

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/31

Paper 3 Pure Mathematics 3 (**P3**)

October/November 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO **NOT** WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



- 1** Given that $\ln(1 + e^{2y}) = x$, express y in terms of x . [3]

[illegible]

2 Solve the inequality $|2x - 3| > 4|x + 1|$.

[4]

[illegible]

3 The parametric equations of a curve are

$$x = 2t + \sin 2t, \quad y = \ln(1 - \cos 2t).$$

Show that $\frac{dy}{dx} = \operatorname{cosec} 2t$.

[5]

[illegible]

- 4 The number of insects in a population t weeks after the start of observations is denoted by N . The population is decreasing at a rate proportional to $Ne^{-0.02t}$. The variables N and t are treated as continuous, and it is given that when $t = 0$, $N = 1000$ and $\frac{dN}{dt} = -10$.

(i) Show that N and t satisfy the differential equation

$$\frac{dN}{dt} = -0.01e^{-0.02t}N. \quad [1]$$

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Solve the differential equation and find the value of t when $N = 800$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

[1]

- [illegible]

- (ii) Verify by calculation that p lies between 2.2 and 2.6.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Use an iterative formula based on the equation in part (i) to determine p correct to 2 decimal places. Give the result of each iteration to 4 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 6 (i) By differentiating $\frac{\cos x}{\sin x}$, show that if $y = \cot x$ then $\frac{dy}{dx} = -\operatorname{cosec}^2 x$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Show that $\int_{\frac{1}{4}\pi}^{\frac{1}{2}\pi} x \operatorname{cosec}^2 x \, dx = \frac{1}{4}(\pi + \ln 4)$. [6]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 7** Two lines l and m have equations $\mathbf{r} = a\mathbf{i} + 2\mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$ and $\mathbf{r} = 2\mathbf{i} + \mathbf{j} + 2\mathbf{k} + \mu(2\mathbf{i} - \mathbf{j} + \mathbf{k})$ respectively, where a is a constant. It is given that the lines intersect.

(i) Find the value of a . [4]

[illegible]

[illegible]

8 Let $f(x) = \frac{x^2 + x + 6}{x^2(x + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.

[illegible]

- 9 (i) By first expanding $\cos(2x + x)$, show that $\cos 3x \equiv 4\cos^3 x - 3\cos x$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (ii) Hence solve the equation $\cos 3x + 3\cos x + 1 = 0$, for $0 \leq x \leq \pi$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(iii) Find the exact value of $\int_{\frac{1}{6}\pi}^{\frac{1}{3}\pi} \cos^3 x \, dx$. [4]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- 10 (a)** The complex number u is given by $u = -3 - (2\sqrt{10})i$. Showing all necessary working and without using a calculator, find the square roots of u . Give your answers in the form $a + ib$, where the numbers a and b are real and exact. [5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple sets of three horizontal dotted lines, providing a guide for letter height and placement. The lines are evenly spaced across the entire page, leaving ample room for writing practice. There is no text or other markings on the page.

- (b) On a sketch of an Argand diagram shade the region whose points represent complex numbers z satisfying the inequalities $|z - 3 - i| \leq 3$, $\arg z \geq \frac{1}{4}\pi$ and $\operatorname{Im} z \geq 2$, where $\operatorname{Im} z$ denotes the imaginary part of the complex number z . [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/32

Paper 3 Pure Mathematics 3 (P3)

October/November 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



BLANK PAGE

- 1** Solve the equation $5 \ln(4 - 3^x) = 6$. Show all necessary working and give the answer correct to 3 decimal places. [3]

[illegible]

-
- This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 3** The polynomial $x^4 + 3x^3 + ax + b$, where a and b are constants, is denoted by $p(x)$. When $p(x)$ is divided by $x^2 + x - 1$ the remainder is $2x + 3$. Find the values of a and b . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 4 (i) Express $(\sqrt{6}) \sin x + \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. State the exact value of R and give α correct to 3 decimal places. [3]

[illegible]

[4]

[illegible]

- 5** The equation of a curve is $2x^2y - xy^2 = a^3$, where a is a positive constant. Show that there is only one point on the curve at which the tangent is parallel to the x -axis and find the y -coordinate of this point. [7]

[illegible]

for $0 < \theta < \pi$. It is given that $x = 1$ when $\theta = \frac{1}{3}\pi$. Solve the differential equation and obtain an expression for x in terms of $\cos \theta$. [8]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and extend across the width of the page, providing a template for handwriting practice or general writing. There are no margins, text, or other markings on the page.

