)$, dep < 0.01
		2	
6(iii)	$e^{-7.0} \left(1 + 7 + \frac{7^2}{2} + \frac{7^3}{3!} + \frac{7^4}{4!} \right)$	M1	Correct expression for $P(X \leq 4 \mid \lambda = 7.0)$
	0.173 (3 sf)	A1	
		2	

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1(i)	9.6, 12.4	B1 B1	
		2	
1(ii)	6.6, 49.6	B1 B1	
		2	

Question	Answer	Marks	Guidance
2(i)	$(\lambda (= 2 \times 2.4) = 4.8)$ $e^{-4.8} \left(1 + 4 + \frac{4.8^2}{2} + \frac{4.8^3}{3!} \right)$	M1	Any λ
	0.294 (3 sf)	A1	
		2	
2(ii)	$(\lambda (= 60 \times 2.4) = 144)$ N('144', '144')	M1	N and $\sigma^2 = \mu$ SOI
	$\frac{139.5 - '144'}{\sqrt{'144'}} (= -0.375)$	M1	Allow with no continuity correction
	$\phi('0.375')$	M1	Correct area consistent with their working
	0.646 (3 sf)	A1	
		4	

Question	Answer	Marks	Guidance
3(i)	Assume population is normally distributed	B1	
	$\bar{x} = 25.9$	B1	Allow $\frac{259}{10}$
	$z = 2.17$	B1	
	$'25.9' \pm z \times \frac{3}{\sqrt{10}}$	M1	Must have correct form and z.
	23.8 to 28.0 (3 sf)	A1	CWO
		5	
3(ii)	0.03^2 (=0.0009)	B1	
		1	

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Question	Answer	Marks	Guidance
4	Assume trains are independent OR probability of being on time is constant	B1	Must be in context
	$H_0: P(\text{on time})=0.92$ $H_1: P(\text{on time})<0.92$	B1	Both. Allow ' p ' or π
	$1 -$ $\left({}^{20}C_{17} \times 0.92^{17} \times 0.08^3 + {}^{20}C_{18} \times 0.92^{18} \times 0.08^2 + 20 \times 0.92^{19} \times 0.08 + 0.92^{20} \right)$	M1	Allow one end error Must have $1 - \dots$
	$=0.0706$ (3 sf)	A1	
	Compare with 0.05	M1	Valid comparison needed
	No evidence that percentage less than 92%	A1FT	OE No contradictions. <u>Method using normal approximation:</u> If the first B1B1 is earned then: $CV - 1.566 \left(\text{from } \frac{16.5 - 20 \times 0.92}{\sqrt{20 \times 0.92 \times 0.08}}, \text{ with continuity correction} \right)$ or $CV = 1.978$ (without continuity correction) comp $z = 1.645$ No evidence that % decreased (1.566) or evidence that % decreased (1.978) is awarded SC2 after B marks
		6	

Question	Answer	Marks	Guidance
5(i)	Po(3)	B1	SOI
	$e^{-3} \left(\frac{3^3}{3!} + \frac{3^4}{4!} + \frac{3^5}{5!} \right)$	M1	Allow one or two extra terms (2 or 6 or both)
	0.493 (3 sf)	A1	
		3	
5(ii)	A correct equation from $P(0) = P(2)$ $\left(\text{leading to } 1 = \frac{\lambda^2}{2} \right)$	M1	
	$\lambda = \sqrt{2}$ or 1.41 (3 sf)	A1	CWO
		2	

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Question	Answer	Marks	Guidance
5(iii)(a)	Correct inequality $\left(\text{leading to } \frac{5.2^n}{n!} < \frac{5.2^{n+1}}{(n+1)!} \right)$	B1	
		1	
5(iii)(b)	$n + 1 < 5.2$ or $1 < \frac{5.2}{n+1}$	M1	Simplify to a stage without exponentials, powers or factorials.
	Largest n is 4	A1	
		2	

Question	Answer	Marks	Guidance
6(i)	$k \int_0^3 (3x - x^2) dx = 1$	M1	Attempt to integrate $f(x)$ and = 1
	$k \left[\frac{3}{2}x^2 - \frac{x^3}{3} \right]_0^3$ $k \left(\frac{27}{2} - \frac{27}{3} \right) = 1$	A1	Correct integral and limits
	$k = \frac{2}{9}$	A1	AG No errors seen
		3	

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Question	Answer	Marks	Guidance
6(ii)	$\frac{2}{9} \int_1^2 (3x - x^2) dx = \frac{2}{9} \left[\frac{3}{2}x^2 - \frac{x^3}{3} \right]_1^2 = \frac{2}{9} \times \left(6 - \frac{8}{3} - \frac{3}{2} + \frac{1}{3} \right)$	M1	Attempt to integrate $f(x) dx$ with limits 1 and 2 OE
	$\frac{13}{27}$ or 0.481 (3 sf)	A1	
		2	
6(iii)	$y = 3x - x^2$ symmetrical about $x = \frac{3}{2}$	M1	Attempt $\frac{2}{9} \int_0^3 (3x^2 - x^3) dx$
	$E(X) = \frac{3}{2}$	A1	
	$\frac{2}{9} \int_0^3 (3x^3 - x^4) dx$	M1	Attempt to integrate $x^2 f(x)$
	$= \frac{2}{9} \left[\frac{3x^4}{4} - \frac{x^5}{5} \right]_0^3 (= \frac{2}{9} \times \frac{243}{20} = \frac{27}{10})$ $\frac{27}{10} - \left(\frac{3}{2} \right)^2$	M1	Subtract their $(E(X))^2$ from their integral $x^2 f(x)$ with correct limits substituted
	$\frac{9}{20}$ or 0.45	A1	
		5	

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Question	Answer	Marks	Guidance
7(i)	H_0 : Pop mean=546 H_1 : Pop mean>546	B1	Both. Allow just μ , but not just ‘mean’
	$\frac{581 - 546}{\frac{120}{\sqrt{40}}}$	M1	Standardising. Need $\frac{120}{\sqrt{40}}$
	=1.845 allow 1.844	A1	Allow 1.84 or 1.85 AWRT
	1.845<1.96	M1	OE. Or area comparison 0.0325>0.025 or large probabilities
	No evidence that mean weekly income has increased	A1FT	No contradictions. If H_1 : $\neq$, and 2.241 used, max B0M1A1M1A0
		5	
7(ii)	$\frac{a - 546}{\frac{120}{\sqrt{40}}} = 1.96$	M1	Standardise to find a . Need $\frac{120}{\sqrt{40}}$ and 546 and a value of z
	$a = 583.19$	A1	Allow 583 to 3sf
	$\frac{'583.19' - 595}{\frac{120}{\sqrt{40}}} (= -0.622)$	M1	Standardise. Need $\frac{120}{\sqrt{40}}$ and 595
	$\phi(-0.622) = 1 - \phi(0.622)$	M1	Consistent area
	0.267	A1	
		5	