- 7 (a)** Find the complex number z satisfying the equation

$$z + \frac{iz}{z^*} - 2 = 0,$$

where z^* denotes the complex conjugate of z . Give your answer in the form $x + iy$, where x and y are real. [5]

[illegible]

- (b) (i) On a single Argand diagram sketch the loci given by the equations $|z - 2i| = 2$ and $\text{Im } z = 3$, where $\text{Im } z$ denotes the imaginary part of z . [2]

- (ii) In the first quadrant the two loci intersect at the point P . Find the exact argument of the complex number represented by P . [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

8 Let $f(x) = \frac{2x^2 + x + 8}{(2x - 1)(x^2 + 2)}$.

(i) Express $f(x)$ in partial fractions.

[5]

This image shows a full page of a handwriting practice worksheet. It consists of multiple rows of horizontal dotted lines spaced evenly down the page, providing a guide for letter height and placement. The background is plain white, and there are no other markings or text present.

- (ii) Hence, showing full working, find $\int_1^5 f(x) dx$, giving the answer in the form $\ln c$, where c is an integer. [5]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9 It is given that $\int_0^a x \cos \frac{1}{3}x \, dx = 3$, where the constant a is such that $0 < a < \frac{3}{2}\pi$.

(i) Show that a satisfies the equation

$$a = \frac{4 - 3 \cos \frac{1}{3}a}{\sin \frac{1}{3}a}. \quad [5]$$

This image shows a single page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- (ii) Verify by calculation that a lies between 2.5 and 3.

[2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) Use an iterative formula based on the equation in part (i) to calculate a correct to 3 decimal places. Give the result of each iteration to 5 decimal places.

[3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

10 The line l has equation $\mathbf{r} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k} + \lambda(\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$. The plane p has equation $2x + y - 3z = 5$.

(i) Find the position vector of the point of intersection of l and p . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Calculate the acute angle between l and p . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

- (iii) A second plane q is perpendicular to the plane p and contains the line l . Find the equation of q , giving your answer in the form $ax + by + cz = d$. [5]

[illegible]

[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.

CANDIDATE
NAME

--

CENTRE
NUMBER

--	--	--	--	--

CANDIDATE
NUMBER

--	--	--	--

MATHEMATICS

9709/33

Paper 3 Pure Mathematics 3 (P3)

October/November 2019

1 hour 45 minutes

Candidates answer on the Question Paper.

Additional Materials: List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

Write your centre number, candidate number and name in the spaces at the top of this page.

Write in dark blue or black pen.

You may use an HB pencil for any diagrams or graphs.

Do not use staples, paper clips, glue or correction fluid.

DO NOT WRITE IN ANY BARCODES.

Answer **all** the questions in the space provided. If additional space is required, you should use the lined page at the end of this booklet. The question number(s) must be clearly shown.

Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.

The use of an electronic calculator is expected, where appropriate.

You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.

The number of marks is given in brackets [] at the end of each question or part question.

The total number of marks for this paper is 75.

This document consists of **19** printed pages and **1** blank page.



BLANK PAGE

1 Solve the inequality $2|x + 2| > |3x - 1|$.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 2** The polynomial $6x^3 + ax^2 + bx - 2$, where a and b are constants, is denoted by $p(x)$. It is given that $(2x + 1)$ is a factor of $p(x)$ and that when $p(x)$ is divided by $(x + 2)$ the remainder is -24 . Find the values of a and b . [5]

This image shows a full page of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page, typical of notebook or legal stationery. There are no margins, text, or other markings on the page.

- 3** Showing all necessary working, solve the equation $\frac{3^{2x} + 3^{-x}}{3^{2x} - 3^{-x}} = 4$. Give your answer correct to 3 decimal places. [4]

This image shows a single sheet of white paper with horizontal ruling lines. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

- 4 (i) By first expanding $\tan(2x + x)$, show that the equation $\tan 3x = 3 \cot x$ can be written in the form $\tan^4 x - 12 \tan^2 x + 3 = 0$. [4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(ii) Hence solve the equation $\tan 3x = 3 \cot x$ for $0^\circ < x < 90^\circ$.

[3]

[illegible]

- 5 (i) By sketching a suitable pair of graphs, show that the equation $\ln(x + 2) = 4e^{-x}$ has exactly one real root. [2]

- (ii) Show by calculation that this root lies between $x = 1$ and $x = 1.5$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

6 Throughout this question the use of a calculator is not permitted.

The complex number with modulus 1 and argument $\frac{1}{3}\pi$ is denoted by w .

- (i) Express w in the form $x + iy$, where x and y are real and exact. [1]

.....

.....

.....

.....

.....

.....

.....

.....

The complex number $1 + 2i$ is denoted by u . The complex number v is such that $|v| = 2|u|$ and $\arg v = \arg u + \frac{1}{3}\pi$.

- (ii) Sketch an Argand diagram showing the points representing u and v . [2]

- (iii) Explain why v can be expressed as $2uw$. Hence find v , giving your answer in the form $a + ib$, where a and b are real and exact. [4]

[illegible]

- 7 The plane m has equation $x + 4y - 8z = 2$. The plane n is parallel to m and passes through the point P with coordinates $(5, 2, -2)$.

(i) Find the equation of n , giving your answer in the form $ax + by + cz = d$. [2]

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) Calculate the perpendicular distance between m and n . [3]

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

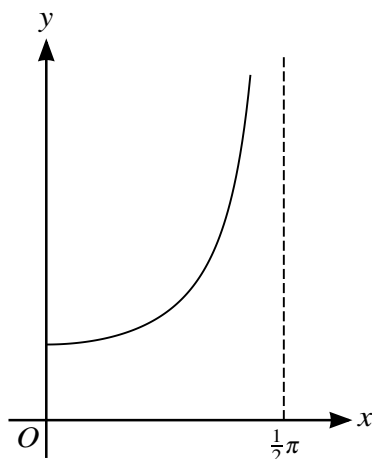
.....

.....

- (iii) The line l lies in the plane n , passes through the point P and is perpendicular to OP , where O is the origin. Find a vector equation for l . [4]

[illegible]

8



The diagram shows the graph of $y = \sec x$ for $0 \leq x < \frac{1}{2}\pi$.

- (i) Use the trapezium rule with 2 intervals to estimate the value of $\int_0^{1.2} \sec x \, dx$, giving your answer correct to 2 decimal places. [3]

.....

.....

.....

.....

.....

.....

.....

- (ii) Explain, with reference to the diagram, whether the trapezium rule gives an overestimate or an underestimate of the true value of the integral in part (i). [1]

.....

.....

.....

.....

.....

.....

.....

- [6]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting practice. There are no margins, text, or other markings on the page.

- 9** The variables x and t satisfy the differential equation $5\frac{dx}{dt} = (20 - x)(40 - x)$. It is given that $x = 10$ when $t = 0$.

(i) Using partial fractions, solve the differential equation, obtaining an expression for x in terms of t . [9]

This image shows a full page of white paper with horizontal dotted lines. The lines are evenly spaced and run across the width of the page, providing a guide for handwriting or typing. There are no margins, text, or other markings on the page.

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

.....

(ii) State what happens to the value of x when t becomes large.

[1]

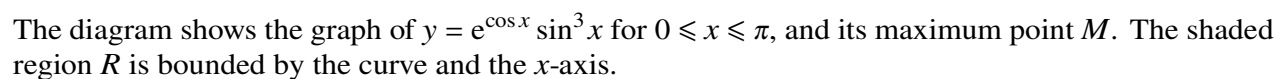
.....

.....

.....

.....

.....



- [illegible]

[illegible]

[illegible]

Cambridge Assessment International Education is part of the Cambridge Assessment Group. Cambridge Assessment is the brand name of the University of Cambridge Local Examinations Syndicate (UCLES), which itself is a department of the University of Cambridge.